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When skilled and unskilled labor are mobile: a new economic geography approach*

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Abstract

This paper develops an analytically solvable new economic geography model of the 'footloose entrepreneur' class in which not only skilled labor is mobile, but also unskilled labor. Allowing unskilled labor to move freely between different regions increases the agglomeration incentive of skilled labor. Depending on the level of unskilled labor mobility, the geographical distribution of economic activity is either a 'pitchfork' or a 'tomahawk'. If unskilled labor is very mobile, complete agglomeration is the only stable outcome. When trade costs are high, skilled and unskilled labor migration reinforce each other leading to agglomeration of both types of labor in the same region. For lower levels of trade cost, unskilled labor returns to its region of origin, whereas skilled labor remains concentrated.

JEL classification: F12; F15; F22; R22
Keywords: agglomeration; migration; economic geography; bifurcation pattern

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1 Introduction

Models of the new economic geography (NEG) need three main ingredients to explain the agglomeration of economic activity: firstly, increasing returns to scale and monopolistic competition, secondly, impediments to trade, and thirdly, factor mobility or production linkages. Standard NEG models with mobile labor as proposed by Krugman (1991), Forslid (1999), Forslid and Ottaviano (2003), Ottaviano *et. al.* (2002), Pflüger (2004) and Pflüger and Südekum (2008a) assume two types of individuals - mobile and immobile workers. Mobile workers are usually referred to as skilled labor and can move freely between regions. Immobile workers - usually labeled as unskilled workers - are totally immobile and are bound to their region of origin. Looking at the empirical side of the coin, it becomes obvious that the assumption of total immobility of unskilled labor is far from realistic. According to Carrington and Detragiache (1998) and Docquier and Marfouk (2006) who created two of the most important databases on international migration and educational attainment, in the year 2000 only 34.6% of the total stock of migrants in OECD countries had tertiary education (13 years of education and above) whereas 65.4% were less educated. The facts show unambiguously that unskilled labor migration is substantial and should not be neglected for simplicity's sake. But the assumption of immobile unskilled labor is crucial to these models. From Robert-Nicoud (2003) we know that if there is no immobile factor of production in a standard NEG framework, complete agglomeration is the only stable outcome. This is intuitive - if there are benefits from agglomeration but no disadvantages, why then should people be dispersed? And indeed, Helpman (1998) and Murata (2003) have developed NEG models where labor is perfectly mobile between regions. But they need to introduce a dispersion force to obtain equilibria different from total agglomeration.

Consequently, there are two types of NEG models with mobile labor which represent extremes: on the hand there are model which assume the total immobility of unskilled labor, on the other hand there are very few approaches where labor is entirely mobile. The model presented here is a first approach to overcome this gap and develops a **synthesis** of these extreme cases. More precisely, it presents an analytically solvable NEG model, in which both skilled and unskilled labor are mobile. The aim of this essay is to work out the effects mobile unskilled labor adds to standard NEG labor mobility models. Furthermore, it is a first approach to deal with **two**

mobile factors of production in a NEG framework. The existing literature has concentrated on models with one mobile factor - be it a certain type of labor or capital. But other factors of production have been assumed to be bound to a certain region. Having more than one 'footloose' input raises interesting questions with respect to their interactions, to the causality of mobility as well as to the stability of the model equilibria. This paper offers a first proposal to handle these issues.

The novelty of the model presented here is that it assumes unskilled labor to be imperfectly mobile, instead of being perfectly (im-)mobile. More precisely, this paper accounts for taste heterogeneity within the working force as proposed by Ludema and Wooton (1999), Tabuchi and Thisse (2002) and Murata (2003). In these papers it is assumed that skilled workers can freely migrate between regions, but have idiosynchronic preferences about locations. According to Russek (2008) taste heterogeneity is analytically equivalent to the assumption of migration costs. Consequently, heterogeneous preferences about locations partially impede skilled labor migration. The unskilled workforce is assumed to be entirely immobile. Different from these papers, the present approach assumes that unskilled labor has heterogeneous tastes and, consequently, incurs costs when migrating from one region to the other. Skilled labor, instead, is assumed to be perfectly mobile between regions. The latter assumption is not crucial to the analysis, but is made for two reasons. On the one hand, it helps to isolate the effects mobile unskilled labor add to this type of new economic geography models. Therefore, it is easier to compare the results to the literature. And on the other hand, there is a body of empirical evidence which supports the assumption that skilled people have less mobility costs and have greater ease to adapt to new regions, cultures, *etc.* With respect to international migration, Carrington and Detragiache (1998) and Docquier and Marfouk (2006) find that in most countries migration rates by educational category are highest for highly educated workers (for some countries the emigration rate of skilled labor is greater than 80%). Studies about internal migration come to the same conclusion. In an empirical study about migrants in Spain, Antonlin and Bover (1997) find that the probability of migration increases with the level of education. Hunt (2000) and Borjas *et. al.* (1992) report the same for the US and Germany, respectively.

The main results of the model presented are characterized as follows.

Firstly, the mobility of unskilled labor adds an additional agglomeration force. The strength of this force is determined by the degree of unskilled labor mobility. Therefore, (partial) agglomeration is stable at higher levels of trade costs in comparison to models where unskilled labor is immobile. Secondly, the extent of unskilled labor mobility has an effect on the agglomeration pattern of skilled labor. If unskilled labor is not mobile at all or relatively immobile, the agglomeration pattern of skilled labor is a 'pitchfork' and agglomeration is smooth and reversible. Consequently, the outcome of Pflüger (2004) is replicated. If the unskilled workforce is relatively mobile, agglomeration forces gain strength so that the resulting bifurcation pattern is a 'tomahawk' as described by the seminal core-periphery model by Krugman (1991). Consequently, we find catastrophic agglomeration and hysteresis of skilled labor. If unskilled workers are very or even perfectly mobile between regions, complete agglomeration of both types of labor in either region is the only stable equilibrium of the model. This is in keeping with the above mentioned models by Helpman (1998) and Murata (2003), if these models are considered without dispersion force. Thirdly, when trade costs are high, the model predicts synchronous migration flows of both skilled and unskilled labor into the same region. During this process of agglomeration skilled and unskilled labor migration reinforce each other. During the ongoing process of economic integration, unskilled labor remigrates to its region of origin while skilled labor either continuous to agglomerate or remains concentrated. There seems to be some empirical support for this theoretical phenomenon. In a study about internal migration in Spain Bover and Velilla (1999) show that rich regions have have become net outmigration areas, whereas poorer regions are net receivers of migrants. Giannetti (2003) reports that the high-skilled agglomeration area Silicon Valley loses its unskilled workers. In a study about international migration Borjas and Bratsberg (1996) find that foreign-born return migrants leaving the US are negatively selected and are seldom among the highly skilled. Cohen and Haberfeld (2001) find that Isrealis returning from the US have been less successful in terms of income than those remaining. Reagan and Olsen (2000) concentrate on the length of stays of US immigrants. They come to the conclusion that the length of stay increases with the level of education. Bauer and Gang (1998) and Steiner and Velling (1994) support this result in studies on migrants from Egypt and migrants in Germany, respectively. And fourthly, as unskilled labor is mobile, it can react to differences in real wages. Consequently, unskilled labor in the periphery is no longer worse off than unskilled workers in the agglomeration core, which is in contrast to models where unskilled labor is immobile. Later in this paper it will be shown, that migration flows of unskilled workers come to a halt when the difference in indirect utilities equals migration costs. The marginal migrant then bears migration costs which set off the benefits of being in the agglomeration core. Consequently, unskilled workers remaining in the periphery can not improve their situation by migration.

To the best of my knowledge the models developed by Helpman (1998) and Murata (2003) are the only NEG models with mobile labor which neglect the assumption of an immobile factor production, but which exhibit crucial differences to the model presented here. In these models there is only one factor of production which enters the production of goods both as fixed and variable factor of production, whereas the model presented here assumes two of them. As a prerequisite for production skilled labor is needed as a fixed factor of production (think of headquarter services or R&E) and is perfectly mobile, whereas unskilled labor is required in the production process and is partially mobile. Furthermore, there are differences with respect to the homogeneous good (which is usually referred to as agricultural good). Helpman (1998) assumes that the homogeneous good is exogenously supplied to both regions and is non-tradable (a stock of housing). Immigration then leads to rising housing prices and, therefore, it serves as dispersion force. In Murata (2003) there is no such good. In his model dispersion comes from taste heterogeneity of the working force. In contrast to these models, the model in this paper assumes that the homogeneous good is endogenously supplied and can freely be traded between the regions.

This paper is organized as follows: section 2.1 to section 2.3 describes the basic assumptions of the model as well as the preference structure of households and the production side of the economies. To conclude, the short-run equilibrium is derived for any given distribution of skilled and unskilled labor. Section 3 is dedicated to the long-run equilibrium of the model where the breakpoint and the corresponding bifurcation patterns of skilled and unskilled workers are determined. Section 4 concludes.

2 The model

2.1 The basic set-up

The basic structure of this model is based on the analytically solvable footloose entrepreneur model developed by Pflüger (2004). The crucial difference to his paper is that unskilled labor is assumed to be heterogeneous and imperfectly mobile instead of being entirely immobile. There are two countries in the economy named home (H) and foreign (F). Both countries are identical with respect to tastes, production technologies and the (initial) endowment of factors of production. There are two types of households, skilled and unskilled. The world population of unskilled labor is given by L which is the sum of unskilled labor living in home (L_H) and foreign (L_F). The world-wide mass of skilled people is formalized by K and is composed of skilled people of both regions, K_H and K_F (the subindex indicates the region of residence). Each type inelastically supplies one unit of factor input and receives unskilled wages (W) or skilled wages (R) as income, respectively. This income is entirely spent for the consumption of goods from which people derive utility. There are two types of goods. The homogeneous good (A) is produced under perfect competition with a linear constant returns to scale technology using unskilled labor as the only input. The homogeneous good can be traded without trade costs and serves as the numéraire. Furthermore, there is a set of heterogeneous goods (X) which shall be called manufacturing goods. Each variety is produced under monopolistic competition and increasing returns to scale using both skilled and unskilled labor. Unskilled labor is the only variable factor of production. The marginal input requirement is constant and is given by c. Furthermore, each firm needs one unit of skilled labor as fixed input (*e.g.*, headquarter services or R&D). Varieties of heterogeneous goods incur trade costs when traded between the regions, within a region trade is costless.

