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Emission standards vs. taxes: The case of asymmetric Cournot duopoly and uncertain control costs*

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Abstract. It is well known that uncertainty concerning firms' costs as well as market power of the latter have to be taken into account in order to design and choose environmental policy instruments in an optimal way. As a matter of fact, in the most actual regulation settings the policy maker has to face both of these complications simultaneously. However, hitherto environmental economic theory has restricted itself to either of them when submitting conventional policy instruments to a comparative analysis. The article at hand accounts for closing this gap by investigating the welfare effects of emission standards and taxes against the background of uncertain emission control costs and a polluting asymmetric Cournot duopoly.

Keywords: *Asymmetric Cournot duopoly, external diseconomies of pollution, cost uncertainty, emission standard, emission tax*

JEL Classification: *D62, D89, L13, Q58*

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1. Introduction

The starting point of the present paper is the environmental problem in terms of external diseconomies. More precisely, it considers the standard case of several firms emitting a harmful pollutant in the course of their production and thus causing external damage costs. Trying to internalise the latter, the policy maker – below for simplification referred to as environmental policy agency, short EPA – will frequently face the following two complications: Firstly, it is not equipped with all the information required to generate the welfare maximising allocation. Secondly, the polluting firms exhibit market power at least to some extent. Environmental economists realised at an early stage that these hitches crucially influence the optimal[1] design and choice of emission control instruments. One stream of literature, initiated by the seminal papers of Weitzman (1974) along with Adar and Griffin (1976), deals with the question how uncertainty concerning damage and emission control costs affects the relative performance of emission standards, tradable emission licences and emission taxes. They showed that an additive shock to the marginal damage cost function leaves the basically given equivalence of these instruments unaffected,[2] while a congruent shock to the marginal control cost function makes their comparative advantage dependant on the relative slopes of the two aforesaid functions.[3] Another branch of literature analyses how market power affects internalisation strategies. Buchanan (1969) started the discussion by pointing out that Pigouvian taxation might lead to suboptimal allocations when the polluting firms possess market power, which means an additional distortion to the external diseconomies of pollution. A further milestone was set by Barnett (1980), who derived a rigorous (second best) Pigouvian tax rule tailored to the actuality of a monopolistic polluter, followed by more sophisticated rules for symmetric (Ebert, 1992) as well as asymmetric Cournot oligopoly (Simpson, 1995). However, there were very few attempts to compare emission control instruments against the background of market power, like e.g. Requate (1993a and b) undertook.

All in all, the review of the respective literature reveals a considerable shortfall: It has very well been detected that both information problems on the part of the EPA and market power of the polluting firms play an essential role for the optimal design and choice of conventional environmental policy instruments (Requate, 2005, pp. 85ff). Though, it has been constantly ignored that they emerge in virtually every real problem of environmental regulation simultaneously. Recent work of Heuson (2008) took a first step in closing this gap by investigating the welfare effects of emission standards and taxes within a static partial framework subject to the EPA's frequent information problem of uncertain emission control costs and a polluting symmetric Cournot oligopoly. It showed that the interaction between information problems and market power has indeed an impact on optimal environmental policy.

The present paper sets on this research effort through abolishing the restrictive assumption of identical polluting firms. Considering the case of an asymmetric Cournot

[1] In the remainder, "optimal" is used synonymously to "accomplishing the EPA's goal of welfare maximisation", if applicable subject to market power of the polluting firms and/or incomplete information.

[2] Amongst others, the equivalence between standards, licences and taxes particularly presumes identical cost functions of the regulated firms; see e.g. Tisato (1994).

[3] More precisely, the according policy rule first derived by Weitzman (1974) – in the following called the original Weitzman-rule – tells that the ranking of the mentioned instruments is determined by the relation of the marginal damage and the marginal minimised aggregate control cost function's slope; see section 6.

duopoly, it demonstrates that the decision whether to choose standards or taxes[4] might be wrong when based on the original Weitzman-rule, which presupposes perfect competition and symmetry of firms. An adequate modification of this policy rule guaranteeing the optimal instrument choice is derived. In doing so, two further insights emerge: Firstly, the basically given superiority of tax policy in case of asymmetric polluters, which is owed a higher abatement efficiency compared to standards, can be completely reversed by the Weitzman-effect.[5] Secondly, the degree of uncertainty plays a decisive role for the optimal instrument choice in opposite to the original Weitzman-setting.

The analysis starts out by contemplating the reference case of complete information. Section 2 sets up the model and displays the basic characteristics of the policy intervention game. In section 3, the comparative analysis of instruments reveals the superiority of the tax policy in the absence of uncertainty. Section 4 introduces the information problem, whereupon section 5 investigates the instruments' relative performance subject to uncertain control costs and derives the modification of the original Weitzman-rule. Section 6 opposes the original to the modified rule, and section 7 finally summarises results and gives a brief outlook on possible further research.

2. The basic problem

Consider a Cournot duopoly with firms $i = 1,2$ producing x_1 and x_2 units of a homogeneous good at costs amounting to $cp_i(x_i) = \zeta_i x_i + (1/2)x_i^2$, whereas $\zeta_1 > \zeta_2$. The price of the good is determined by the linear inverse market demand function $p(X) = a - bX$, given the aggregate output $X = x_1 + x_2$. In the course of production occur emissions of a harmful pollutant proportionally to the output level. Assume that the pollutant only emerges in the industry under consideration.

Each firm can abate emissions either by decreasing the output level or by adopting an end-of-pipe technology, i.e. implementing a filter system. Thus, the individual amount of emissions actually discharged into the environment is $em_i(x_i, v_i^e) = \varepsilon x_i - v_i^e$, ε denoting the emission coefficient and v_i^e the firm specific end-of-pipe abatement effort. The latter causes costs according to the function $cc^e(v_i^e) = \gamma v_i^e + (1/2)v_i^{e2}$. Consequently, the asymmetry solely affects the firms' production technology in a way that the related marginal costs of the inefficient firm $i = 1$ run parallel above the ones of the efficient firm $i = 2$, given any output level.[6] The monetary value of the damage emanating from the firms' emissions is captured by the damage cost function $DC(EM) = \alpha EM + (1/2)\beta EM^2$, $EM = \sum_i em_i(x_i, v_i^e)$ denoting the aggregate emission level.

[4] Naturally, it seems obvious to incorporate tradable licences which certainly belong to the most important conventional instruments. Yet, a realistic modelling of the former has to account for strategic effects on the licence market – due to the firms' market power – and thus goes beyond the scope of the analysis.

[5] The Weitzman-effect denotes the mechanism driving the difference between standards' and taxes' "costs of uncertainty"; see section 5.

[6] Of course there is no reason why a firm should choose the technology causing higher production costs and providing no ulterior advantage. However, as the analysis focuses rather on the asymmetry's impact on the optimal instrument choice than on its causes, this question is omitted for the sake of simplification. The same applies for modelling asymmetry moreover w.r.t. the end-of-pipe technology.

After all, let both the inputs for production and end-of-pipe abatement be produced at an exogenously given price on a perfectly competitive market. Thus each firm's costs of production and end of-pipe abatement $cp_i(x_i)$ and $cc^e(v_i^e)$ coincide with the associated costs incurred by the society.
Note that the specification of the demand and cost functions was chosen for two reasons: On the one hand, it allows for an explicit solution of the model at all, on the other hand, it is necessary to reproduce the basic characteristics of Weitzman's original setting (see Weitzman 1974),[7] which shall be exposed to an asymmetric Cournot duopoly.

The Cournot-Nash-Equilibrium, short CNE, in the absence of regulation $(\mathbf{x}^{CN}, \mathbf{v}^{eCN} = \mathbf{0})$[8] results from the simultaneous solution of the firms' profit maximisation problems

$$[1] \qquad \max_{\{x_i, v_i^e\}} \pi_i(x_i, v_i^e) = p(X)x_i - cp_i(x_i) - cc^e(v_i^e), \qquad \forall i = 1,2$$

taking into account the Cournot conjecture.[9] Clearly, the former deviates from the allocation of the output and end-of-pipe abatement level $(\mathbf{x}^{**}, \mathbf{v}^{**})$ which maximises the partial welfare, defined as the sum of consumers' and producers' surplus less costs of production, end-of-pipe abatement and environmental damage

$$[2] \qquad \max_{\{\mathbf{x}, \mathbf{v}^e\}} W(\mathbf{x}, \mathbf{v}^e) = \int p(X)dX - \sum_i (cp_i(x_i) + cc^e(v_i^e)) - DC(EM).$$

This problem can be alternatively stated as to minimise the sum of aggregate control and damage costs, which will be helpful for the further proceeding:

$$[3] \qquad \min_{\{\mathbf{x}, v_i^e\}} \sum_i (CC_i^x(x_i) + cc^e(v_i^e)) + DC(EM),$$

$$\text{whereas } CC_i^x(x_i) = \left(\int_0^{X^{CN}} p(X)dX - \sum_i cp_i(x_i^{CN}) \right) - \left(\int_0^{x_i + x_j^{CN}} p(X)dX - cp_i(x_i) - cp_j(x_j^{CN}) \right),$$

$$x_i < x_i^{CN}, \forall i, j = 1,2, i \neq j$$

denotes the aggregate costs of firm i's output-reduction – the according loss of the consumers' and producers' surplus.[10] The failure of the market mechanism is initially driven by the external diseconomies of pollution and the firms' market power, whose combined impact on the relation between the unregulated CNE and the first best solu-

