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Acquisitions in a Patent Contest Model with Large and Small Firms *

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Abstract

Big companies and small innovation factories possess different advantages in a patent contest. While large firms typically have a better access to product markets, small firms often have a superior R&D efficiency. In this paper I model a patent contest with asymmetric firms. In a pre-contest acquisition game large firms bid sequentially for small firms to combine respective advantages. Sequential bidding allows the first large firm to wait strategically and let the other firm acquire. For low efficiencies this leads to an asymmetric market structure even though the initial situation is symmetric. Furthermore, acquisitions increase the chances for a successful innovation.

Keywords: patent contest, sequential acquisitions

JEL Classification: O31, L24, G34

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1 Introduction

In R&D intensive industries firms often do not rely only on their own research capabilities but use the results of competitors via licensing or even acquire other firms to directly benefit from their innovation efforts. Theoretical literature, based on transaction costs economies and property rights (Coase (1937), Arrow (1962)), argues that internal development and external sourcing are substitutes. Empirical evidence is mixed. Veugelers and Cassiman (1999) show that – especially large companies – use both internal and external sources for innovations. Roberts and Berry (1985) on the other hand provide evidence that acquisition of small technology based companies is often used to get access to innovations.

Different types of firms are likely to possess different advantages in a patent contest. While large firms typically have better access to product markets, small firms often have a superior R&D efficiency. The aim of this paper is to model these advantages and use them to explain acquisitions of small firms in innovative industries.

There are several empirical studies showing the superior R&D efficiency of small firms. Acs and Audretsch (1990) show empirically that small firms contribute as many innovations as large firms and – in terms of innovations per employee – outperform large firms. They use a dataset by the U.S. Small Business Administration consisting of 8,074 innovations introduced in the United States in 1982. Van Dijk et al. (1995), Rothwell (1989) and Santarelli and Sterlacchini (1990) provide similar results for Netherlands, UK and Italy, respectively.

Other studies from Cohen and Levin (1989) and Kamien and Schwartz (1982) show that small firms can keep up with large firms in the field of innovation. They spend relatively more on R&D and are more efficient in using this R&D for innovative output.

An explanation for this superior R&D efficiency may be the less bureaucratic atmosphere in small firms. Creative minds tend to dislike paperwork and may be frustrated and therefore less effective in a large firm where reporting and formal documentation is necessary (see Ernst and Vitt (2000)). Furthermore, the organization of internal communication – which can be less formal in small companies – encourages faster and more creative ways of problem solving and a fast adaption to changes in the external environment (see Rothwell and Zegveld (1982)).

Rothwell (1989) explains the better position in the product market of large firms with their comprehensive distribution networks and service facilities. Economies of scope puts them in a superior position. They can offer a range of complementary products and use their market power of existing products. Moreover, in case of a patent infringement they can afford to litigate to defend their patents and avoid misuse. What is more, the marketing of complex high technology goods often requires a qualified after sales service difficult to provide for small firms.

These different advantages immediately lead to the question of cooperations between these firms. Lindholm (1996a, 1996b) shows for a Swedish case that small firms actively

engage in being taken over by large firms to get access to international markets and use their global market presence. A recent analysis of an acquisition in the optics industry also shows the problem of small firms (Competition Commission (2004)). Bio Rad's microscopes business unit was acquired by Zeiss. The main reason for this acquisition was a patent dispute between Zeiss and Bio Rad which has been solved through this merger. Although Bio Rad's business unit was in possession of important licences in 3D light microscope systems the unit was not profitable. One of the reasons was that this unit was not providing own microscope stands but only equipping competitors' stands with their technique. A very critical detail in the commission's analysis is the conclusion that Bio Rad would not remain an independent supplier in the long run whether or not the merger takes place. The small business unit had not enough capacities to survive in this market alone. Therefore – if the merger was blocked – Bio Rad's microscopes business unit would either drop out of the market or eventually get acquired by a different competitor.