Unskilled labor is assumed to be mobile across regions, but incurs costs when migrating from one region to the other. These costs differ between individuals. Skilled labor is perfectly mobile between regions. Within one region both types of workers are perfectly mobile between sectors. $\lambda = K_H/K$ and $(1-\lambda) = K_F/K$ express the share of skilled workers living in home (foreign) in relation to the world population of skilled workers. The share of unskilled workers residing in home (foreign) with respect to the world population of skilled labor is denoted by $\rho = L_H/K$

and $(\overline{\rho} - \rho) = L_F/K$. The parameter $\overline{\rho} = L/K$ is the world population of unskilled workers relative to the world population of skilled labor.

2.2 Preferences and demand

Preferences are homogeneous and are given by a logarithmic quasi-linear utility function. The homogeneous good enters the utility function in the form of the linear extension, whereas the aggregate of heterogeneous goods enters logarithmically and is modeled as a CES bundle:

$$\begin{aligned} U &= \alpha \ln C_X + C_A \qquad \text{where } C_X \equiv \left[\int_{i=0}^{N_H} x_i^{\frac{\sigma-1}{\sigma}} \mathrm{d}n + \int_{j=N_H}^{N_F} x_j^{\frac{\sigma-1}{\sigma}} \mathrm{d}n\right]^{\frac{\sigma}{1-\sigma}} \\ \alpha &> 0, \qquad \sigma > 1 \end{aligned} \tag{1}$$

C_X (C_A) is the quantity consumed of the heterogeneous aggregate (homogeneous good), σ measures the elasticity of substitution between any pair of heterogeneous goods and is assumed to be greater one. The positive parameter α measures the weight of heterogeneous goods in the utility function. x_i (x_j) represents the per capita consumption of a domestic (imported) heterogeneous good. N_H and N_F stand for the number of domestic and foreign firms producing each one variety of the manufacturing good. Households maximize their utility given the budget constraint defined as follows:

$$PC_X + C_A = Y \qquad \text{where } \; P \equiv \left[\int_{i=0}^{N_H} p_i^{1-\sigma} \mathrm{d}n + \int_{j=N_H}^{N_F} (\tau p_j)^{1-\sigma} \mathrm{d}n\right]^{\frac{1}{1-\sigma}}, \tau > 1 \tag{2}$$

P is the optimal CES price index where the price of the domestic (imported) variety is given by p_i (p_j). As the homogeneous good is the numéraire, its price is normalized to one. The parameter τ is greater one and captures the (iceberg) trade costs. The income per household is given by Y which is W for unskilled and R for skilled labor. Utility maximization with respect to quantities consumed yields the following demands and the indirect utility function V:

$$\begin{aligned} C_X &= \alpha/P, \qquad C_A = Y - \alpha \\ x_i &= \alpha\ p_i^{-\sigma}\ P^{(\sigma-1)}, \qquad x_j = \alpha\ (\tau p_j)^{-\sigma}\ P^{(\sigma-1)} \\ V &= Y - \alpha \ln P + \alpha(\ln \alpha - 1) \end{aligned} \tag{3}$$

To guarantee that both types of goods are consumed, α is assumed to be less than Y[1].

2.3 Production and short-run equilibrium

The homogeneous good is produced under constant returns to scale and perfect competition. As the production technology of the numéraire is linear and unskilled labor is the only variable factor of production, the wage of unskilled workers equals one.

Each variety of the heterogeneous good is produced under increasing returns to scale with a linear production technology using unskilled labor as the only variable input. To produce one unit of the good, c units of unskilled labor is needed. Furthermore, one unit of skilled labor is required as fixed input to produce at all. Firms serve both the domestic and the foreign market. Exporting goods incurs trade costs which are formalized by iceberg trade costs. Hence, if τx units are sent away, only x units arrive at the foreign market. Firms aim to maximize their profit function Π which for firm i is given by

$$\Pi_i = (p_i^H - c)(L_H + K_H)\ x_i^H\ +\ (p_i^F - c)(L_F + K_F)\ \tau x_i^F - R_i \tag{4}$$

The first (second) term on the LHS is the demand of the domestic (foreign) market. Maximizing profits with respect to the prices p_i^H and p_i^F leads to the following equilibrium prices:

$$p_i^H = p_i^F = p = c\frac{\sigma}{\sigma - 1} \tag{5}$$

Equilibrium prices are characterized by a constant mark-up over marginal costs (mill pricing). Due to free market entry and exit of firms, profits are zero in equilibrium. Setting the equilib-

[1]As p_A is set one, α has to be less than one. See chapter 2.3

rium price equal to average production costs reveals the equilibrium relation between firm size and skilled wages:

$$R_i = \frac{X_i c}{\sigma - 1} \tag{6}$$

where X_i is the aggregate production of variety i. In equilibrium aggregate production has to be equal aggregate demand by all skilled and unskilled workers. As prices are given by Eq. (5), the market clearing condition is uniquely determined by:

$$X_i = \frac{\alpha(\sigma-1)(L_H + K_H)}{\sigma c[K_H + \phi K_F]} + \frac{\alpha(\sigma-1)(L_F + K_F)\phi}{\sigma c[\phi K_H + K_F]} \tag{7}$$

where the RHS of Eq. (7) is the aggregate demand from domestic and foreign consumers. ϕ measures the freeness of trade and is commonly given by $\phi = \tau^{1-\sigma}$. If trade costs tend to infinity, ϕ tends to zero. If trade is costless, ϕ is one. As X_i is identical for all firms i, the subindex of X and R can be omitted. Substituting Eq. (7) into Eq. (6) and dividing both the denominator and the enumerator by K yields the equilibrium wage for skilled workers in region H for any given domestic share of skilled and unskilled labor (for region F the analogous holds true):

$$\begin{aligned} R_H &= \frac{\alpha}{\sigma}\left[\frac{\lambda+\rho}{\lambda+(1-\lambda)\phi} + \frac{\phi[(1-\lambda)+\overline{\rho}-\rho]}{\phi\lambda+1-\lambda}\right] \\ R_F &= \frac{\alpha}{\sigma}\left[\frac{\phi(\lambda+\rho)}{\lambda+(1-\lambda)\phi} + \frac{1-\lambda+\overline{\rho}-\rho}{\phi\lambda+1-\lambda}\right] \end{aligned} \tag{8}$$

Once the goods market equilibrium is determined, the labor market equilibrium can be characterized. The demand for unskilled labor in the manufacturing sector of region H which is related to the equilibrium aggregate production of each variety X is given by $N_H c X$. Putting Eq. (6) into this expression yields the following expression for the labor demand of the domestic manufactoring sector:

$$L^D = N_H R_H(\sigma - 1) \tag{9}$$

Unskilled workers who are not employed in the manufacturing sector find employment in the homogeneous good sector. The demand for unskilled labor given by Eq. (9) is assumed to be less than the regional supply of unskilled labor L_H so that in either region both types of goods are produced. Different from Pflüger (2004), the regional supply of unskilled is not exogenously given and fixed. Rather unskilled labor is mobile. In section 3 it is shown that the regional supply of unskilled labor is a function of trade costs and the geographical distribution of skilled workers. Taking the mobility of unskilled labor into account, the assumption of regional non-specialization is fulfilled for any given level of trade costs and for any geographical distribution of skilled labor whenever $\alpha < \sigma/2(\sigma-1)$ and $\overline{\rho} > \alpha(\sigma-1)/(\sigma/2-\alpha(\sigma-1))$. Furthermore, unskilled labor must not too mobile[2].

Substituting equilibrium prices from Eq. (5) into the CES- price index yields:

$$P_H = p^*[\lambda + (1-\lambda)\phi]^{\frac{1}{1-\sigma}} \qquad P_F = p^*[\lambda\phi + (1-\lambda)]^{\frac{1}{1-\sigma}} \tag{10}$$

3 Long-run equilibrium

In the long run, skilled and unskilled labor are mobile across regions.

Factor movements of skilled labor are determined by the following equation of motion:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = (V_H - V_F)\lambda(1-\lambda) \tag{11}$$

Consequently, people respond to the difference in indirect utilities ΔV which is given by using Eq.(3):

$$\Delta V \equiv V_H - V_F = -\alpha(\ln P_H - \ln P_F) + (Y_H - Y_F) \tag{12}$$

[2]Due to the analytical expression of the labor supply curve the points of intersection between the labor supply and demand curve cannot be determined analytically. But as both the labor demand and the labor supply are increasing in λ, it is possible to focus on $\lambda = \{0, 0.5, 1\}$. At these points the labor supply is always greater than the labor demand, if the above parameter restriction hold. Assuming that μ, which is a measure of the mobility costs, is not too small ensures that the labor supply of unskilled workers is greater than the demand for any $\lambda \in [0,1]$.

A spatial equilibrium of skilled labor distribution arises when $\mathrm{d}\lambda/\mathrm{d}t = 0$. Using Eq. (8) and (10) in (12) the condition for spatial equilibria is fulfilled whenever

$$\Delta V_S \equiv \frac{\alpha}{1-\sigma} \ln \frac{\lambda\phi + 1 - \lambda}{\lambda + \phi(1-\lambda)} + \frac{\alpha(1-\phi)}{\sigma} \left(\frac{\rho + \lambda}{\lambda + (1-\lambda)\phi} - \frac{\overline{\rho} - \rho + 1 - \lambda}{\phi\lambda + 1 - \lambda} \right) = 0 \tag{13}$$

where ΔV_S is the migration incentive of skilled labor. The logarithmic expression captures the difference in prices, whereas the second term is the difference in skilled wages.