[7] Amongst others, the original setting implies linear aggregate marginal control and damage costs. The present framework generalises the original setting by explicitly distinguishing between two abatement options.
[8] Subsequently, bold print variables denote vectors of firm specific variables, and the superscript "CN" marks the variables' occurrence in the CNE. Note that due to the firms' asymmetry it holds that $x_1^{CN} < x_2^{CN}$.
[9] For the second order condition of [1], the existence and uniqueness of the unregulated CNE see appendix A1.
[10] For the second order conditions of [2] and [3] see appendix A2.

tion is ambiguous.[11] In addition, the asymmetric Cournot duopoly is noted for inefficiently allocating production between the firms: $\partial cp_1(x_1)/\partial x_1|_{x_1=x_1^{CN}} > \partial cp_2(x_2)/\partial x_2|_{x_2=x_2^{CN}}$.[12] However, as will be seen in section 4, neither standards nor taxes are capable of resolving this inefficiency. Thus, solely the two first-mentioned distortions give reason for the welfare maximising EPA to implement the instruments under consideration, except for the negligible case that they just cancel each other out. The procedure of the optimal instrument choice can be described as a sequential game, whose timing is depicted in the following figure:

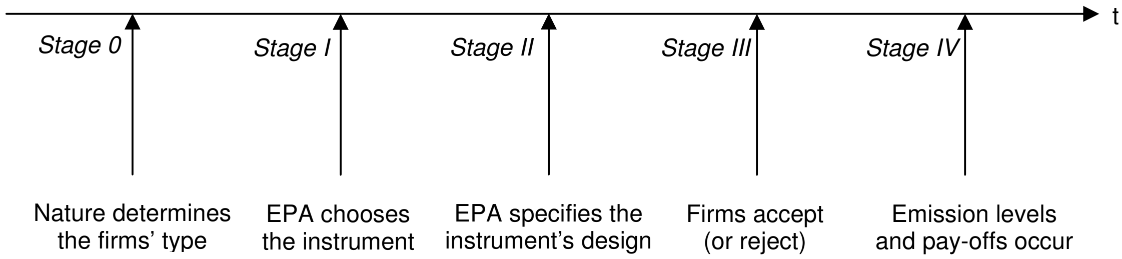


Figure 1: Timing of the policy intervention game

First of all, the nature determines the firms' type, i.e. their cost functions. The EPA, since usually endowed with sovereign authority and thus in the position of the Stackelberg-leader, chooses whether to implement emission standards or taxes (stage I). After having specified the design of the chosen instrument in stage II, the EPA offers the pursuant occurrence of the standard or tax in terms of a contract to the firms, whose single decision is to comply with environmental regulation or not – i.e. to accept or reject the contract (stage III). Subsequently, it is taken for granted that the EPA can monitor the firms' emission levels without any costs and enforce compliance by charging a sufficiently high fine in case that the firms reject.[13] So any kind of moral-hazard problem can be ruled out and the firms will always accept the contract, which enables to skip stage III throughout the further analysis. Finally (stage IV), the firms meet the demands of the instrument at hand, i.e. render the adequate abatement effort and if applicable pay the tax obligation. Beyond, the according welfare level occurs. The analysis commences by supposing that all the functions and moves listed above are common knowledge to the parties involved. Naturally, the game will be solved for the subgame perfect equilibrium, short SPE, using backwards induction.

[11] The stronger the distortion of pollution, i.e. the higher β (or ε), the smaller/larger is the welfare maximising output/end-of-pipe abatement level relatively to the one of the unregulated CNE: $\left(\partial\left(x_i^{**} - x_i^{CN}\right)/\partial\beta\right) < 0$ / $\left(\partial\left(v_i^{e**} - v_i^{eCN}\right)/\partial\beta\right) > 0, \forall i = 1,2$.

Contrary, as the distortion of output shortage gains weight, which is the case for an increasing level of the marginal willingness to pay (fading out pollution), the welfare maximising output grows relatively to the unregulated CNE: $\left(\partial\left(x_i^{**} - x_i^{CN}\right)/\partial a\right)\Big|_{\varepsilon=0} > 0, \forall i = 1,2$ (while of course $v_i^e, \forall i = 1,2$ remains unchanged).

[12] For the proof see appendix A3.

[13] To prevent rejection, the fine has to meet at least the firms' compliance costs of the respective instrument. As the specification of the adequate fine yields tedious expressions but no further insight it is omitted.

3. Optimal instrument choice with complete information

The analysis restricts to that SPE which comprises positive output and end-of-pipe abatement levels of both firms (inner solution), since this is the sole constellation allowing for the simultaneous appearance of the complications under regard: The distortions arising from the duopolistic market structure and, later on, control cost uncertainty.[14] Moreover, this kind of inner solution implies that the distortion of pollution outweighs the one of duopolistic output shortage – both a subgame perfect standard and tax inducing a positive end-of-pipe abatement effort necessarily lead to a decrease of the firms' output compared to the unregulated CNE as well.[15]

Backwards induction starts out with stage IV, deriving the firms' responses and equilibrium quantities for a given standard or tax. The firms aim at minimising their overall compliance costs of regulation. In case of emission standards, the EPA prescribes a particular emission quantity q to each firm,[16] whose problem consequently reads

$$[4] \qquad \min_{\{x_i, v_i^e\}} \; cc_i^x(x_i) + cc^e(v_i^e) \quad \text{s.t.} \quad em_i(x_i, v_i^e) = q\,.$$

$$\text{Thereby } cc_i^x(x_i) = \pi_i(\mathbf{x}^{\mathbf{CN}}, v_i^{eCN}) - \pi_i(x_i, x_j^{CN}, v_i^{eCN}), \qquad x_i < x_i^{CN},\ \forall i, j = 1,2,\ i \neq j$$

represents the firms' control costs concerning the option of output reduction, which simply amount to the respective loss of profit relatively to the unregulated CNE. The corresponding first order condition

$$[5] \qquad -\partial cc_i^x(x_i)/\partial x_i = \varepsilon\left(\partial cc^e(\varepsilon x_i - q)/\partial(\varepsilon x_i - q)\right), \qquad \forall i = 1,2$$

stipulates to equal the firm specific marginal costs of the two abatement options. By solving for the response functions $rx_i(q, x_j), rv_i^e(q, x_j), \forall i, j = 1,2, i \neq j$, it becomes evident that the firms fix the output and end-of-pipe abatement effort simultaneously, such that both serve as channels of strategic interaction. Define the quantities occurring in the standard regulated CNE as $x_i^{CN}(q), v_i^{eCN}(q)$ and $em_i^{CN}(q) = q, \forall i = 1,2$.[17]
Facing an emission tax, the firms have to pay a fee t per emission unit discharged into the environment and thus aspire to

$$[6] \qquad \min_{\{x_i, v_i^e\}} \; cc_i(x_i) + cc^e(v_i^e) + t\,em_i(x_i, v_i^e), \qquad \forall i = 1,2$$

[14] As will be seen in section 4, uncertainty only emerges for a positive end-of-pipe abatement effort due to the modelling.
[15] The specification of the parameter restrictions which guarantee the inner solution is neglected with the same reasoning as in footnote 13.
[16] The modus of a uniform absolute emission standard is chosen for two reasons: Firstly, it is the standards' prototype taken for granted in the respective literature (see Helfand, 1991, p. 622) and thus allows for the comparability of results. Secondly, it keeps the model tractable and yet enables to capture the standards' inherent inefficiency which emerges when polluters are heterogeneous.
[17] For the second order condition of [4], the existence and uniqueness of the standard regulated CNE see appendix A4.