This paper is related to the patent races and patent contest literature. Reinganum (1989) provides a very good overview of that literature up to that point. More recent literature on patent races focuses on optimal patent design (Denicolò (1996)) and races with multiple prizes (Dorazelski (2003), Hörner (2004)). In the latter literature, firms' past R&D efforts influence their chances in a new race. This improves the description of the dynamics of innovation competition. Zachau (1987) analyzes mergers using a simple patent race model. Firms are symmetric at start but a merger is assumed to have synergetic effects and increases R&D efficiency of the merged firm. He allows for mergers of any numbers of firms and shows that in a n -firm industry the number of firms merging that maximizes merged firm's profits is typically less than n . Jost and van der Velden (2006) provide a similar analysis using a contest model proposed by Dixit (1987). As in Zachau's paper a merger is assumed to increase R&D efficiency of the merged firm. They show that only little efficiency gains are needed to make a bilateral merger profitable. They do also show that in the presence of knowledge spillovers more efficiency gains are needed to make a merger profitable. In contrast to both these papers, my model starts with an asymmetric situation where firms possess different advantages. Furthermore, I allow for a sequence of acquisitions to analyze the strategic effect of a merger.

Economic and management literature seeks to explain why mergers often happen in waves. There are nonstrategic explanations of merger waves that attribute the occurrence of these waves to exogenous factors like changes in competition policy (Stigler (1950), Bittlingmayer (1985) or the business cycle (van Wegberg (1994)). On the other hand there are strategic explanations how past acquisitions affect future mergers. A recent example from the telecommunications industry is the acquisition of Tele Atlas by TomTom¹ which at least partly triggered the acquisition of Tele Atlas' competitor Navteq

¹See for example Tele Atlas' press release under:
http://www.teleatlas.com/WhyTeleAtlas/Pressroom/PressReleases/TA_CT018103.

by Nokia.² Nilssen and Sørsgard (1998) analyze sequential acquisitions, in particular the strategic impact of an acquisition on subsequent merger decisions. The authors show in a linear Cournot model that the decision to merge or not to merge can either trigger or prevent follow-up merger. I do also analyze the strategic effect of an acquisition but with focus on the innovation market.

In this paper, I model a patent contest with asymmetric competitors. There are small research firms that have a better R&D efficiency and large firms that benefit from their better access to the product market. I use the Dixit (1987) model to describe the contest but in contrast to Jost and van der Velden (2006) I allow for initial differences in R&D efficiency. To the best of my knowledge there exists no paper that deals with this kind of asymmetry in an R&D contest.³ The firms compete for an exclusive patent that gives them a fixed return in the product market. In contrast to Jost and van der Velden, the amount of this return depends on the product market power of the winning firm. Prior to the innovation stage, I allow for acquisitions of small firms by large firms. Large firms sequentially bid for small firms. After an acquisition the merged firm combines the respective advantages of large and small firms.

The paper is organized as follows. The next section presents the patent contest and the way respective advantages for large and small firms are modeled. Then the game structure is described and the game is solved. The robustness section analyzes the impact of the timing on results of the game. The final section concludes, discusses limitations of my model and points out interesting topics for further research.

2 Model

Firms are competing for an exclusive prize V_i . They simultaneously choose an effort level z_i which affects their own - and thus competitors' - chances of winning the contest. The probability of winning is the own effort level divided by the sum of all efforts multiplied by an innovation efficiency parameter, thus

$$p_i = \alpha_i \cdot \frac{z_i}{\sum z_j} \quad i = 1, \dots, n.$$

Throughout the paper I assume that large firms have advantages in the product market by realizing a value V_L if they win the contest, while small firms only realize a value $V_S < V_L$. On the other hand, small firms have a greater chance of winning the contest with the same investment in R&D, i.e. $\alpha_S > \alpha_L$. I assume that overall efficiency is the

²See for example Nokia's press release under: <http://www.nokia.com/A4136002?newsid=-6233>.

³Asymmetry in an R&D race is often modeled with an incumbent and an entrant in a market, usually with some advantages for the incumbent. See for example Gilbert and Newberry (1982) or Harris and Vickers (1985).

same for large and small firms⁴

$$\alpha_S \cdot V_S = \alpha_L \cdot V_L$$

or

$$\alpha_S = x \cdot \alpha_L \text{ and } V_S = \frac{V_L}{x}, \quad (2.1)$$

where $x > 1$ is a measure of difference between large and small firms which is the source of efficiencies of mergers. Throughout the paper, x is therefore called efficiency parameter. In the following section I first describe the time structure of the game. Then the R&D games for different outcomes of the acquisition game are analyzed. Finally, the game is solved by backward induction to find sub-game perfect strategies for the bidding of large firms and acceptance decisions of small firms.