Unskilled workers are assumed to have heterogeneous tastes. These tastes are broadly defined and embrace factors of influence like preferences about locations or the value of being integrated in certain social or cultural network as suggested by migration theory. Following Tabuchi and Thisse (2002) and Murata (2003), taste heterogeneity can be modeled by a stochastic term ϵ which is part of the utility function of unskilled labor. Consequently, we have $\overline{V}_{rj} = V_r + \epsilon_{rj}$ where V_r is the indirect utility of region $r \in [H, F]$ given by Eq. (3). $\overline{V}_{rj}$ is the perceived utility of person j in region r. ϵ_{rj} is assumed to be identically and independently Gumbel (*i.e.*, double exponentially) distributed within the population with a location parameter of 0 and a variance of $\pi^2\mu^2/6$ with π being the circular constant. The parameter μ is a positive scale parameter and a direct measure of taste heterogeneity. The greater μ, the greater is the variance of tastes. An unskilled worker settles down in the region where his perceived indirect utility $\overline{V}_{rj}$ is greatest. According to Anderson *et. al.* (1992), the assumptions about the iid random component in the utility function lead to a probability of choosing region r which is given by

$$P_r = \frac{\exp(V_r/\mu)}{\exp(V_H/\mu) + \exp(V_F/\mu)} \tag{14}$$

In this set-up, the population of unskilled workers changes according to the following equation of motion:

$$\frac{\mathrm{d}(\rho/\overline{\rho})}{\mathrm{d}t} = \rho/\overline{\rho}\ P_F - (1 - \rho/\overline{\rho})\ P_H \tag{15}$$

where $\rho/\overline{\rho}$ is the share of unskilled labor in H with respect to the world population of unskilled labor. The first term on the LHS of Eq. (15) are the gross migration flows from region H to region F, whereas the second term expresses the gross migration flows from region F to region H. A spatial equilibrium arises when $\mathrm{d}\,(\rho/\overline{\rho})/\mathrm{d}t = 0$. Net migration flows are then zero. As the denominators of P_H and P_H are identical, the equation of motion reduces to $\rho/\overline{\rho}\ \exp(V_F/\mu) = (1 - \rho/\overline{\rho})\ \exp(V_H/\mu)$. Taking the logarithm on both side and rearranging the expression yields the following condition for spatial equilibria associated with Eq. (15):

$$G \equiv \Delta V_U - \mu \ln \frac{\rho/\overline{\rho}}{1 - \rho/\overline{\rho}} = 0 \tag{16}$$

ΔV_U is the migration incentive of unskilled labor in form of the difference in indirect utilities given by Eq. (12). The second term on the RHS of Eq. (15) is the migration cost function K derived from taste heterogeneity. This curve in upward sloping in $\rho/\overline{\rho}$ and tends toward (negative) infinity, when $\rho/\overline{\rho}$ approaches (zero) one. Taking into account that in both regions unskilled wages are normalized to one and using Eq. (10) in (**??**), the migration incentive of unskilled labor ΔV_U is given by:

$$\Delta V_U = \frac{\alpha}{1 - \sigma} \ln \frac{\lambda\phi + 1 - \lambda}{\lambda + \phi(1 - \lambda)} \tag{17}$$

Hence, unskilled labor responds to differences in price levels which are a function of trade costs and the distribution of skilled labor. When $\lambda = 0.5$, we find that $\Delta V_U = 0$ for any degree of trade freeness.

A spatial equilibrium is obtained **unambiguously** by any combination of λ and ρ which simultaneously satisfies the conditions given by Eq. (16) and (13). To analyze the stability of these equilibria I will proceed as follows: In a first step (section 3.1), I analyze the effects of a distributional shock of skilled labor taking into account the reaction of unskilled workers. In a second step (section 3.2), the effect of a distributional shock of unskilled labor is examined taking into account the reaction of skilled labor.

3.1 Skilled labor as first-mover

In this section skilled workers are assumed to be the first-movers. They take the initiative to deviate from a spatial equilibrium, whereas unskilled workers are assumed to follow. Analytically, the equilibrium share of unskilled labor for any given distribution of skilled labor is given by solving Eq. (16) with respect to ρ:

$$\rho(\Delta V_U) = \frac{\overline{\rho}}{1 + e\ \exp(-\Delta V_U/\mu)} \tag{18}$$

where $\Delta V_U = \Delta V_U(\lambda, \phi)$ as given by Eq. (17). Consequently, we have $\rho = \rho(\lambda, \phi)$. Note that $\rho(\Delta V_U)$ is a logistic probability function multiplied by $\overline{\rho}$. Consequently, when the migration incentive tends to (negative) infinity, the ratio of unskilled workers in H tends to (0) $\overline{\rho}$. If $\Delta V_U = 0$, unskilled labor is equally split over both regions. Furthermore, it becomes obvious that greater levels of taste heterogeneity reduce the willingness to migrate. When μ tends toward infinity, unskilled labor is immobile and equally spread over both regions for any given migration incentive. When μ approaches 0, unskilled workers are perfectly mobile so that even a marginal difference in indirect utilities induces them to migrate.

Using Eq. (18) in (13) yields the migration incentive of skilled labor taking into the reaction of unskilled workers $\Delta V_S(\lambda) = \Delta V_S(\lambda,\ \rho(\lambda, \phi),\ \phi)$.

3.1.1 Model forces and breakpoint of skilled labor

The symmetric allocation of skilled and, consequently, unskilled labor is always an equilibrium ($\Delta V_S(\lambda = 0.5) = 0$). The stability of this equilibrium is revealed by the sign of the first derivative of $\Delta V_S(\lambda)$ with respect to λ evaluated at symmetry, which is given by:

$$\frac{\mathrm{d}\Delta V_S(\lambda)}{\mathrm{d}\lambda}\Bigg|_{\lambda=1/2} = \frac{\partial(-\alpha\Delta \ln P)}{\partial\lambda} + \frac{\partial\Delta R}{\partial\lambda} + \frac{\partial\Delta R}{\partial\rho}\frac{\mathrm{d}\rho}{\mathrm{d}\lambda}\Big|_{\lambda=1/2} \tag{19}$$

where $\Delta \ln P \equiv \ln P_H - \ln P_F$ and $\Delta R \equiv R_H - R_F$. The first two terms of the RHS of Eq. (19) are known from Pflüger (2004) and Pflüger and Südekum (2008b). The first expression is the supply linkage. When λ rises the price index in H falls, because more varieties are produced domestically and do not have to be imported. In F the opposite holds true, which leads to a greater migration incentive for skilled labor toward H. Following Pflüger and Südekum (2008b), the derivate of ΔR with respect to λ can be decomposed into two different forces. Firstly, holding the individual demand per good constant, an increase in λ leads to a bigger domestic market and higher profits. This increases the attractiveness of region H (demand linkage by skilled labor). Secondly, holding the market size constant, the lower price index in H relatively increases the price of a variety in region H. Consequently, people demand less units per variety, which lowers the profit of domestic firms making the region less attractive (competition effect). The third term of the LHS of Eq. (19) is novel in the literature and originates in the mobility of unskilled labor. If unskilled workers were immobile as in standard models (*i.e.*, $\partial\rho/\partial\lambda = 0$), the effect would be zero. Here, an increase in λ raises the migration incentive of unskilled workers as the price index drops in H and rises in F. The gap in regional price levels then increases the share of unskilled labor residing in H and increases the domestic market. This in turn raises domestic profits and the wages of the skilled workforce. As the mechanism at work is equivalent to the demand linkage associated with skilled labor, this new agglomerative force shall be called demand linkage by unskilled labor. The analytical expressions of the linkages can be found in appendix A.

Setting Eq. (19) evaluated at symmetry equal to zero and solving it for ϕ gives the breakpoint of the model:

$$\phi_b = \frac{\alpha\overline{\rho} + \mu\overline{\rho} + \mu\sigma - \mu\overline{\rho}\sigma}{\alpha\overline{\rho} + \mu\overline{\rho} + \mu\sigma - \mu\overline{\rho}\sigma - 2\mu(2\sigma - 1)} \tag{20}$$

As long as trade costs are greater than the breakpoint ($\phi < \phi_b$), the slope of the difference in indirect utilities is negative around $\lambda = 0.5$, so that dispersion is stable. For levels of trade costs less than this threshold ($\phi > \phi_b$), symmetry becomes instable leading to agglomeration in either region. To guarantee that ϕ_b lies between 0 and 1 and, therefore, dispersion is stable when trade costs are (infinitively) high, two parameter restrictions have to be imposed (no-black-hole conditions). The necessary condition is that μ has to be greater than $\alpha/(\sigma - 1)$. Intuitively, the restriction assures that unskilled labor is not too mobile and, hence, the demand linkage of unskilled labor not too great. The (dispersive) competition effect is then stronger than the demand linkage of unskilled workers for any level of trade costs. If μ is smaller than the critical threshold, complete agglomeration is the only stable equilibrium even for infinitively high values of trade costs. Once the necessary condition is fulfilled, it is sufficient to assume that $\overline{\rho}$ is greater than $\mu\sigma/[\mu(\sigma-1)-\alpha]$. This assumption guarantees that the competition effect is strong enough to overcome the agglomeration forces generated by the supply and demand linkage of skilled labor. The analytical derivation of the no-black-hole conditions can be found in appendix A. In comparison to the model without mobile unskilled labor, agglomeration is induced at higher levels of trade costs. This is intuitive, because mobile unskilled labor adds an additional agglomeration force.

The comparative statics of the breakpoint are straightforward: $\partial\phi_b/\partial\alpha < 0$ which is due to stronger agglomerative forces as heterogeneous goods get more weight in the utility function, $\partial\phi_b/\partial\mu > 0$ meaning weaker agglomerative forces as the unskilled demand linkage becomes less important, $\partial\phi_b/\partial\sigma > 0$ since agglomeration forces become weaker as firms have less market power and lower mark-ups over marginal costs. And finally, we have $\partial\phi_b/\partial\overline{\rho} > 0$, if the no-black-hole condition with respect to μ (which is $\mu > \alpha/(\sigma-1)$) holds true. The competition effect then dominates the demand linkage of unskilled labor, so that a greater number of unskilled labor strengthens the dispersive competition effect.