Contrary to standards, the firms now choose the output and end-of-pipe level in two separate steps, so as to the related marginal costs are equalised with t:

[7] $-\partial cc_i^x(x_i)/\partial x_i = \varepsilon t; \quad \partial cc^e(v_i^e)/\partial v_i^e = t. \qquad \forall i = 1,2$

Consequently, there is only strategic interaction w.r.t. the output decision. Solving [7] yields the firms' response functions $rx_i(t, x_j), rv_i^e(t), \forall i, j = 1,2, i \neq j$ and at last the tax regulated CNE's quantities $x_i^{CN}(t), v_i^{eCN}(t), em_i^{CN}(t), \forall i = 1,2$.[18]

Next (stage II), the EPA determines the instruments' design which minimises the total costs [3], anticipating the equilibrium outcome of the final stage. The optimal standard solves the problem

[8] $$\min_q \quad \sum_i \left(CC_i^x\left(x_i^{CN}(q)\right) + cc^e\left(v_i^{eCN}(q)\right)\right) + DC(2q),$$

satisfying the first order condition[19]

[9] $$-\sum_i \left(\frac{\partial CC_i^x\left(x_i^{CN}(q^*)\right)}{\partial\left(x_i^{CN}(q^*)\right)} \frac{\partial x_i^{CN}(q^*)}{\partial q^*} + \frac{\partial cc^e\left(v_i^{eCN}(q^*)\right)}{\partial\left(v_i^{eCN}(q^*)\right)} \frac{\partial\left(v_i^{eCN}(q^*)\right)}{\partial q^*} \right) = \frac{\partial DC(2q^*)}{\partial q^*},$$

which tells to choose q in a way, that the additional aggregate control costs of a marginal decrease in q (left hand side of [9]) are balanced with the according saving of damage costs (right hand side of [9]).
Analogously, the optimal tax can be derived out of

[10] $$\min_t \quad \sum_i \left(CC_i^x\left(x_i^{CN}(t)\right) + cc^e\left(v_i^{eCN}(t)\right)\right) + DC\left(EM^{CN}(t)\right),$$

at which it is assumed that the tax revenue will be redistributed lump sum to the consumers. The resulting first order condition[20]

[11] $$\sum_i \left(\frac{\partial CC_i^x\left(x_i^{CN}(t^*)\right)}{\partial\left(x_i^{CN}(t^*)\right)} \frac{\partial x_i^{CN}(t^*)}{\partial t^*} + \frac{\partial cc^e\left(v_i^{eCN}(t^*)\right)}{\partial\left(v_i^{eCN}(t^*)\right)} \frac{\partial\left(v_i^{eCN}(t^*)\right)}{\partial t^*} \right) = -\frac{\partial DC(\cdot)}{\partial t^*}$$

has exactly the same meaning as [9] – of course referred to an marginal increase in t.

Finally (stage I), the EPA decides whether to implement taxes or standards by comparing the level of total costs associated with t^* and q^*. The respective cost difference is[21]

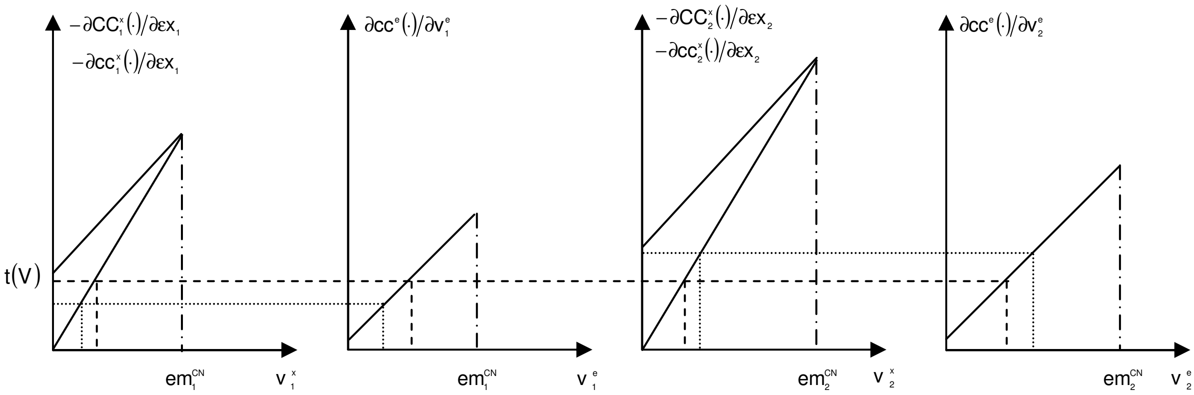
[18] For the second order condition of [6], the existence and uniqueness of the tax regulated CNE see appendix A5.
[19] For the second order condition and the explicit solution see appendix A6.
[20] For the second order condition and the explicit solution see appendix A7. Furthermore it can be shown that the optimal tax is below the Pigou-level, which is owed to the additional distortion of market power and corresponds to the standard result of Barnett (1980). Simpson (1995) found that in an asymmetric Cournot duopoly the opposite case can apply under certain conditions. However, these do not apply here.
[21] For the proof see appendix A8.

$$
\begin{aligned}
\text{[12]}\qquad \Delta^{C} = & \left(\sum_i \left(CC_i^x\left(x_i^{CN}\left(t^*\right)\right) + cc^e\left(v_i^{eCN}\left(t^*\right)\right)\right) + DC\left(EM\left(t^*\right)\right)\right) - \\
& - \left(\sum_i \left(CC_i^x\left(x_i^{CN}\left(q^*\right)\right) + cc^e\left(v_i^{eCN}\left(q^*\right)\right)\right) + DC\left(2q^*\right)\right) \qquad < 0,
\end{aligned}
$$

leading immediately to

Proposition 1: *With complete information, the optimal emission tax is strictly superior to the optimal emission standard (SPE).*

The reasoning for the superiority of taxes in case of complete information can be followed by means of the subsequent figure:



Firms' optimisation facing q(V), leading to quantities $v_i^{xCN}(q(V)), v_i^{eCN}(q(V)), \forall i = 1,2$

Firms' optimisation facing t(V), leading to quantities $v_i^{xCN}(t(V)), v_i^{eCN}(t(V)), \forall i = 1,2$

Figure 2: Instrument specific abatement effort of the firms

Figure 2 depicts the marginal costs of the two abatement options at choice occurring on the firm and aggregate level, contingent on the individual abatement efforts of the firms v_i^e and $v_i^x = em_i^{CN} - \varepsilon x_i,\ x_i < x_i^{CN}, \forall i = 1,2$.[22] Thereby the latter represents the amount of emissions that firm i abates by diminishing output. The maximal amount of emission that can be abated by each firm with any option is naturally $em_i^{CN}, \forall i = 1,2$. Remember that due to the assumption of a perfectly competitive market for the end-of-pipe inputs a marginal increase of each firm's end-of-pipe abatement effort raises the firms' and the aggregate costs by the same amount: $\partial cc^e(\cdot)/\partial v_i^e, \forall i = 1,2$. Additionally applies that $\partial cc^e(\cdot)/\partial v_1^e = \partial cc^e(\cdot)/\partial v_2^e$ for $v_1^e = v_2^e$, because both firms implement the same end-of-pipe technology. In contrast, abating one additional emission unit via output reduction causes a higher cost augmentation in aggregate terms (loss of consumers' and producer's surplus) than for the single firm (loss of producer's surplus): $-\partial CC_i^x(\cdot)/\partial \varepsilon x_i > -\partial cc_i^x(\cdot)/\partial \varepsilon x_i,\ \forall i = 1,2$. Logically, it is more costly for the society when the efficient firm accomplishes this measure, i.e. $-\partial CC_2^x(\cdot)/\partial \varepsilon x_2 > -\partial CC_1^x(\cdot)/\partial \varepsilon x_1$, whereas $-\partial^2 CC_2^x(\cdot)/\partial(\varepsilon x_2)^2 = -\partial^2 CC_1^x(\cdot)/\partial(\varepsilon x_1)^2$. Due to the type of production technologies used by the firms, the latter's marginal control costs of output reduction coin-

[22] For the proof of the functions' progress see appendix A9.

cide: $\partial cc_1^x(\cdot)/\partial \varepsilon x_1 = \partial cc_2^x(\cdot)/\partial \varepsilon x_2$ for $x_1 = x_2$.[23] After all, the specification of the model leads to linearity of all the marginal control cost functions described above.
Next, in order to make standards and taxes comparable, define $q(V)$ and $t(V)$ as that specifications of the former which enforce an overall abatement effort to the amount of V. These can be computed by solving $V(q) = EM^{CN} - 2q$ for q respectively $V(t) = EM^{CN} - EM^{CN}(t)$ for t.[24] In case of $t(V)$, the emissions abated via end-of-pipe as well as the ones abated via output reduction coincide for both firms, i.e. $v_1^{eCN}(t(V)) = v_2^{eCN}(t(V))$ and $v_1^{xCN}(t(V)) = v_2^{xCN}(t(V))$, because these adopt their identical marginal control costs to the uniform tax rate t. On the contrary, the emission standard, prescribing the same emission level to both firms, induces a relatively higher abatement effort of firm 2, since it produces and thus emits more in the unregulated CNE: $v_2^{eCN}(q(V)) > v_{i=1,2}^{eCN}(t(V)) > v_1^{eCN}(q(V))$ and $v_2^{xCN}(q(V)) > v_{i=1,2}^{xCN}(t(V)) > v_1^{xCN}(q(V))$.