2.1 Sequential acquisitions

In this section I analyze sequential acquisitions. An acquisition is assumed to have the following effect: The large firm quits its own research and only uses the small firm's research facilities. Thus, the firm can now operate with the high research ability α_S and at the same time using the large firm's market power, meaning that winning the contest leads to a prize V_L . The number of active firms in the market is reduced by one. This way of modeling the acquisition can also be interpreted as ex ante licensing or contract research.⁶ The acquisition has both a positive and a negative effect on the merged company. The positive effect is that it operates now more efficiently than its competitors in the innovation or the product market. The negative effect is that the former two firms now only operate as one.⁷ Without efficiencies, this leads to a loss in terms of expected profits for the merged company. The reason is that before the merger, both the large and the small firm had a chance to win the contest while now there is only one firm competing in the contest. The outsiders benefit from the reduction of firms but they suffer from the created efficiencies by the merged company. A second acquisition by the same large firm has no effect on its R&D efficiency. It just reduces the number of competitors again and prevents the other large firm from enjoying efficiencies of an acquisition. It is assumed that a large firm cannot buy the other large firm for reasons of competition law.

In the acquisition game large firms successively bid for small firms. After each bid the small firm decides whether to accept the offer or not. I assume that the game starts with

⁴This assumption is made to keep the model analytically tractable and it is clearly restricting. However, from my results tendencies for more general cases can be deviated. If overall efficiency is different, the efficiency effect of an acquisition is increased.

⁵ To simplify notation in the following $\alpha_S := \alpha$ and $V_S := V$ is used.

⁶Important for my way of modeling is that the small firm still performs R&D in the same way as before the acquisition. Otherwise, the superior R&D efficiency would be lost.

⁷Without efficiencies investment of the merged firm is smaller than combined investment of the two former independent firms. The result is comparable to the output reduction and subsequent reduced profit of a merged firm in the Salant et al. (1983) model without cost efficiencies.

the first large firm's bid for the first small firm. Each large firm only bids one time for each small firm. The acquisition game ends either when both small firms are acquired or both large firms made their bids for both small firms.⁸ In the following I call the large firms L_1 and L_2 and the small firms S_1 and S_2 . After the acquisition game the remaining independent firms make their investments in the patent contest. The game structure is summarized in figure 1. Each decision point is numbered to make reference in the equilibrium section. Large firms have the possibility to bid continuously, $p_{ij} \in [0, \infty)$. Strategies for small firms are A(cepting) and D(eny)ing) the offers.