3.1.2 The bifurcation pattern of skilled labor

Following Grandmont (1988) and Pflüger and Südekum (2008a) the type of bifurcation is determined by the sign of the third derivative of $\Delta V_S(\lambda)$ with respect to λ evaluated at $\lambda = 1/2$ and $\phi = \phi_b$. Furthermore, the second derivative of $\Delta V_S(\lambda)$ with respect to λ evaluated at these points must be zero, which in this model holds true. The third derivative of the difference of indirect utilities evaluated at symmetry and ϕ_b is given by:

$$\frac{\mathrm{d}^3\Delta V_S(\lambda)}{\mathrm{d}\lambda^3} = -\frac{32\alpha\mu(2\sigma-1)^3[2\mu^3(\overline{\rho}+1)(\sigma-1)^3\sigma + \alpha^3\overline{\rho}(2\sigma-1) - \alpha\mu^2\overline{\rho}(\sigma-1)^2(4\sigma-1)]}{[\mu(\overline{\rho}+1)(\sigma-1)-\alpha\overline{\rho}]^4(\sigma-1)^3\sigma} \tag{21}$$

Whether this term is positive or negative depends on the sign of the numerator. Rearranging the term in square brackets reveals that it is a linear function in $\overline{\rho}$ with a slope of $2\mu^3(\sigma-1)^3\sigma - \alpha\mu^2(\sigma-1)^2(4\sigma-1) + \alpha^3(2\sigma-1)$ and a positive constant of $2\mu^3(\sigma-1)^3\sigma$. If μ is greater than a critical value μ_{crit}, the slope is positive leading to a negative third derivative irrespectively of $\overline{\rho}$ (see appendix B for further details). The corresponding bifurcation is a 'pitchfork' as in Pflüger (2004). Consequently, partial agglomeration is stable and the transition from dispersion to total agglomeration is smooth and reversible. This situation is shown in Figure 1. The diagrams on the left represent the evolution of the difference of indirect utilities with respect to falling trade costs (increasing trade freeness), whereas the diagram on the right is the corresponding bifurcation pattern.

If μ is smaller than μ_{crit} and the number of unskilled labor $\overline{\rho}$ is greater than a critical amount $\overline{\rho}_{crit}$, the term in square brackets is negative leading to a positive third derivative (see appendix B). This results in a 'tomahawk' bifurcation[3]. In other words: if unskilled labor is relatively mobile and the number of unskilled migrants is relatively large, agglomeration forces are strong leading to a core-periphery pattern and a bang-bang solution as in Krugman (1991). Therefore, once skilled labor becomes completely concentrated in either region, the model exhibits hysteresis for increasing trade costs until the sustain point ϕ_s with $\phi_s < \phi_b$ is reached. The utility difference curves and the corresponding bifurcation pattern are shown in Figure 2.

[Figures 1 and 2 about here]

The transition from a pitchfork to a tomahawk bifurcation is surprising. If unskilled labor is relatively mobile ($\mu < \mu_{crit}$) and its global stock is close to but smaller than the critical amount of $\overline{\rho}_{crit}$, the wiggle diagram is locally concave around $\lambda = 0.5$, but becomes convex for higher levels of skilled labor agglomeration. This situation is shown in Figure 3. When trade costs are sufficiently high, $\Delta V_S(\lambda)$ is downward sloping in λ (figure *a*). Once trade costs have fallen below a critical threshold, the difference of indirect utilities remains concave around symmetry, but becomes convex at the corners of the interval $\lambda \in [0, 1]$. This allows four asymmetric equilibria despite the symmetric one and the two corner solutions[4]. The corner solutions (total agglomeration) and two interior equilibria (partial agglomeration) are stable, whereas the symmetric allocation and the remaining asymmetric spatial equilibria are not. Figures *b* show this phenomenon. When trade costs continue to fall, $\Delta V_S(\lambda)$ is upward sloping and convex in λ as shown by figure *c*. The corresponding bifurcation pattern can be found in the lower right corner.

[Figure 3 about here]

[3]Grandmont (1988) calls this kind of bifurcation pattern 'subcritical' pitchfork bifurcation. This label is adopted by several papers. To avoid confusion and to clearly distinguish the 'subcritical' from the 'supercritical' pitchfork bifurcation, the 'subcritical' bifurcation is referred to as 'tomahawk' whereas the the 'supercritical' is referred to as 'pitchfork' bifurcation. For an introduction into bifurcations see Grandmont (1988) or Fujita *et. al.* (1999).

[4]The greatest relevant exponent of λ in $\mathrm{d}\Delta V_S(\lambda)/\mathrm{d}\lambda$ can be shown to be greater than two. Consequently, more than two extrema may exist leading to more than five spatial equilibria (incl. corner solutions). In numerical evaluations no more than four extrema were found.

When $\mathrm{d}^3\Delta V_S(\lambda)/\mathrm{d}\lambda^3 = 0$, the wiggle diagram is **locally** linear around the symmetric distribution of skilled labor and the breakpoint, leading to catastrophic agglomeration. Unlike in standard NEG models with immobile labor the local linearity cannot be generalized over the whole interval of $\lambda \in [0, 1]$ and $\phi \in [0, 1]$ as proposed by Pflüger and Südekum (2008a). Instead, the bifurcation pattern is a tomahawk with $\phi_s < \phi_b$.

3.1.3 The breakpoint and bifurcation pattern of unskilled labor

Once the equilibrium location of skilled workers is determined, the geographical distribution of unskilled workers can be derived. Figure 4 illustrates the relation between skilled and unskilled labor agglomeration and trade costs. Diagram I in the lower right corner shows the migration incentive of skilled labor $\Delta V_S(\lambda)$ for $\lambda \in [0.5, 1]$ and for a given level of trade costs ϕ as introduced by figures 1 or 2. The equilibrium distribution of skilled labor λ^* then determines the migration incentive of unskilled labor $\Delta V_U = \Delta V_U(\lambda^*(\phi), \phi)$ which is depicted in the upper right corner (diagram II). Given the migration incentive, the share of unskilled labor residing in H is uniquely determined in diagram III in the upper left corner.

[Figure 4 about here]

As long as the symmetric allocation of skilled labor is stable, unskilled labor is dispersed, too (see the bold black lines in fig. 4). When trade costs haven fallen below the breakpoint, skilled labor agglomeration becomes stable which breaks the symmetric allocation of unskilled labor. Consequently, the breakpoint of unskilled labor is identical to the breakpoint defined by ϕ_b.

The shape of the bifurcation pattern of unskilled workers depends on the pattern of skilled labor agglomeration. If parameters are such that we observe a pitchfork bifurcation of skilled labor, partial agglomeration of skilled labor is stable. Falling trade costs then have two opposing effects on the migration incentive of unskilled labor (the analytical expressions are shown in appendix C)

$$\frac{\mathrm{d}\Delta V_U}{\mathrm{d}\phi} = \frac{\partial \Delta V_U}{\partial \lambda^*}\frac{\mathrm{d}\lambda^*}{\mathrm{d}\phi} + \frac{\partial \Delta V_U}{\partial \phi} \tag{22}$$

On the one hand, falling trade costs lead to an increasing share of skilled labor in either region (compare $\lambda^*(\phi_1)$ and $\lambda^*(\phi_2)$ in diagram I with $\phi_1 < \phi_2$). Therefore, the gap in regional price levels widens and increases the migration incentive of unskilled labor. This relocation effect is captured by the first term on the RHS of Eq. (22). Analytically, we have $\mathrm{d}\lambda/\mathrm{d}\phi \geq 0$ and $\partial \Delta V_U / \partial \lambda^* > 0$. On the other hand, there is a direct trade costs effect which is expressed by the second term on the RHS. A lower level of trade costs makes the distance between markets less important. Consequently, the difference in regional price levels decreases and unskilled labor has less incentives to migrate. This is shown in diagram II, where the dotted gray line (which is $\Delta V_U(\phi_1)$) is below the dotted black line (which plots $\Delta V_U(\phi_2)$). Analytically, we find that $\mathrm{sgn}(\partial \Delta V_U / \partial \phi) = -\,\mathrm{sgn}(\lambda^* - 0.5)$.

Which of these two effects dominates, depends on the level of trade costs. Around the breakpoint and symmetry of skilled labor, the migration incentive ΔV_U increases. This holds true since at symmetry the trade costs effect is zero, whereas the relocation effect is positive. Once trade costs have fallen below a certain threshold ϕ_c, complete agglomeration of skilled labor is the only stable equilibrium so that falling trade costs reduce the gap in regional price levels, but leave the equilibrium distribution of skilled labor unchanged. Consequently, the migration incentive ΔV_U decreases leading to a continuous redispersion of the unskilled work force. Such a situation is shown by the bold gray lines in Figure 4. Figure 5 shows the corresponding bifurcation pattern of unskilled labor. The parameter ϕ_{crit} expresses the critical level of trade costs at which the trade costs effect dominates the relocation effect. From the analysis we can conclude that $\phi_b < \phi_{crit} \leq \phi_c$.

[Figure 5 about here]

If parameters are such that we observe a tomahawk bifurcation, the equilibrium distribution of skilled labor jumps discontinuously from symmetry to complete agglomeration once trade costs have reached the breakpoint. Consequently, the migration incentive of unskilled labor discontinuously increases, leading to catastrophic agglomeration of the unskilled workforce. Note that we do not observe total agglomeration of unskilled labor due to the existence of migration costs. Falling trade costs then reduce the gap in regional price levels. Hence, ΔV_U becomes smaller inducing a continuous redispersion of unskilled labor. Skilled labor, instead, remains agglomerated.

In analogy to skilled labor agglomeration, unskilled labor agglomeration exhibits hysteresis. Consequently, once catastrophic agglomeration occurred, an increase in trade costs does not lead to catastrophic redispersion. But different from skilled labor hysteresis, the share of unskilled labor does not remain unchanged. Rather, increasing trade costs widen the gap in regional price levels, leading to further inflows of unskilled labor. These inflows of unskilled labor induced by increasing trade costs come to halt, when trade costs have reached the sustain point ϕ_s. At this point, skilled labor agglomeration becomes instable so that both types of labor catastrophically remigrate and are equally split over both regions. Figure 6 shows the corresponding bifurcation pattern of unskilled labor. Appendix C offers the analytical expressions of ρ for a tomahawk bifurcation pattern of skilled labor.