This reveals immediately the higher abatement efficiency of taxes compared to standards for given V. $t(V)$ yields identical end-of-pipe abatement efforts and thus balances the associated marginal costs. Indeed the firms' output shortage induced by taxation is congruent as well, entailing however that the allocative inefficiency of production within the unregulated CNE remains unchanged – recall that both firms' marginal costs of production possess the same slope. Hence, the aggregate marginal costs of output reduction are higher in case of the efficient firm 2 as against to the inefficient firm 1. Enforcing V through standard policy causes higher aggregate abatement costs compared to taxes for two reasons: Firstly, the marginal end-of-pipe costs diverge as the firms render different end-of-pipe abatement efforts, which provides a cost saving potential. Secondly, standards increase the allocative inefficiency of production, since the efficient firm decreases and the inefficient firm increases the output opposite to the levels under taxation.[25]

Based on the linearity and congruence of the firms' marginal control costs and by definition of $q(V)$ along with $t(V)$, it holds that $\sum_i v_i^{eCN}(t(V)) = \sum_i v_i^{eCN}(q(V))$ and $\sum_i v_i^{xCN}(t(V)) = \sum_i v_i^{xCN}(q(V))$. These relations combined with the progress of the option specific aggregate marginal control cost functions imply that a marginal raise of V causes the same augmentation in aggregate control costs – no matter if induced by standards or taxes:[26]

[13]
$$\frac{\partial q(V)}{\partial V}\sum_i\left(\frac{\partial CC_i^x\left(x_i^{CN}(q(V))\right)}{\partial\left(x_i^{CN}(q(V))\right)}\frac{\partial x_i^{CN}(q(V))}{\partial q(V)}+\frac{\partial cc^e\left(v_i^{eCN}(q(V))\right)}{\partial\left(v_i^{eCN}(q(V))\right)}\frac{\partial\left(v_i^{eCN}(q(V))\right)}{\partial q(V)}\right)=$$
$$=\frac{\partial t(V)}{\partial V}\sum_i\left(\frac{\partial CC_i^x\left(x_i^{CN}(t(V))\right)}{\partial\left(x_i^{CN}(t(V))\right)}\frac{\partial x_i^{CN}(t(V))}{\partial t(V)}+\frac{\partial cc^e\left(v_i^{eCN}(t(V))\right)}{\partial\left(v_i^{eCN}(t(V))\right)}\frac{\partial\left(v_i^{eCN}(t(V))\right)}{\partial t(V)}\right)=MCC(V).$$

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[23] This congruence is owed to the fact that the firms' marginal costs of production possess the same slope.
[24] Note that $t(V)$ corresponds to the (horizontal) aggregation of the firms' marginal control costs because of [7].
[25] For the proof and comparison of the allocative inefficiency w.r.t. the standard and tax regulated CNE see appendix A10.
[26] For the proof see appendix A11.

Thereby $MCC(V)$ denotes the aggregate marginal control cost function which is realisable for the EPA when adopting standard or tax policy. As of course $q(V)$ and $t(V)$ evoke the same damage costs $DC(V) = \alpha(EM^{CN} - V) + (1/2)\beta(EM^{CN} - V)^2$, [9] and [11] can be alternatively posed as to equate $MCC(V)$ with $MDC(V) = -\partial DC(V)/\partial V$, leading to the direct corollary

[14] $$V(q^*) = V(t^*) = V^*$$

Consequently, the advantage in terms of abatement efficiency of $t(V)$ over $q(V)$ applies for the optimally designed instruments as well, as both t^* and q^* induces the same overall abatement effort, which is the rationale for proposition 1.[27]

Opposing the marginal control costs occurring on the firm and aggregate level enables furthermore to set up

Proposition 2: *Both standards and the taxes fail to enforce the welfare maximising allocation.*

Achieving the welfare maximum obviously requires to render the overall abatement effort efficiently from an aggregate perspective. This again is grounded on two conditions that are both hurt by standards as well as taxes.
The first condition is intra-firm-efficiency, demanding that the aggregate marginal control costs of the two options are balanced for each firm: $-\partial CC_i^x(\cdot)/\partial \varepsilon x_i = \partial cc^e(\cdot)/\partial v_i^e$, $\forall i = 1,2$. However, the firms' cost minimisation, no matter whether facing the standard or tax policy, leads to $-\partial cc_i^x(\cdot)/\partial \varepsilon x_i = \partial cc^e(\cdot)/\partial v_i^e$, $\forall i = 1,2$ (see [5], [7]) and thus hurts the intra-firm-efficiency condition: $-\partial CC_i^x(\cdot)/\partial \varepsilon x_i > \partial cc^e(\cdot)/\partial v_i^e$, for $v_i^x = v_i^{xCN}(q(V))$ respectively $v_i^x = v_i^{xCN}(t(V))$ and $v_i^e = v_i^{eCN}(q(V))$ respectively $v_i^e = v_i^{eCN}(t(V))$, $\forall i = 1,2$.[28]
The market power enables the firms to shift a part of the control costs upon the consumers via the abatement option of output shortage. Hence, the latter is overused, against what the end-of-pipe abatement effort is too low from a welfare perspective.
Secondly, the overall abatement has to be allocated efficiently among the firms, such that $-\partial CC_1^x(\cdot)/\partial \varepsilon x_1 = -\partial CC_2^x(\cdot)/\partial \varepsilon x_2$ and $\partial cc^e(\cdot)/\partial v_1^e = \partial cc^e(\cdot)/\partial v_2^e$ (inter-firm-efficiency). Yet, as demonstrated both standards and taxes lead to $-\partial CC_1^x(\cdot)/\partial \varepsilon x_1 < -\partial CC_2^x(\cdot)/\partial \varepsilon x_2$ for $v_i^x = v_i^{xCN}(q(V))$ respectively $v_i^x = v_i^{xCN}(t(V))$, $\forall i = 1,2$. Moreover, standards additionally hurt the second equity, since they entail $\partial cc^e(\cdot)/\partial v_2^e > \partial cc^e(\cdot)/\partial v_1^e$ for $v_i^e = v_i^{eCN}(q(V))$, $\forall i = 1,2$.

[27] Of course the damage costs are the same for t^* and q^*.
[28] This is an immediate consequence of the marginal cost functions' progress proved in appendix A9.

4. Introducing uncertainty

One information problem that the EPA faces frequently when it tries to determine the optimal standard and tax is uncertainty with respect to the firms' control costs. The latter is modelled following the seminal works of Weitzman (1974) along with Adar and Griffin (1976), who investigated an additive shock to the aggregate marginal control cost function.[29] Suppose that the uncertainty enters through the end-of-pipe control costs as follows:[30] $cc^e(v_i^e,u)=(\gamma+u)v_i^e+(1/2)v_i^{e2}, \forall i=1,2,$ whereas u is a continuous random variable with expectation $E[u]=0$ and thus $Var[u]=E[u^2]$.[31] Hence, the uncertainty solely affects the level, but not the slope of the marginal end-of-pipe control cost function $\partial cc^e(v_i^e,u)/\partial v_i^e=(\gamma+u)+v_i^e, \forall i=1,2$. It is straightforward to show that this kind of uncertainty also translates into an additive shock to the aggregate marginal control costs $MCC(V,u)$, see [13], and thus corresponds to the original Weitzman-setting.[32] Concerning the precise information structure, it is taken for granted that the EPA merely knows the distribution of u – which needs not to be specified for the further analysis – against what the firms are acquainted with the latter's true specification: So the firms can perfectly observe the nature's move in stage I against what the EPA builds beliefs concerning the firms' type. Consequently, now the relevant equilibrium concept is the Bayes-Nash equilibrium (short BNE). The structure of the policy intervention game does neither allow for Bayesian updating nor for problems of asymmetric information, because the firms cannot exploit their advance in information. It is presumed that the feasible value range of the random variable is sufficiently small in order to guarantee that the BNE comprises an inner solution as well.[33] The modelling of uncertainty along with this assumption guarantee apparently that the second order conditions, the existence and uniqueness of both the standard and the tax regulated CNE still hold with analogous reasoning as under complete information and thus need not to be proved any more.

5. Optimal instrument choice with uncertain control costs

As a matter of course, the firms' problem of minimising their compliance costs remains unchanged since the uncertainty only pertains to the EPA. Subsequently, the equilibrium outcome of stage IV can be calculated according to [4] - [7]. From the EPA's view, in case of standards the equilibrium output and end-of-pipe abatement level are random while the emission level is certain $(x_i^{CN}(q,u), v_i^{eCN}(q,u), em_i^{CN}(q)=q, \forall i=1,2)$. Though, tax policy leads to a deterministic output, but to an uncertain end-of-pipe

[29] An additive shock does not only facilitate the exposition, but "…has also considerable economic content since factor price variations in inputs subject to a Leontief production function would produce additive errors" (Adar and Griffin, 1976, p. 184).
[30] Note that modelling the uncertainty to enter the end-of-pipe costs reduces the analysis' degree of complexness considerably: It involves the introduction of one only random variable, as both firms use the same end-of-pipe technology, unlike the case of production cost uncertainty.
[31] Not only u itself, but every function entered by u is a random variable. In order to highlight this insight, u will be explicitly listed as an argument of these functions.
[32] For the proof see appendix A12.
[33] Listing u's feasible range of values explicitly is very cumbersome but yields no further insights.

abatement and emission level $(x_i^{CN}(t), v_i^{eCN}(t,u), em_i^{CN}(t,u), \forall i = 1{,}2)$. This difference arises for the reason that the firms choose the output quantity and the end-of-pipe abatement effort simultaneously when facing standards, but separately when facing taxation, as seen above.
Presumed that the EPA is risk neutral, it now chooses q and t in stage II such that the expectation of total costs is minimised, given the firms' (in part) random responses of stage IV. The respective problem for standards reads:[34]

$$[15] \qquad \min_q \quad E\left[\sum_i \left(CC_i^x\left(x_i^{CN}(q,u)\right) + cc^e\left(v_i^{eCN}(q,u),u\right)\right)\right] + DC(2q).$$