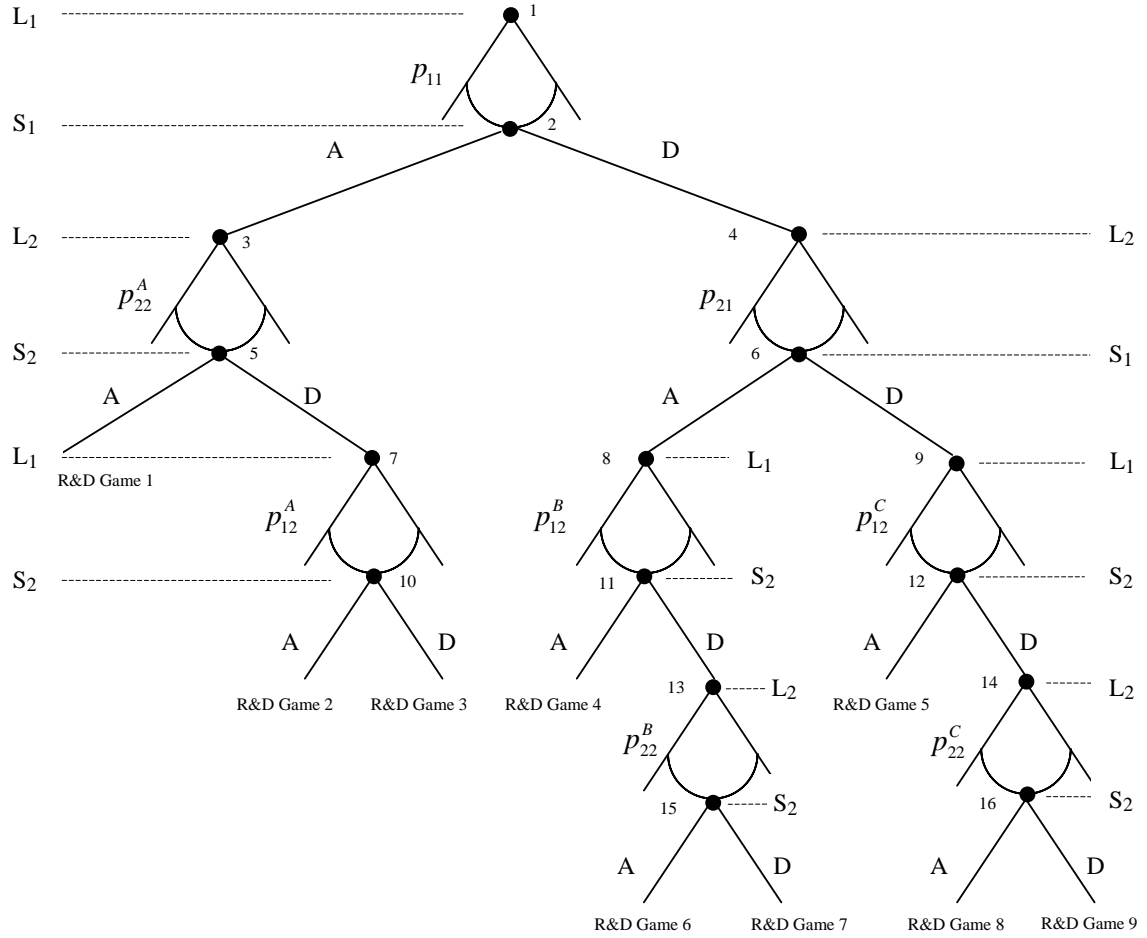


Figure 1: Game Structure

⁸The resulting market structure in equilibrium is the same for most parameter values if the acquisition game is simplified in a way that large firms just pay the current expected profits of small firms when it wants to acquire. Thus, if small firms do not expect further changes in the market structure and act myopic in the acquisition game.

2.1.1 R&D Games

In this section optimal investment strategies and resulting expected profits of the R&D game for all possible outcomes of the acquisition game are derived. Firms are assumed to maximize their expected profits

$$E[\pi_i] = p_i \cdot V_i - z_i = \alpha_i \cdot \frac{z_i}{\sum z_j} \cdot V_i - z_i \quad i = 1, \dots, n \quad (2.2)$$

which yields the first order conditions:

$$\alpha_i \cdot \frac{\sum_{j \neq i} z_j}{(\sum_j z_j)^2} \cdot V_i - 1 \stackrel{!}{=} 0 \quad i = 1, \dots, n. \quad (2.3)$$

Firms thus increase their investments in R&D to improve their chances to win the contest until marginal benefits of the investments equal marginal costs.

If no firm decides to merge, there are 4 active firms in the market. Two identical small and large firms, respectively. This is the situation in R&D game 9. With (2.1) first order conditions (2.3) can be solved for

$$z_i = \frac{\alpha \cdot V}{16} \quad i = 1, \dots, 4.$$

Thus, both types of firms invest the same in R&D as their overall efficiency is the same. Inserting equilibrium investments in the profit functions (2.2) yields⁹

$$E[\pi_S^{R\&D_9}] = E[\pi_L^{R\&D_9}] = \frac{\alpha V}{16}.$$

In R&D games 1 and 4 each of the small firms is acquired by one large firm, i.e. we have a situation where only 2 identical firms remain in the market. Both have small firms' R&D efficiency α and large firms' product market efficiency $V_L = x \cdot V$. Using these parameters in (2.3) yields the first order conditions

$$\alpha \frac{z_j}{(z_i + z_j)^2} - \frac{1}{x \cdot V} \stackrel{!}{=} 0 \quad i = 1, 2. \quad (2.4)$$

With $z_1 = z_2$ due to symmetry equations (2.4) can be solved for

$$z_1 = z_2 = \frac{\alpha x V}{4}.$$

Resulting expected profits are

$$E[\pi_M^{R\&D_1}] = \frac{\alpha x V}{4}.$$

Small firms get the price the large firms bid in the acquisition game as payoffs.