[Figure 6 about here]

3.2 Unskilled labor as first-mover

Different from the analysis above, it is now assumed that unskilled labor takes the initiative to deviate from a spatial equilibrium, whereas skilled labor follows according to its equation of motion. Consequently, solving Eq. (13) with respect to λ yields $\lambda = \lambda(\rho/\overline{\rho}, \phi)$. Using this expression in Eq. (16) we get $G(\rho/\overline{\rho}) = G(\rho/\overline{\rho},\ \lambda(\rho/\overline{\rho}, \phi),\ \phi)$.

The symmetric allocation of unskilled and, therefore, skilled labor is a spatial equilibrium ($G(0.5) = 0$). But as before, the symmetric equilibrium does not necessarily have to be stable. In analogy to the above, differentiation of $G(\rho/\overline{\rho})$ with respect to $\rho/\overline{\rho}$ and evaluation at symmetry shows the effects of distributional shock of unskilled labor

$$\frac{\mathrm{d}G}{\mathrm{d}\ (\rho/\overline{\rho})}\bigg|_{\rho/\overline{\rho}=1/2} = \frac{\partial \Delta V_U}{\partial \lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\ (\rho/\overline{\rho})} - \frac{\partial K}{\partial\ (\rho/\overline{\rho})}\Big|_{\rho/\overline{\rho}=1/2} \tag{23}$$

where K denotes the migration cost function as defined by Eq. (16). On the one hand, a greater stock of unskilled labor leads to higher skilled wages in the immigration region. This in turn induces skilled labor to follow the unskilled workforce, which widens the gap in regional price levels and raises the migration incentive of unskilled workers. Analytically, we have

$\mathrm{d}\lambda/\mathrm{d}(\rho/\overline{\rho}) \geq 0$ and $\mathrm{d}\Delta V_U/\mathrm{d}\lambda > 0$. Appendix D offers further analytical details. On the other hand, the agglomeration force is reduced by migration costs which are increasing in $\rho/\overline{\rho}$. Setting Eq. (23) equal to zero and solving it with respect to ϕ yields the breakpoint, which is identical to the breakpoint determined in section 3.1.1. When trade costs are greater (less) than ϕ_b, the symmetric allocation of unskilled and skilled labor is (in)stable.

As the second derivative of G can be shown to be zero, the third derivative of G with respect to $\rho/\overline{\rho}$ reveals the type of bifurcation:

$$\frac{\mathrm{d}^3 G}{\mathrm{d}\ (\rho/\overline{\rho})^3} = -\frac{32\mu\left[2\mu^3(\overline{\rho}+1)(\sigma-1)^3\sigma + \alpha^3\overline{\rho}(2\sigma-1) - \alpha\mu^2\overline{\rho}(\sigma-1)^2(4\sigma-1)\right]}{\alpha^3\overline{\rho}(2\sigma-1)} \tag{24}$$

The sign of this expression is uniquely determined by the sign of the numerator. As the expression in square brackets is identical to the expression in section 3.1.2, the results are identical. Consequently, if μ is greater than μ_{crit} (defined in appendix B), partial agglomeration of unskilled and skilled labor is stable, so that the transition from dispersion to agglomeration is smooth and reversible. The corresponding bifurcation patterns are shown in figure 5. If μ is less than μ_{crit} and $\overline{\rho}$ is greater than $\overline{\rho}_{crit}$ (see appendix B), the agglomeration of both types of labor is catastrophic once trade costs have fallen below the breakpoint. Figure 6 shows the corresponding bifurcation patterns.

4 Conclusion

Allowing skilled and unskilled workers to be mobile in a ‘footloose entrepreneur’ new economic geography model adds an additional agglomerative force. Besides the (positive) supply and demand linkage by skilled labor migration and the (negative) competition effect, this paper identifies a demand linkage by unskilled labor mobility. If unskilled workers migrate from one region to another, they enlarge the market of the immigration country. The increase in market size raises the profitability of domestic firms and thus the wages of skilled labor. If skilled labor responds to the increase in wages and migrates toward this region, production is shifted. Consequently, more varieties are produced domestically and do not have to be imported. This saves trade costs and lowers the domestic price level, whereas the price level in the emigration region rises for the same reason. The increasing gap in regional price levels in turn induces unskilled labor to migrate which leads to the forward-linkage described.

Depending on the level of unskilled labor mobility, the geographical distribution of economic activity has different shapes. If unskilled labor is relatively immobile, the increase in domestic demand by a relocation of unskilled labor is relatively unimportant compared to the demand by workers remaining in the distant market. The corresponding bifurcation pattern then is a pitchfork as proposed by Pflüger (2004) and exhibits a smooth transition from dispersion to total agglomeration. If unskilled labor is relatively mobile, the number of unskilled migrants will be great even for small migration incentives. Consequently, the aggregate demand in the immigration region increases significantly and compensates the effects of fiercer competition. The corresponding bifurcation pattern then is a tomahawk as in the seminal core-periphery model developed by Krugman (1991) and agglomeration is catastrophic. If the degree of unskilled labor mobility is very high so that the necessary no-black-hole condition is violated, complete agglomeration of skilled labor is the only stable equilibrium for any level of trade costs. This is in line with Helpman (1998) and Murata (2003), if these models are considered without the additional dispersion force.

The transition from a pitchfork to a tomahawk bifurcation is surprising: as shown in section 3.1.2, there exist up to seven equilibria (including border solutions of complete agglomeration). This differs from what has been worked out by the literature. Robert-Nicoud (2003) shows that standard NEG models with immobile unskilled labor display at most five equilibria. Borck and Pflüger (2006) prove that the same holds true in Pflüger (2004), which is the underlying model of the present paper. As the demand linkage by unskilled workers is the crucial difference between Pflüger (2004) and the model presented here, the reason for the increase in spatial equilibria must originate in the reaction of skilled labor wages induced by the immigration of unskilled workers. In Pflüger (2004) at medium trade costs the competition effect becomes stronger in comparison to the agglomerative supply and demand linkage by skilled labor. Consequently, the difference in real wages is decreasing in the number of skilled labor for high levels of agglomeration. Instead, in the model presented the inflow of unskilled labor raises the domestic aggregate income which leads to higher skilled wages. The increase in skilled wages attenuates the effects of fiercer competition and leads to stable partial **and** total agglomeration equilibria.

The patterns of the geographical distribution of skilled and unskilled labor are tightly related to each other. If skilled (unskilled) labor agglomeration is smooth and reversible, then the same holds true for unskilled (skilled) labor agglomeration. If skilled (unskilled) labor agglomeration is catastrophic, unskilled (skilled) labor agglomeration will be catastrophic, too. Furthermore, the model predicts migration flows which depend on the level of economic integration. For high levels of trade costs, both types of labor are dispersed. Falling trade costs then lead to synchronous migration flows of both factors of production into the same region. During this process skilled and unskilled labor migration reinforce each other. During the ongoing process of economic integration, unskilled labor remigrates to its region of origin while skilled labor either continues to agglomerate or remains concentrated.

There is another crucial difference to standard versions of this model. When unskilled labor is immobile, the unskilled workforce in the periphery is worse off than unskilled workers in the agglomeration core. Wages are normalized in both regions, but in the periphery the price index is higher, because more products have to be imported. In the present approach unskilled labor can react to this imbalance in indirect utilities by migration. Migration movements come to a halt when the difference in indirect utilities equals migration costs. The marginal migrant then bears migration costs which set off the benefits of being in the agglomeration core. Consequently, unskilled workers remaining in the periphery cannot improve their utility net of migration costs and are no longer worse off than unskilled labor in the agglomeration core.

Furthermore, the results of the presented model allow the interesting technical conclusions that a pitchfork bifurcation can be transformed into a tomahawk by adding an additional agglomeration force. This supports the analysis by Pflüger and Südekum (2008a). In a generalized new economic geography model with mobile labor, they identify the absence (Pflüger (2004)) or accordingly the existence (Krugman (1991)) of an income effect as the source of the different bifurcation patterns. The model developed in this paper substitutes the agglomerative income effect by a demand linkage by unskilled labor. The advantage of this model is that the strength of the unskilled demand linkage can be controlled for and, consequently, replicates a variety of different bifurcation patterns.

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A Model forces and breakpoint

The first derivative evaluated at $\lambda = 0.5$ can be decomposed into the following forces:

- Supply linkage:

$$\left.\frac{\partial(-\alpha \ln \Delta P)}{\partial \lambda}\right|_{\lambda=1/2} = \frac{4\alpha(1-\phi)}{(\sigma-1)(1+\phi)} > 0 \tag{25}$$

- Demand linkage of skilled labor:

$$\frac{\partial \Delta R}{\partial \lambda}\bigg|_{\lambda=1/2, x^* fixed} = \frac{4\alpha(1-\phi)}{\sigma(1+\phi)} > 0 \tag{26}$$

- Competition effect:

$$\frac{\partial \Delta R}{\partial \lambda}\bigg|_{\lambda=1/2, marketsize fixed} = -\frac{4\alpha(\overline{\rho}+1)(1-\phi)^2}{\sigma(1+\phi)^2} < 0 \tag{27}$$

- Demand linkage of unskilled labor:

$$\frac{\partial \Delta R}{\partial \rho}\frac{\partial \rho}{\partial \Delta V_U}\frac{\mathrm{d}\Delta V_U}{\mathrm{d}\lambda}\bigg|_{\lambda=1/2} = \frac{4\alpha^2\overline{\rho}(1-\phi)^2}{\mu(\sigma-1)\sigma(1+\phi)^2} > 0 \tag{28}$$

Demand linkage of skilled labor and competition effect together - Eq. (26)+Eq. (27):

$$\frac{\partial \Delta R}{\partial \lambda}\bigg|_{\lambda=1/2} = -\frac{4\alpha(1-\phi)[\overline{\rho}(1-\phi)-2\phi]}{\sigma(1+\phi)^2} \tag{29}$$

This curve is negative for $0 < \phi < \overline{\rho}/(2+\overline{\rho})$ and greater than zero for $\overline{\rho}/(2+\overline{\rho}) < \phi < 1$

The sum of all 4 forces Eq. (25) to Eq. (28) is given by:

$$\frac{\mathrm{d}\Delta V_S(\lambda)}{\mathrm{d}\lambda}\bigg|_{\lambda=1/2} = \frac{4\alpha(1-\phi)[\alpha\overline{\rho}(1-\phi)+\mu(\sigma-\overline{\rho}(\sigma-1)(1-\phi)+(3\sigma-2)\phi)]}{\mu(\sigma-1)\sigma(1+\phi)^2} \tag{30}$$

The model developed by Pflüger (2004) arises as a special case for $\mu \to \infty$.