Naturally, the first order condition tells to equate the expected aggregate marginal control costs and the marginal damage costs related to an infinitesimal decrease in q:

$$[16] \qquad -E\left[\sum_i \left(\frac{\partial CC_i^x\left(x_i^{CN}\left(q^{E*},u\right)\right)}{\partial\left(x_i^{CN}\left(q^*,u\right)\right)} \frac{\partial x_i^{CN}\left(q^{E*},u\right)}{\partial q^*} + \frac{\partial cc^e\left(v_i^{eCN}\left(q^{E*},u\right)\right)}{\partial\left(v_i^{eCN}\left(q^{E*},u\right)\right)} \frac{\partial\left(v_i^{eCN}\left(q^{E*},u\right)\right)}{\partial q^{E*}} \right)\right] = \\ = \frac{\partial DC\left(2q^{E*}\right)}{\partial q^{E*}}.$$

Implementing taxation, the EPA solves

$$[17] \qquad \min_t \quad \sum_i \left(CC_i^x\left(x_i^{CN}(t)\right) + E\left[cc^e\left(v_i^{eCN}(t,u),u\right)\right]\right) + DC\left(E\left[EM^{CN}(t,u)\right]\right)$$

by setting t such that

$$[18] \qquad \sum_i \left(\frac{\partial CC_i^x\left(x_i^{CN}\left(t^{E*}\right)\right)}{\partial\left(x_i^{CN}\left(t^{E*}\right)\right)} \frac{\partial x_i^{CN}\left(t^{E*}\right)}{\partial t^{E*}} + E\left[\frac{\partial cc^e\left(v_i^{eCN}\left(t^{E*},u\right),u\right)}{\partial\left(v_i^{eCN}\left(t^{E*},u\right)\right)} \frac{\partial\left(v_i^{eCN}\left(t^{E*},u\right)\right)}{\partial t^{E*}} \right]\right) = \\ = -\frac{\partial DC\left(E\left[EM^{CN}\left(t^{E*},u\right)\right]\right)}{\partial\left(E\left[EM^{CN}\left(t^{E*},u\right)\right]\right)} \frac{\partial\left(E\left[EM^{CN}\left(t^{E*},u\right)\right]\right)}{\partial t^{E*}},$$

with an analogous interpretation as [16].[35]

The relevant criterion for the optimal instrument choice under uncertainty (stage I) is obviously the expected level of total costs generated by the – in terms of expectation – optimal tax and standard. Setting up the associated difference leads to

$$[19] \qquad \Delta^{EC} = E\left[\begin{array}{l} \left(\sum_i \left(CC_i^x\left(x_i^{CN}\left(t^{E*}\right)\right) + cc^e\left(v_i^{eCN}\left(t^{E*},u\right),u\right)\right) + DC\left(EM^{CN}\left(t^{E*},u\right)\right)\right) - \\ \quad - \left(\sum_i \left(CC_i^x\left(x_i^{CN}\left(q^{E*},u\right)\right) + cc^e\left(v_i^{eCN}\left(q^{E*},u\right),u\right)\right) + DC\left(2q^{E*}\right)\right) \end{array} \right] =$$

[34] "E" denotes subsequently the expectation operator, and – when used as a superscript – means "in terms of expectation".
[35] Note that due to the assumption $E[u] = 0$ the optimal design of the instruments coincides for complete and incomplete information, i.e. $q^* = q^{E*}$ and $t^* = t^{E*}$.

$$= \phi^2 Var[u]\left(\frac{\beta-\mu}{2\mu^2}\right) - \Delta^C,$$

with $\mu > 0$ denoting slope of the aggregate marginal control cost function and $\phi > 0$ representing a term influencing the latter's level.[36]
As can be seen easily, the sign of the expected cost difference is ambiguous contrary to [12], implying the policy rule (in what follows referred to as the modified Weitzman-rule)

$$[20] \qquad \Delta^{EC} \begin{cases} > 0, & \text{if} \quad \beta > \mu + \dfrac{2\mu^2\Delta^C}{\phi^2 Var[u]} \quad \Rightarrow \quad \text{quota} \succ \text{tax} \\ < 0, & \text{if} \quad \beta < \mu + \dfrac{2\mu^2\Delta^C}{\phi^2 Var[u]} \quad \Rightarrow \quad \text{quota} \prec \text{tax} \end{cases} \qquad \text{(BNE)}$$

which stipulates to prefer standards to taxes when β is sufficiently large, i.e. the marginal damage cost function runs sufficiently steep, and vice versa. All in all, the ranking of the instruments is determined by three effects:
Firstly, the lower abatement efficiency of standards $(\Delta^C > 0)$ which has been revealed for complete information still applies in the event of uncertainty and thus affects the instruments' relative performance in favour of taxes.
The second effect relates to the instrument specific costs of uncertainty – the costs emerging from the deviation between the random variable's true value and the expectation of the EPA. As Weitzman (1974) showed first, these costs depend crucially on the relation between the absolute slopes of the aggregate marginal control and damage cost function, μ and β. The intuition for this so-called Weitzman-effect shall be illustrated by the ensuing figure:

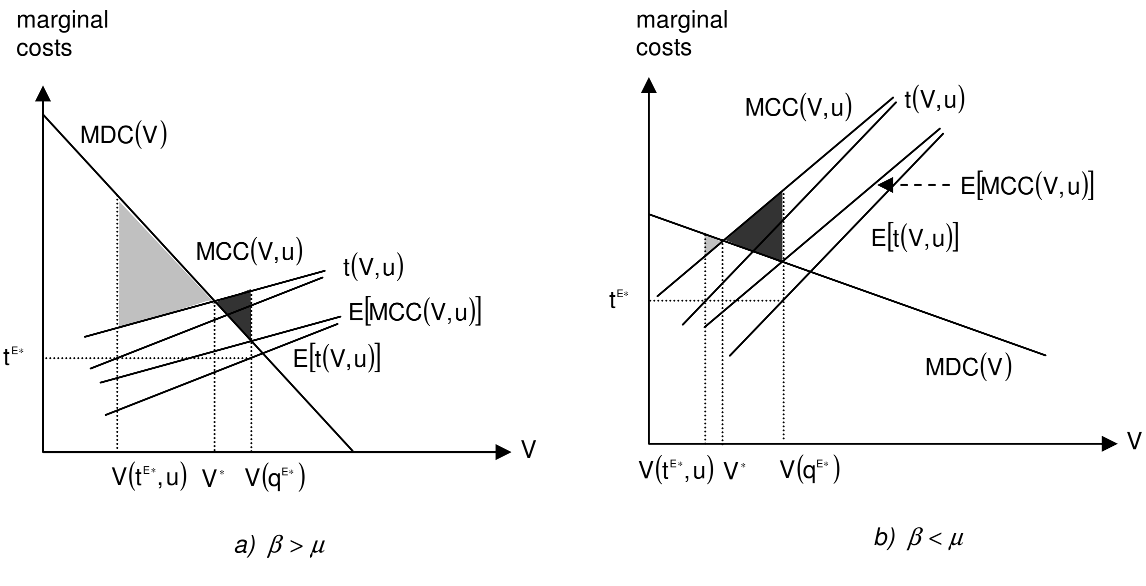

Figure 3: The Weitzman-effect for u>0

[36] More precisely: $\mu = \partial MCC(V,u)/\partial V$, $\phi = \partial MCC(V,u)/\partial u|_{V=0}$, see also appendix A12.

Figure 3 depicts the first order conditions for the optimal standard and tax under uncertainty [16] and [18], alleging for instance that the true value of the random variable is positive $(u > 0)$.[37] With analogous reasoning as for complete information, both q^{E*} and t^{E*} balance the expectation of the aggregate marginal control costs $E[MCC(V,u)]$ and the marginal damage costs $MDC(V)$, each contingent on V. As the aggregate abatement effort can be accurately controlled by standards, $V(q^{E*})$ is determined out of the intersection between $E[MCC(V,u)]$ and $MDC(V)$. Clearly, due the EPA's erroneous expectation $V(q^{E*})$ deviates from the optimal aggregate abatement effort V^* that could be obtained by means of standards (and taxes) under complete information through equating the true marginal aggregate control costs $MCC(V,u)$ with $MDC(V)$. The resulting overall cost saving potential, respectively the standard's costs of uncertainty, is congruent to the dark shaded area and can be computed as follows

[21] $$CU(q^{E*},u) = \int_{V^*}^{V(q^{E*})} (MCC(V,u) - MDC(V))dV \, .$$

As the firms' and the aggregate marginal control costs diverge, t^{E*} results from plugging the aggregate abatement effort which is expected to be optimal, $V(q^{E*})$, into the horizontal aggregation of the expected firm specific marginal control costs $E[t(V,u)]$. For the reason that the firms' cost minimisation endeavour involves the congruence of t and $t(V,u)$, V^* is failed again which becomes manifest in the tax's costs of uncertainty

[22] $$CU(t^{E*},u) = \int_{V(t^{E*},u)}^{V^*} (MDC(V) - MCC(V,u))dV \, ,$$

corresponding to the light shaded area. Now consider the linkage between the instrument specific costs of uncertainty and the slopes of $MCC(V,u)$ and $MDC(V)$. If the marginal damage costs run steeper than the aggregate marginal control costs ($\beta > \mu$, figure 3a) it holds that $CU(q^{E*},u) < CU(t^{E*},u)$: In case of μ being relatively small, already an insignificant error of the EPA, i.e. the true value of u only slightly deviates from zero, leads to a rather large difference between V^* and $V(t^{E*},u)$. Since moreover the damage costs react sensitively to variations of the aggregate abatement effort due to comparatively large β, the Weitzman-effect brings forward standards, which enable an accurate control of V. The economic intuition for the opposite constellation $CU(q^{E*},u) > CU(t^{E*},u)$ when $\beta < \mu$ (figure 3b) becomes clearest through conceiving the extreme case of a horizontal marginal damage cost function $(\beta = 0)$. Here the EPA can enforce V^* despite its lack of information by simply adjusting the tax rate to the axis intercept of the marginal damage cost function α. The firms will then for sure generate

[37] For the proof of the functions' progress see appendix A13.