In R&D games 3, 5, 7 and 8 only one large firm acquires one small firm, the other 2 firms stay independent. The respective first order conditions for the small, merged and

⁹I use the indices S, L and M to denote small, large and merged firms respectively.

large firm are

$$\begin{aligned}\alpha \frac{z_L + z_M}{(z_S + z_L + z_M)^2} - \frac{1}{V} &\stackrel{!}{=} 0 \\ \alpha \frac{z_S + z_L}{(z_S + z_L + z_M)^2} - \frac{1}{xV} &\stackrel{!}{=} 0 \\ \frac{\alpha}{x} \frac{z_S + z_M}{(z_S + z_L + z_M)^2} - \frac{1}{xV} &\stackrel{!}{=} 0.\end{aligned}$$

Solving for z_S, z_L and z_M yields

$$\begin{aligned}z_S &= z_L = \frac{2\alpha x V}{(1 + 2x)^2} \\ z_M &= \frac{2\alpha x V}{(1 + 2x)^2} (2x - 1).\end{aligned}$$

It is easy to see that the merged firm invest more than its competitors. Resulting profits are

$$\begin{aligned}E[\pi_S^{R\&D_3}] &= E[\pi_L^{R\&D_3}] = \frac{\alpha V}{(1 + 2x)^2} \\ E[\pi_M^{R\&D_3}] &= \frac{\alpha x V}{(1 + 2x)^2} (2x - 1)^2.\end{aligned}$$

As with investments, the expected profit of the merged firm is larger since $x > 1$. The acquired small firm gets the price bid in the acquisition game as payoff.

In R&D games 2 and 6 one large firm acquires both small firms. There is thus one firm with high and one firm with low R&D efficiency in the market. Both firms have high product market efficiency. That yields the first order conditions

$$\begin{aligned}\frac{\alpha}{x} \frac{z_M}{(z_L + z_M)^2} - \frac{1}{xV} &\stackrel{!}{=} 0 \\ \alpha \frac{z_L}{(z_L + z_M)^2} - \frac{1}{xV} &\stackrel{!}{=} 0.\end{aligned}$$

Solving for z_L and z_M yields

$$\begin{aligned}z_L &= \frac{\alpha x V}{(1 + x)^2} \\ z_M &= \frac{\alpha x^2 V}{(1 + x)^2}.\end{aligned}$$

Resulting expected profits are

$$\begin{aligned}E[\pi_L^{R\&D_2}] &= \frac{\alpha V}{(1 + x)^2} \\ E[\pi_M^{R\&D_2}] &= \frac{\alpha x^3 V}{(1 + x)^2}.\end{aligned}$$

As before in the triopoly game, the more efficient firm invests more in R&D and has higher expected profits. Small firms get the price the acquiring large firm bid in the acquisition game as payoffs.

If a large firm acquired one or two small firms, the price it payed has to be subtracted from its expected profits in order to make payoffs comparable.

2.1.2 Equilibria

Large firms decide on their bid in each decision point; small firms make their acceptance decisions.¹⁰ In the acquisition game there are two critical barriers for the bids of large firms and acceptance decisions of small firms. The first is the profit of an independent firm if no acquisition takes place: $p_N = \frac{\alpha V}{16}$, the second is the profit of an outsider in a triopoly: $p_O = \frac{\alpha V}{(1+2x)^2}$.¹¹ These are the two possible expected profits a small firm can achieve if it has not been acquired and remains an independent firm in the R&D game.

Proposition 1. *In equilibrium the acquisition game ends in a symmetric duopoly with accepted bids $p_{11} = p_{22} = p_O$ for $x \geq 1.18731$, in an asymmetric duopoly with accepted bids $p_{21} = p_{22} = p_O$ for $1.18731 > x \geq 1.05678$ and accepted bids $p_{21} = p_N$, $p_{22} = p_O$ for $1.0333 > x$, and in a triopoly where only S_2 is acquired for $p_{22} = p_N$ if $1.05678 > x \geq 1.0333$.*

Proof. To prove proposition 1, the acquisition game is solved by backward induction. At decision point 16 S_2 has to decide whether to accept the price p_{22}^C offered by L_2 or denying the offer and getting a payoff of $\frac{\alpha V}{16}$. Thus, S_2 accepts every offer $p_{22}^C \geq p_N$ and denies any other offer.