To guarantee that dispersion is stable when trade costs are infinitively high, parameters have be set such that the slope of ΔV_S with respect to λ evaluated at symmetry and $\phi = 0$ is less than zero. Consequently, we have

$$\lim_{\phi\to 0}\frac{\mathrm{d}\Delta V_S(\lambda)}{\mathrm{d}\lambda}\bigg|_{\lambda=1/2} = \frac{4\alpha(\alpha\overline{\rho}+\mu\overline{\rho}-\mu\sigma\overline{\rho}+\mu\sigma)}{\mu\left(\sigma^2-\sigma\right)} \overset{!}{<} 0 \tag{31}$$

The sign of Eq. (31) depends on the sign of numerator. Consequently, the full-form no-black-hole condition is given by $(\alpha\overline{\rho}+\mu\overline{\rho}-\mu\sigma\overline{\rho}+\mu\sigma) < 0$. To understand the role of unskilled labor mobility, it is of great use to modify this inequality. Assuming α and σ to be given, solving the full-form no-black-hole condition with respect to $\overline{\rho}$ reveals the no-black-hole conditions as stated in the paper:

$$(\alpha\overline{\rho}+\mu\overline{\rho}-\mu\sigma\overline{\rho}+\mu\sigma) < 0 \qquad \Leftrightarrow \qquad \mu > \frac{\alpha}{\sigma-1} \qquad \text{and} \qquad \overline{\rho} > \frac{\mu\sigma}{\mu(\sigma-1)-\alpha}$$

If μ less than the critical threshold, $\overline{\rho}$ has to be smaller than a negative value to ensure that Eq. (31) is negative. Therefore, the condition with respect to μ ($\overline{\rho}$) is necessary (sufficient).

B The bifurcation pattern of skilled labor

The slope of the function in $\overline{\rho}$ can at most have three roots in μ. One root is given by $\mu = \alpha/(\sigma-1)$. Reducing the slope by polynomial division yields a positively quadratic function in μ. The critical level of unskilled labor mobility is given by

$$\mu_{crit} \equiv \frac{\alpha(2\sigma-1)}{4(\sigma-1)\sigma} + \frac{1}{4}\sqrt{\frac{\alpha^2(\sigma(20\sigma-12)+1)}{(\sigma-1)^2\sigma^2}} \tag{32}$$

which is greater than the lower bound of $\mu = \frac{\alpha}{\sigma-1}$. Straightforward analysis shows that μ_{crit} is the only relevant root of the slope, if the no-black-hole condition with respect to unskilled labor mobility holds true.
If $\mu < \mu_{crit}$, the total number of unskilled labor $\overline{\rho}$ has to be greater than

$$\overline{\rho}_{crit} \equiv -\frac{2\mu^3(\sigma-1)^3\sigma}{2\mu^3(\sigma-1)^3\sigma - \alpha\mu^2(\sigma-1)^2(4\sigma-1) + \alpha^3(2\sigma-1)} \tag{33}$$

to ensure that the linear function in $\overline{\rho}$ takes on negative values. It can easily be shown that $\overline{\rho}$ is greater than the lower bound given by the sufficient no-black-hole condition $\overline{\rho} = \frac{\mu\sigma}{\mu(\sigma-1)-\alpha}$.

C The geographical distribution of unskilled labor

The trade cost effect is given by:

$$\frac{\partial \Delta V_U}{\partial \phi} = -\frac{2\alpha(\lambda^*-0.5)}{(\sigma-1)(\lambda^*\phi+1-\lambda^*)(\lambda^*+(1-\lambda^*)\phi)} \tag{34}$$

For $\lambda^* > 0.5$ ($\lambda^* < 0.5$) the trade cost effect is less (greater) than zero.

The general supply linkage is analytically given by:

$$\frac{\partial \Delta V_U}{\partial \lambda^*} = \frac{\alpha(1-\phi^2)}{(\sigma-1)(\lambda^*\phi+1-\lambda^*)(\lambda^*+(1-\lambda^*)\phi)} > 0 \tag{35}$$

If skilled labor agglomeration is catastrophic, the migration incentive and the proportion of unskilled labor for complete agglomeration of skilled workers (here $\lambda^* = 1$) is analytically given by:

$$\Delta V_U(\lambda^* = 1) = \frac{\alpha}{1-\sigma}\ln\phi \tag{36}$$

$$\rho/\overline{\rho}\ (\lambda^* = 1) = \frac{1}{1+\phi^{\alpha/[\mu(\sigma-1)]}} \tag{37}$$

The derivative of $\rho/\overline{\rho}(\lambda^* = 1)$ with respect to ϕ is negative for any level of trade costs:

$$\frac{\mathrm{d}\ (\rho/\overline{\rho})}{\mathrm{d}\phi} = -\frac{\alpha}{\mu(\sigma-1)}\phi^{[\alpha-\mu(\sigma-1)]/[\mu(\sigma-1)]} < 0 \tag{38}$$

D Unskilled labor as first mover

Differentiating the migration incentive of skilled labor ΔV_S given by Eq. (13) with respect to $\rho/\overline{\rho}$ yields:

$$\frac{\partial \Delta V_S}{\partial\ (\rho/\overline{\rho})} = \frac{\alpha\overline{\rho}(1-\phi)(1+\phi)}{\sigma(\lambda+(1-\lambda)\phi)(\lambda\phi+1-\lambda)} > 0 \tag{39}$$

As the migration incentive increases for any given distribution of skilled labor and for any level of trade costs, from the equation of motion given by Eq. (11) it follows that $\mathrm{d}\lambda/\mathrm{d}(\rho/\overline{\rho}) \geq 0$. Consequently, if skilled labor agglomeration is partial ($\lambda < 1$), skilled labor migrates toward H. Once total agglomeration of skilled labor arises ($\lambda = 1$), the proportion of skilled labor remains unchanged. Analytically, the reaction of skilled labor can be determined by implicit derivation of Eq. (13).

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}\ (\rho/\overline{\rho})}\right|_{\rho/\overline{\rho}=1/2} = \frac{\overline{\rho}(\sigma-1)(1+\phi)}{\overline{\rho}(\sigma-1)(1-\phi)+2\phi-3\phi\sigma-\sigma} \tag{40}$$

The derivative of the migration cost function is given by

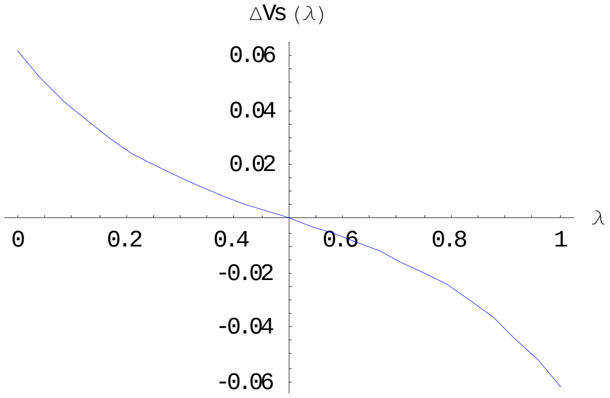
$$\frac{\partial K}{\partial\ (\rho/\overline{\rho})}\Big|_{\rho/\overline{\rho}=1/2} = 4\mu \tag{41}$$

The first derivative of $G(\rho/\overline{\rho}) = G(\rho/\overline{\rho},\ \lambda(\rho/\overline{\rho},\phi),\ \phi)$ evaluated at symmetry is given by

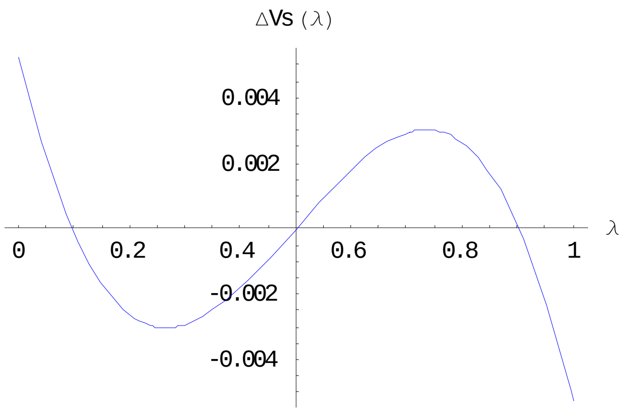
$$\left.\frac{\mathrm{d}G}{\mathrm{d}\ (\rho/\overline{\rho})}\right|_{\rho/\overline{\rho}=1/2} = \frac{4(\alpha\overline{\rho}(\phi-1)-\mu(3\phi\sigma+\sigma-\overline{\rho}(\sigma-1)(1-\phi)-2\phi))}{3\phi\sigma+\sigma-\overline{\rho}(\sigma-1)(1-\phi)-2\phi} \tag{42}$$

Note that the expression in Eq. (42) tends toward infinity when ϕ tends to $\phi_P = [\sigma(\overline{\rho}-1)-\overline{\rho}]/[\sigma(3+\overline{\rho})-\overline{\rho}-2]$. This is the breakpoint worked out by Pflüger (2004). When ϕ is greater than this critical threshold, the sign of Eq. (42) is negative. Consequently, an increase in unskilled labor would induce outmigration flows of the skilled workforce. Although being mathematically correct, it contradicts the migration movements determined by the equation of motion in Eq. (11). Rather, an increase of unskilled labor in either region at levels of trade trade freeness greater than ϕ_P leads to instantaneous total agglomeration of skilled workers in the same area.