V^* in opposite to standards. This advantage of the tax policy persists as long as $\beta < \mu$. Note that the revealed relations between the instruments' costs of uncertainty hold for any allowed occurrence of u and consequently as well for the expected costs of uncertainty as a whole, such that

$$[23] \quad E\left[CU\left(t^{E*},u\right)-CU\left(q^{E*},u\right)\right]=\phi^2 Var[u]\left(\frac{\beta-\mu}{2\mu^2}\right) \begin{cases} >0, & \beta>\mu \\ <0, & \beta<\mu \end{cases}.$$

Thirdly, the degree of uncertainty measured by $Var[u]$ influences the optimal instrument choice through determining the significance of the Weitzman-effect. As [20] discloses, $\beta > \mu$ on its own is not a sufficient condition for the superiority of standards – to overcompensate the advantage of taxation w.r.t. the higher abatement efficiency, $Var[u]$ needs to be sufficiently large.[38]

These insights concerning the optimal instruments' choice under uncertainty can be summed up to

Proposition 3: *The ranking of emission standards and taxes is ambiguous when control costs are uncertain. For a sufficiently steep marginal damage cost function and a sufficiently high degree of uncertainty, the Weitzman-effect can reverse the taxes' basically given superiority, which arises from a higher abatement efficiency.*

6. Original vs. modified Weitzman-rule

Finally, one can gain further insights through opposing the original Weitzman-rule to the modified rule for an asymmetric Cournot Duopoly derived in the previous section. Weitzman (1974) was the first to compare the performance of emission standards and taxes against the background of uncertain control costs on behalf of the EPA, presuming symmetric firms engaging in perfect competition.[39] He came to the result, that standards should be preferred to taxes whenever the marginal damage costs run steeper than the minimised aggregate marginal control costs ($\beta > \mu^{min}$).[40] The reason for the minimised aggregate control cost function $CC^{min}(V,u)$, which results from solving

$$[24] \quad \min_{\{x,v^e\}} \sum_i \left(CC_i^x(x_i)+cc^e\left(v_i^e,u\right)\right) \quad \text{s.t.} \quad EM^{CN}-EM=V,$$

[38] Clearly, given that $\beta < \mu$ taxes dominate standards independently of $Var[u]$.

[39] Weitzman (1974) also incorporated tradable licences into the comparative analysis of instruments which are however not treated within the present paper for the reasons set forth in the introduction.

[40] In deriving this policy rule, Weitzman did not distinguish between different abatement options. Yet, Heuson (2008) showed that the original Weitzman-rule can be generalised for the case of two abatement options.

to be relevant for the original Weitzman-rule is that both standards and taxes accomplish V at minimised aggregate control costs when market power and asymmetry of the firms are ruled out (see Heuson, 2008).[41]

The comparison of the original and modified rule discovers two main differences. Firstly, it is straightforward to show that $\mu > \mu^{min}$.[42] Secondly, of course there is no disparity between standards and taxes w.r.t. the abatement efficiency in the original Weitzman-setting, so there the instruments' ranking solely depends on their costs of uncertainty. Hence it becomes evident that within the original setting standards are the optimal choice for a larger range of values referred to β opposite to the asymmetric Cournot duopoly and thus

Proposition 4: *Both market power and asymmetry of the firms shifts the instruments' relative performance in favour of taxes compared to the original Weitzman setting.*

With the immediate consequence:

Proposition 5: *Applying the original Weitzman-rule to an asymmetric Cournot duopoly comprises the risk of a suboptimal instrument choice.*

Apparently, this risk solely refers to erroneously choosing standards instead of taxes which comes to pass whenever $\mu + 2\mu^2\Delta^C / \phi^2 Var[u] > \beta > \mu^{min}$. An aberrant implementation of taxes implying $\mu^{min} > \beta > \mu + 2\mu^2\Delta^C / \phi^2 Var[u]$ is impossible, for the reason that $\mu > \mu^{min}$ and $2\mu^2\Delta^C / \phi^2 Var[u] > 0$.

7. Conclusion

In almost every actual emission control setting the environmental policy agency (EPA) is confronted with two kinds of complications appearing simultaneously when it pursues the goal of welfare maximisation: On the one hand, the polluting firms nearly always exhibit at least some market power. On the other hand, the EPA is indeed never equipped with all the information needed for reaching the full resolution of distortions with environmental policy instruments, whereas information problems focus particularly on the firms' cost functions. The environmental economic theory so far has admittedly detected that both complications influence the optimal design and choice of conventional instruments decisively, however to date it failed to analyse their mutual impact.

The present paper contributes to overcome this default. It analyses the welfare properties of two frequently used emission control instruments – standards and taxes – within a static partial framework against the background of uncertain emission control costs from the EPA's perspective and assumes concurrently that the polluting firms constitute an asymmetric Cournot duopoly.

[41] More precisely, the output shortage costs of price taking firms coincide with the associated aggregate costs, which guarantees that the firms abate efficiently from a welfare point of view. Beyond, the inherent inefficiency of standards does not appear when polluters are symmetric.

[42] The causes for this relation are rather technical and thus not exposed in detail at this point. For this purpose see appendix A14 which also contains the proof.

The main insight is that the combined emergence of cost uncertainty, market power and asymmetry of firms indeed plays a crucial role for the optimal instrument choice. Hence, the renowned policy rule derived by Weitzman (1974), which tells whether to implement standards or taxes when control costs are uncertain under the premise that the polluting firms are symmetric and engage in perfect competition, might give wrong suggestions within this setting. The paper provides an adequate modification of the original Weitzman-rule which guarantees the optimal instrument choice. It is shown that market power as well as asymmetry of the firms shifts the relative performance of instruments in favour of taxes compared to the original Weitzman-setting. A further result is that the basically given superiority of taxes in case of asymmetric polluters can be annihilated by the Weitzman-effect when the degree of uncertainty is sufficiently high. Thus the latter directly influences the optimal instrument choice, contrary to the original setting. Needless to say both instruments fail at achieving first best, even in the absence of uncertainty.

As a matter of course, the specificity of the model demands caution in adopting these findings to any actual regulation scenario. Yet, Weitzman (1974) weakened this caveat by arguing that the presumed linearity of the aggregate marginal control cost and damage cost function allows for interpreting the results as an approximation for more general functions – provided that the feasible value range of the random variable is sufficiently small. For a critical discussion of that point see Malcomson (1978) and Weitzman (1978).

Clearly, there are much more combinations of information problems and market forms left which could serve as basis for the comparative analysis of conventional environmental policy instruments. Though, future research should especially focus on those combinations which promise essentially new interdependencies between the complications under regard. One interesting option is to investigate imperfectly competitive markets featuring price competition, which is well known to exhibit a completely different dimension of strategic interaction. Beyond, an important task would be to incorporate the instrument of tradable emission licences into this field of research, which is increasingly used in practical environmental policy.

Appendices

Some of the results listed below presume the parameter restrictions guaranteeing the inner solution. However, as the handling of the latter is very cumbersome but yields no further insights the respective proofs will be neglected.

Appendix A1: Properties of the unregulated CNE

The second order condition of [1] is fulfilled as the Hessian of the firms' profit

$$H^{\pi_i}\left(x_i, v_i^e\right) = \begin{pmatrix} -1-2b & 0 \\ 0 & -1 \end{pmatrix} \qquad \forall i = 1,2$$

is strictly negative definite.

The firms' output response functions show a linear form, decrease in the rival's output and thus guarantee the existence and uniqueness of the unregulated CNE:

$$rx_i\left(x_j\right) = \frac{a-\zeta_i}{1+2b} - \frac{b}{1+2b}x_j \qquad \forall i,j = 1,2,\ i \neq j$$

Note that there is no strategic interaction w.r.t. the end-of-pipe effort, which is zero in the absence of regulation.

Appendix A2: Second order condition for the welfare maximising allocation

It is straightforward to show that the Hessian of the welfare function

$$H^W\left(\mathbf{x}, \mathbf{v^e}\right) = \begin{pmatrix} -1-b-\beta\varepsilon^2 & -b-\beta\varepsilon^2 & \beta\varepsilon & \beta\varepsilon \\ -b-\beta\varepsilon^2 & -1-b-\beta\varepsilon^2 & \beta\varepsilon & \beta\varepsilon \\ \beta\varepsilon & \beta\varepsilon & -1-\beta & -\beta \\ \beta\varepsilon & \beta\varepsilon & -\beta & -1-\beta \end{pmatrix}$$

is strictly negative definite and thus the solution of [2] is a global maximum. Of course, since [2] and [3] are equivalent the solution of [3] is necessarily a global minimum.

Appendix A3: Unregulated CNE and allocative inefficiency of production

Setting up the difference of the firms' marginal production costs given the output of the unregulated CNE yields

$$\partial cp_1(x_1)/\partial x_1\big|_{x_1=x_1^{CN}} - \partial cp_2(x_2)/\partial x_2\big|_{x_2=x_2^{CN}} = \frac{b(\zeta_1-\zeta_2)}{1+b} > 0$$

as per assumption $\zeta_1 > \zeta_2$.

Producing efficiently would thus require firm 1 to decrease and firm 2 to increase the output.

Appendix A4: Properties of the standard regulated CNE

The second order condition for [4] is fulfilled because the Hessian of the associated Lagrangian

$$H^{q_i}(x_i, v_i^e) = \begin{pmatrix} 1+2b & 0 \\ 0 & 1 \end{pmatrix} \qquad \forall i = 1,2$$

is strictly positive definite.

Beyond, the existence and uniqueness of the standard regulated CNE applies due to the firms' response functions concerning output and end-of-pipe abatement being linear and decreasing in the rival's output level:

$$rx_i(x_j, q) = \frac{a-(\gamma-q)\varepsilon-\zeta_i}{1+2b+\varepsilon^2} - \frac{b}{1+2b+\varepsilon^2}x_j$$

$$rv_i^e(x_j, q) = \frac{\varepsilon(a-\gamma\varepsilon-\zeta_i)-q-2bq}{1+2b+\varepsilon^2} - \frac{\varepsilon b}{1+2b+\varepsilon^2}x_j \qquad \forall i, j = 1,2, i \neq j$$

Appendix A5: Properties of the tax regulated CNE

Considering the Hessian of the firms' compliance costs under taxation

$$H^{t_i}(x_i, v_i^e) = \begin{pmatrix} 1+2b & 0 \\ 0 & 1 \end{pmatrix} \qquad \forall i = 1,2$$

which is strictly positive definite and seeing that the firms' response functions

$$rx_i(x_j, t) = \frac{a-\zeta_i-\varepsilon t}{1+2b} - \frac{b}{1+2b}x_j, \qquad rv_i^e(t) = t-\gamma \qquad \forall i, j = 1,2, i \neq j$$

are linear and decrease in the rival's output (in case of $rx_i(x_j, t)$) yields the same results as A4.

Appendix A6: Optimal standard – second order condition and explicit solution

Due to

$$\frac{\partial^2\left(\sum_i\left(CC_i^x(\cdot)+cc^e(\cdot)\right)+DC(\cdot)\right)}{\partial q^2}=4\beta+\frac{2\left((1+3b)^2+(1+2b)\varepsilon^2\right)}{\left(1+3b+\varepsilon^2\right)^2}>0$$

total costs are strictly convex in q which is why the optimal standard

$$q^* = \frac{2\gamma\left((1+3b)^2+(1+2b)\varepsilon^2\right)+\varepsilon\left(1+4b+\varepsilon^2\right)\left(2a-\zeta_1-\zeta_2\right)}{2\left((1+3b)^2+(1+2b)\varepsilon^2+2\beta\left(1+3b+\varepsilon^2\right)^2\right)}$$

$$\in \left\langle 0; em_1^{CN}\right\rangle \text{ for the inner solution}$$

yields a global minimum.

Appendix A7: Optimal tax – second order condition and explicit solution

Due to

$$\frac{\partial^2\left(\sum_i\left(CC_i^x(\cdot)+cc^e(\cdot)\right)+DC(\cdot)\right)}{\partial t^2}=2+4\beta+\frac{2\varepsilon^2(1+2b+4\beta+12b\beta)}{(1+3b)^2}+\frac{4\beta\varepsilon^2}{(1+3b)^2}>0$$

total costs are strictly convex in t which is why the optimal tax

$$t^* = \frac{2\beta\left(1+3b+\varepsilon^2\right)\left((2+6b)\gamma+\varepsilon\left(2a-\zeta_1-\zeta_2\right)\right)+b\varepsilon\left(\zeta_1+\zeta_2-2a\right)}{2\left((1+3b)^2+(1+2b)\varepsilon^2+2\beta\left(1+3b+\varepsilon^2\right)^2\right)}$$

$$>0 \text{ for the inner solution}$$

yields a global minimum.

Appendix A8: Instruments' total cost difference

Computing [12] for the specified functions gives

$$\Delta^C = -\frac{\varepsilon^2\left(1+\varepsilon^2+b\left(4+3b+2\varepsilon^2\right)\right)\left(\zeta_1-\zeta_2\right)^2}{4(1+b)^2\left(1+b+\varepsilon^2\right)^2}<0\,.$$

Appendix A9: Progress of the functions in figure 2

Clearly, as the firms use the identical end-of-pipe technology, their according marginal costs correspond to each other:

$$\partial cc^{e}\left(v_i^{e}\right)/\partial v_i^{e} = \gamma + v_i^{e} \qquad \forall i = 1,2$$

For the reason that the firms' marginal costs of production show the same slope, the marginal costs of abatement via output reduction necessarily coincide for both firms as well:

$$-\partial cc_i^{x}\left(x_i\right)/\partial \varepsilon x_i = \frac{1+2b}{\varepsilon^2} v_i^{x,} \qquad \forall i = 1,2$$

Computing the aggregate marginal control costs w.r.t. firm i's output decrease produces

$$-\partial CC_i^{x}\left(x_i\right)/\partial \varepsilon x_i = \frac{b\varepsilon\left(a + ab - \zeta_i - 2b\zeta_i + b\zeta_j\right)}{\varepsilon^2\left(1+4b+3b^2\right)} + \frac{\left(1+b\right)^2\left(1+3b\right)}{\varepsilon^2\left(1+4b+3b^2\right)} v_i^{x} \qquad \forall i,j = 1,2,\ i \neq j$$

and reveals additionally that

$$-\partial^2 CC_2^{x}(\cdot)/\partial\left(\varepsilon x_2\right)^2 = -\partial^2 CC_1^{x}(\cdot)/\partial\left(\varepsilon x_1\right)^2 = \frac{\left(1+b\right)^2\left(1+3b\right)}{\varepsilon^2\left(1+4b+3b^2\right)} > 0.$$

The aggregate marginal costs of firm 2's output reduction are higher than the one of firm 1:

$$\left(-\partial CC_2^{x}\left(x_2\right)/\partial \varepsilon x_2\right) - \left(-\partial CC_1^{x}\left(x_1\right)/\partial \varepsilon x_1\right) = \frac{b\left(\zeta_1 - \zeta_2\right)}{\varepsilon\left(1+b\right)} > 0.$$

The aggregate marginal costs of output reduction run above the according costs occurring on the firms' level:

$$\left(-\partial CC_i^{x}\left(x_i\right)/\partial \varepsilon x_i\right) - \left(-\partial cc_i^{x}\left(x_i\right)/\partial \varepsilon x_i\right) =$$
$$= \frac{b\left(a\varepsilon - \varepsilon\left(\zeta_i - b\left(a - 2\zeta_i + \zeta_j\right)\right) + \left(1+b\right)\left(1+3b\right)v_i^{x}\right)}{\left(1+b\right)\left(1+3b\right)\varepsilon^2} > 0 \text{ for } v_i^{x} \in \left[0; em_i^{CN}\right]$$
$$\forall i,j = 1,2,\ i \neq j.$$

Apparently, all the marginal cost functions described above feature a linear progress.

Appendix A10: Allocative inefficiency in the tax and standard regulated CNE

Setting up the difference of the firms' marginal production costs given the output of the tax regulated CNE yields

$$\partial cp_1(x_1)/\partial x_1\big|_{x_1=x_1^{CN}(t)} - \partial cp_2(x_2)/\partial x_2\big|_{x_2=x_2^{CN}(t)} = \frac{b(\zeta_1-\zeta_2)}{1+b} > 0$$

as per assumption $\zeta_1 > \zeta_2$

which proves that taxation leaves the allocative inefficiency emerging in the unregulated CNE unchanged – independently of the tax rate.

The difference in the marginal production costs occurring in the standard regulated CNE

$$\partial cp_1(x_1)/\partial x_1\big|_{x_1=x_1^{CN}(q)} - \partial cp_2(x_2)/\partial x_2\big|_{x_2=x_2^{CN}(q)} = \frac{(b+\varepsilon^2)(\zeta_1-\zeta_2)}{1+b+\varepsilon^2} > 0$$

shows that the allocative inefficiency of production is not correlated with q. The comparison of the marginal production costs' differences w.r.t. the standard and tax regulated CNE

$$\left(\partial cp_1(x_1)/\partial x_1\big|_{x_1=x_1^{CN}(t)} - \partial cp_2(x_2)/\partial x_2\big|_{x_2=x_2^{CN}(t)}\right) - \\ - \left(\partial cp_1(x_1)/\partial x_1\big|_{x_1=x_1^{CN}(q)} - \partial cp_2(x_2)/\partial x_2\big|_{x_2=x_2^{CN}(q)}\right) = -\frac{\varepsilon^2(\zeta_1-\zeta_2)}{(1+b)(1+b+\varepsilon^2)} < 0$$

proves that standards increase the allocative inefficiency of production compared to taxes respectively the unregulated CNE.