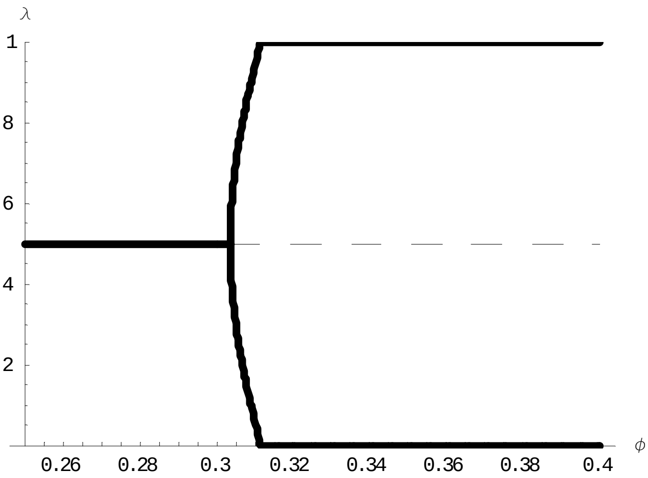
Figure 1 Pitchfork bifurcation

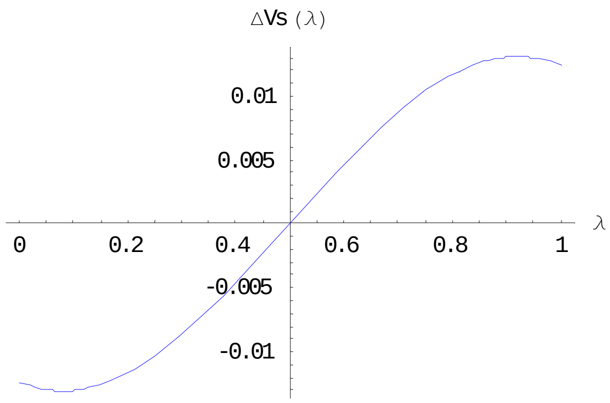


$\phi = 0.2857$



$\phi = 0.3091$

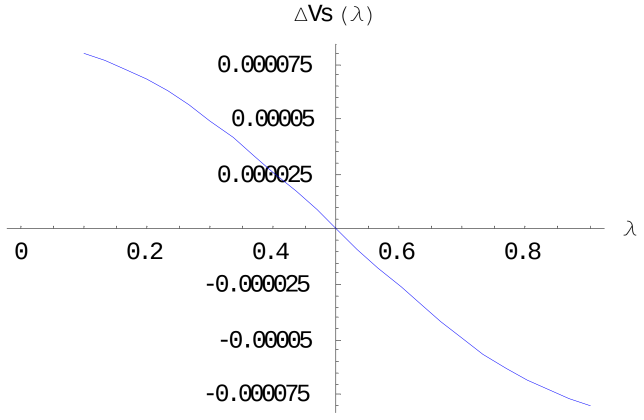




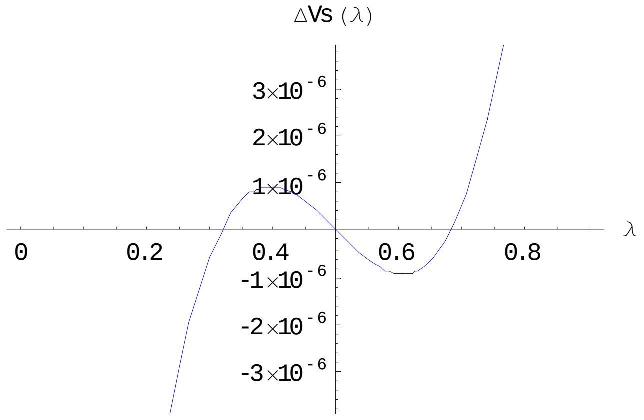
$\phi = 0.3174$

Numerical evaluation for $\alpha = 0.5$, $\sigma = 2$, $\mu = 0.65$, $\overline{\rho} = 20$.

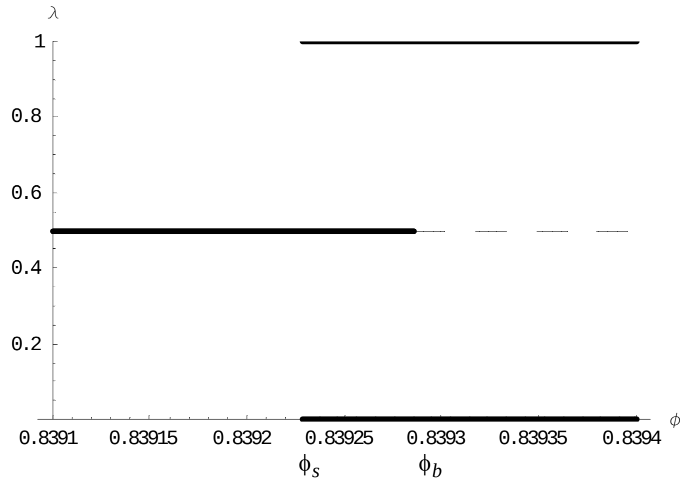
Figure 2 Tomahawk bifurcation

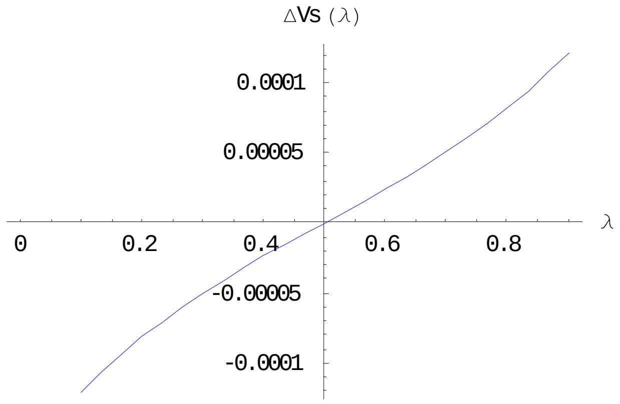


$\phi = 1.1917^{-1}$



$\phi = 1.1915^{-1}$

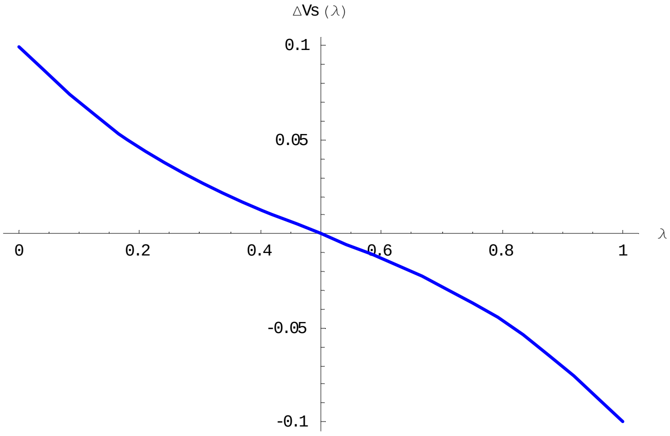




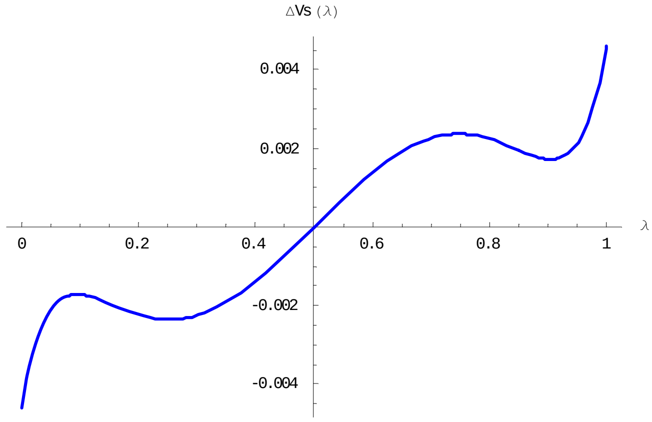
$\phi = 1.1913^{-1}$

Numerical evaluation for $\alpha = 0.5$, $\sigma = 2$, $\mu = 0.6$, $\bar{\rho} = 200$.