Appendix A11: Instrument specific marginal aggregate control cost function

Computing the left and right hand side of [13] for the specified functions yields similarly

$$MCC(V) = \gamma\phi + \frac{b\varepsilon(2a-\zeta_1-\zeta_2)}{2(1+3b)(1+3b+\varepsilon^2)} + \mu V,$$

whereas $\phi = \frac{(1+3b)^2 + (1+2b)\varepsilon^2}{(1+3b+\varepsilon^2)^2}$ and $\mu = \frac{(1+3b)^2 + (1+2b)\varepsilon^2}{2(1+3b+\varepsilon^2)^2}$,

and thus proves that both $q(V)$ and $t(V)$ induce the same (marginal) aggregate control costs.

Appendix A12: Shape of the random aggregate marginal control cost function

Computing [13] for the specified functions and taking into account the additive shock to the end-of-pipe control costs one obtains

$$MCC(V,u) = (\gamma + u)\phi + \frac{b\varepsilon(2a - \zeta_1 - \zeta_2)}{2(1+3b)(1+3b+\varepsilon^2)} + \mu V$$

which proves that $MCC(V,u)$ runs linearly and is affected by u in an additive manner as well. For the specification of ϕ and μ see A11.

Appendix A13: Progress of the functions in figure 3

The damage costs as a function of V read: $DC(V) = \alpha(EM^{CN} - V) + (1/2)\beta(EM^{CN} - V)^2$. Computing the according marginal cost function

$$MDC(V) = -(\partial DC(\cdot)/\partial V) = \alpha + \beta(EM^{CN} - V)$$

proves the latter's linear progress.

For the linearity of $MCC(V,u)$ see A12. Calculating the aggregation of the firm specific marginal control costs for the specified functions results in

$$t(V,u) = \frac{(1+3b)(u+\gamma)}{1+3b+\varepsilon^2} + \frac{1+3b}{2+6b+2\varepsilon^2} V .$$

For the inner solution it holds that $MCC(V,u) > t(V,u)$. Furthermore applies

$$(\partial MCC(\cdot)/\partial V) - ((\partial t(\cdot)/\partial V)) = -\frac{b\varepsilon^2}{2(1+3b+\varepsilon^2)^2} < 0 .$$

Finally note that the slopes of $MCC(V,u)$ and $t(V,u)$ are positively correlated, because they are determined by the same parameters (see above) entering in the identical direction:

$$\frac{\partial^2 MCC(\cdot)}{\partial V \partial b} = \frac{\varepsilon^2(1+6b+\varepsilon^2)}{(1+3b+\varepsilon^2)^3} > 0 , \qquad \frac{\partial^2 t(\cdot)}{\partial V \partial b} = \frac{3\varepsilon^2}{2(1+3b+\varepsilon^2)^2} > 0$$

$$\frac{\partial^2 MCC(\cdot)}{\partial V \partial \varepsilon} = -\frac{\varepsilon(1+\varepsilon^2 + b(7+12b+2\varepsilon^2))}{(1+3b+\varepsilon^2)^3} < 0 , \qquad \frac{\partial^2 t(\cdot)}{\partial V \partial \varepsilon} = -\frac{(1+3b)\varepsilon}{(1+3b+\varepsilon^2)^2} < 0$$

All in all, the relation between $MCC(V,u)$ and $t(V,u)$ is an immediate consequence of the difference between the firms' and the aggregate marginal control costs w.r.t. output reduction depicted in figure 2: The aggregate marginal cost function runs above the one of the firms, yet for increasing abatement effort both functions converge.

Appendix A14: μ vs. μ^{min}

Solving [24] for $(\mathbf{x},\mathbf{v^e})$ and inserting results into $\sum_i \left(CC_i^x(x_i)+cc^e(v_i^e,u)\right)$ yields $CC^{min}(V,u)$, whose first derivative w.r.t. V turns out to be

$$MCC^{min}(V,u)=(\gamma+u)\phi^{min}+\frac{b\varepsilon(2a-\zeta_1-\zeta_2)}{2(1+3b)(1+2b+\varepsilon^2)}+\mu^{min}V,$$

whereas $\phi^{min}=\frac{1+b(5+6b)}{(1+3b)(1+2b+\varepsilon^2)}$ and $\mu^{min}=\frac{1+2b}{2+4b+2\varepsilon^2}$.

Comparing $MCC^{min}(V,u)$ and $MCC(V,u)$ yields

w.r.t. the level

$$\left(MCC^{min}(V,u)-MCC(V,u)\right)\Big|_{V=0}=\frac{b^2\varepsilon(2a-\zeta_1-\zeta_2)}{2(1-3b)(1+2b+\varepsilon^2)(1+3b+\varepsilon^2)}-(\gamma+u)(\phi-\phi^{min})$$

which is strictly positive within the inner solution.

w.r.t. the slope

$$\mu^{min}-\mu=\frac{(b\varepsilon)^2}{2(1+2b+\varepsilon^2)(1+3b+\varepsilon^2)^2}<0\,.$$

So $MCC^{min}(V,u)$ runs on a higher level opposite to $MCC(V,u)$ but exhibits a smaller slope. The causes for this relation become clear through contemplating the progression of the associated aggregate control cost functions within V's feasible value range for the inner solution:

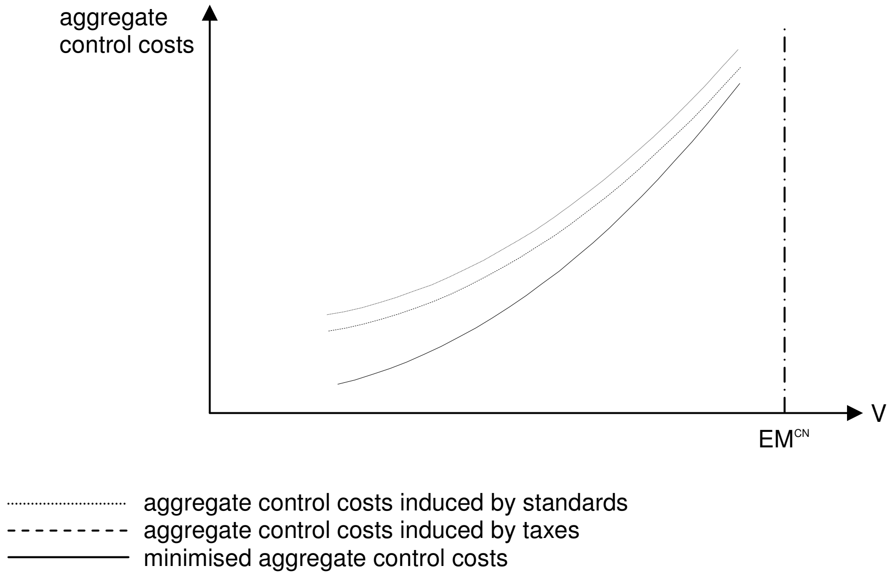

Figure 4: Minimised vs. instrument specific aggregate control costs

First of all it, is easy to see that the aggregate control costs induced by standards $\sum_i \left(CC_i^x\left(x_i^{CN}(q(V),u)\right) + cc^e\left(v_i^{eCN}(q(V),u),u\right)\right)$ run parallel above the ones induced by taxes $\sum_i \left(CC_i^x\left(x_i^{CN}(t(V,u))\right) + E\left[cc^e\left(v_i^{eCN}(t(V,u),u),u\right)\right]\right)$ at a distance of Δ^C – owed to [13], both functions show the same slope which increases in V. Yet, standards cause higher control costs than taxes given any V as a result of their disadvantage concerning the abatement efficiency (see [12] and proposition 1).
Clearly, as both standards and taxes hurt the conditions for efficient abatement, the minimised aggregate control costs $CC^{min}(V,u)$ range on a lower level compared to the associated costs of standards and taxes (the explicit proof is in turn omitted for the familiar reasons).
The crux for $MCC(V,u)$ to grow faster than $MCC^{min}(V,u)$, i.e. $\mu > \mu^{min}$, is the following: As demonstrated in figure 2, the gap between the aggregate marginal control costs w.r.t. output shortage and the firms' costs shrinks with increasing abatement effort. Thus, the higher V, the less impact has the abatement inefficiency coming along with standards and taxes. Consequently, the instruments' and the minimised aggregate control costs converge as V tends to EM^{CN}, the maximal overall abatement effort. This in turn has two kinds of implications: Firstly, the minimised cost function runs steeper than the instruments' cost function and thus necessarily $MCC^{min}(V,u)$ possesses a higher level than $MCC(V,u)$. Secondly, the convergence of the cost functions requires that their slopes assimilate as well for growing V and so $\mu > \mu^{min}$.

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