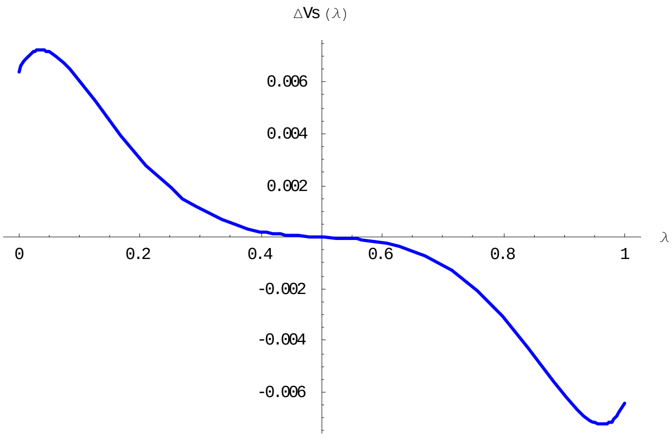
Figure 3 The transition process



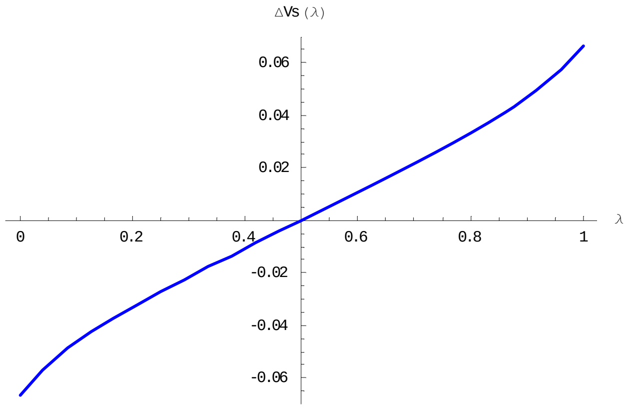
(*a*) $\phi = 0.1666$



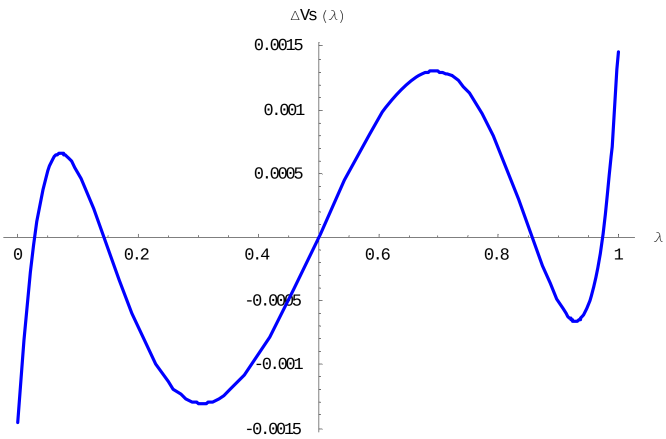
(*b*) $\phi = 0.1980$



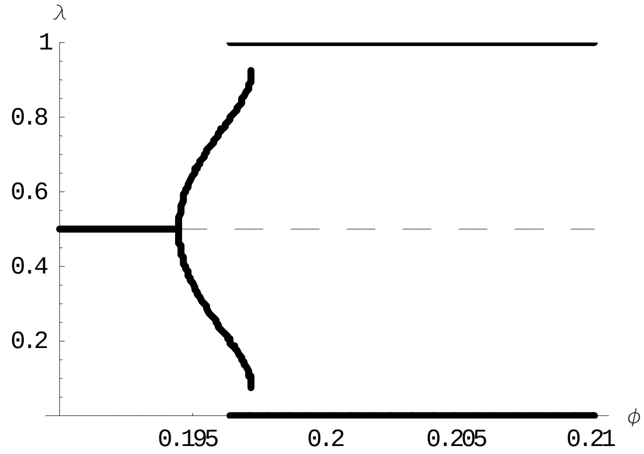
(*b*) $\phi = 0.1942$



(*c*) $\phi = 0.2222$



(*b*) $\phi = 0.1969$



Numerical evaluation for $\alpha = 0.5$, $\sigma = 2$, $\mu = 0.58$, $\bar{\rho} = 25$.

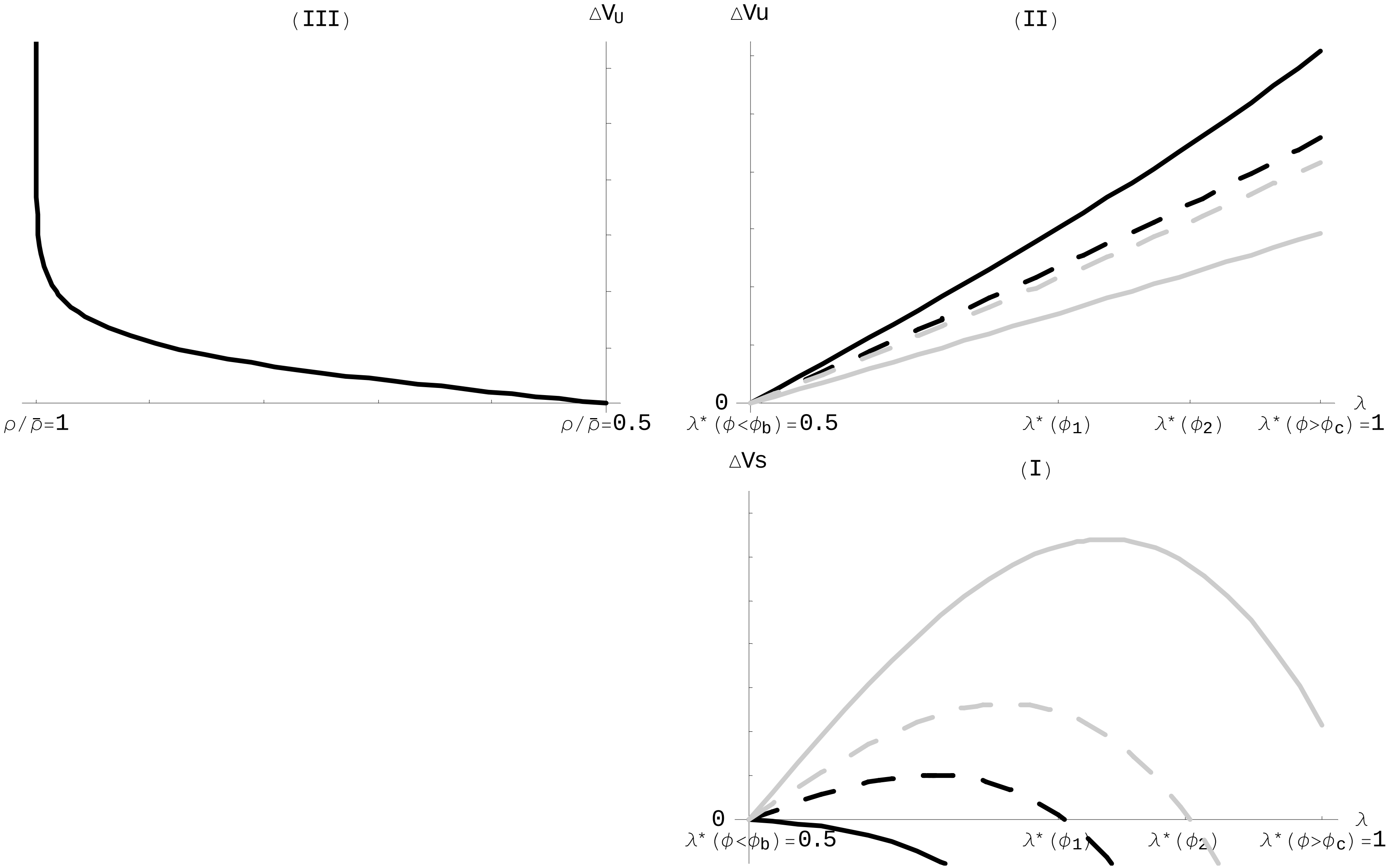


Figure 4 Trade costs, skilled and unskilled labor agglomeration

Figure 5 Bifurcation pattern of unskilled workers

Bold black lines show the bifurcation pattern of unskilled workers, if skilled labor agglomeration is a pitchfork. Gray lines show the distribution of skilled labor:

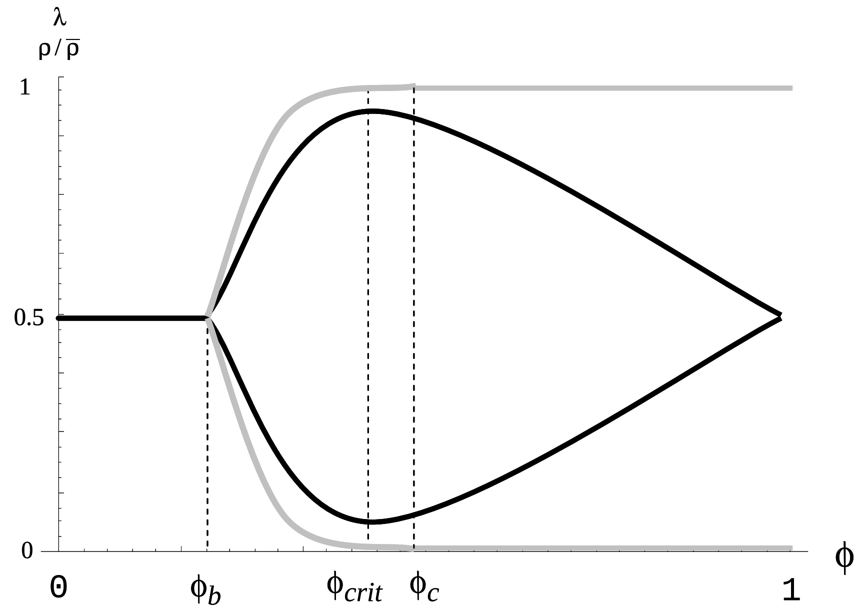
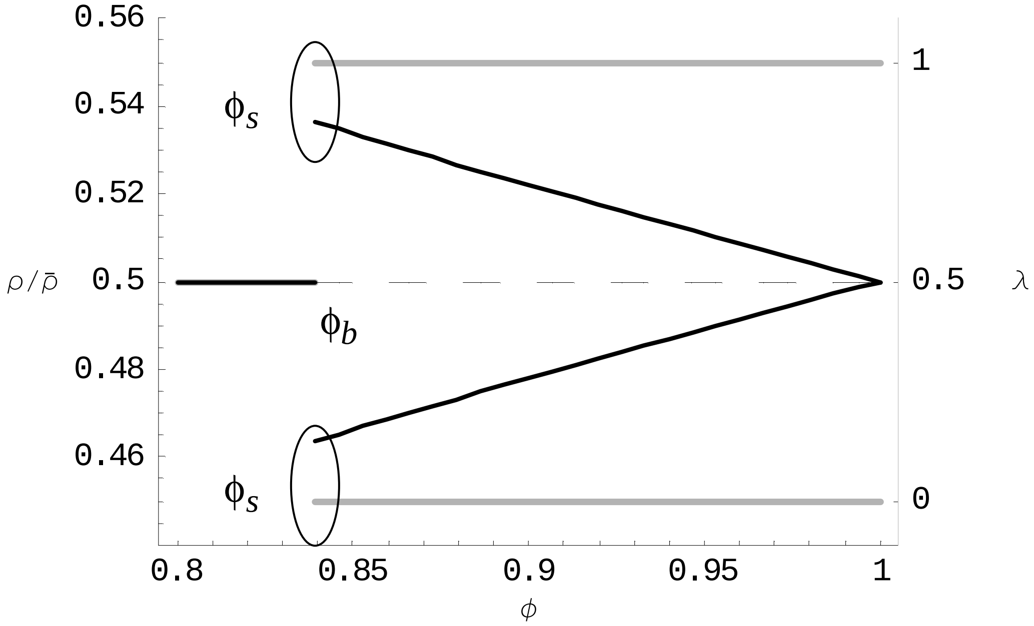


Figure 6 Bifurcation pattern of unskilled workers

Bold black lines show the geographical distribution of unskilled workers, if skilled labor agglomeration is catastrophic. The gray lines are the corresponding agglomeration pattern of skilled labor:



Numerical evaluation for $\alpha = 0.5$, $\sigma = 2$, $\mu = 0.6$, $\bar{\rho} = 200$.