Expecting this decision from S_2 , L_2 makes his offer in decision point 14. If S_2 denies the offer, the game ends without any acquisition and L_2 gets the payoff $\frac{\alpha V}{16}$. If S_2 accepts, L_2 gets the payoff of R&D game 8, the triopoly with one merged, one small and one large firm. If L_2 decides to acquire, it makes the lowest offer S_2 accepts, i.e. $p_{22}^C = p_N$. Thus, L_2 decides to make an offer if

$$\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - \frac{\alpha V}{16} \geq \frac{\alpha V}{16}.$$

This inequality holds for $x \geq 1.0333$.

If S_2 accepts the offer in decision point 12, it gets the payoff p_{12}^C . If not, it either gets $p_{22}^C = p_N$ if $x \geq 1.0333$ resulting from an acceptable offer by L_2 or $\frac{\alpha V}{16}$ if $x < 1.0333$ as an independent firm in R&D game 9. Thus, S_2 accepts any offer by L_1 that is greater or equal than p_N .

In decision point 9 L_1 has to offer at least $p_{12}^C = p_N$ if it wants S_2 to accept. If S_2 accepts the bid, L_1 's payoff is

$$\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - p_{12}^C.$$

If S_2 denies the offer, L_1 gets the payoff of an outsider in R&D game 8 when $x \geq 1.0333$ i.e. in the situation where L_2 decides to acquire in decision point 14. This payoff is

$$\frac{\alpha V}{(1+2x)^2}.$$

¹⁰If a firm is indifferent between acquiring and not acquiring, I assume that the firm is acquiring; if a small firm is indifferent between accepting and denying an offer, I assume it is accepting.

¹¹Note that $p_N \leq p_O \Leftrightarrow x \leq 1.5$. Thus, small firms only prefer p_N to p_O for very high efficiencies.

If $x < 1.0333$, S_1 gets $\frac{\alpha V}{16}$ as payoff. Consider first the case $x \geq 1.0333$: L_1 acquires if

$$\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - \frac{\alpha V}{16} \geq \frac{\alpha V}{(1+2x)^2}.$$

This inequality holds for $x \geq 1.10933$. In the case $x < 1.0333$ L_1 acquires if

$$\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - \frac{\alpha V}{16} \geq \frac{\alpha V}{16}.$$

This inequality never holds since $x < 1.0333$. Thus, L_1 decides to acquire in decision point 9 if $x \geq 1.10933$.

In decision point 15 S_2 gets the payoff of an outsider in R&D game 7 if it denies the offer. Therefore S_2 accepts every offer $p_{22}^B \geq p_O$.

In decision point 13 L_2 has already acquired S_1 for p_{21} . If L_2 acquires, S_1 , the situation is an asymmetric duopoly where L_2 has acquired both small firms. L_2 's payoff if it acquires is

$$\frac{\alpha x^3 V}{(1+x)^2} - p_{21} - p_{22}^B,$$

where $p_{22}^B = p_O$ is the lowest offer S_2 accepts. If L_2 does not acquire, its payoff is the payoff of the merged firm in R&D game 7 minus the price payed for S_1 :

$$\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - p_{21}.$$

Thus, L_2 acquires if

$$\frac{\alpha x^3 V}{(1+x)^2} - p_{21} - \frac{\alpha V}{(1+2x)^2} \geq \frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - p_{21}.$$

This inequality holds for every $x \geq 1$ and therefore L_2 always bids $p_{22}^B = p_O$ and acquires S_2 in decision point 13.

In decision point 11 S_2 accepts any offer $p_{12}^B \geq p_O$.

In decision point 8 L_1 has to offer least $p_{12}^B = p_O$ if it wants S_2 to accept. If S_2 accepts, the resulting situation is R&D game 4, a symmetric duopoly. If S_2 denies the offer, L_1 gets the profit of an outsider in an asymmetric duopoly in R&D game 6 since L_2 always makes an offer that S_2 accepts in decision point 13. L_1 therefore decides to offer $p_{12}^B = p_O$ and to acquire S_2 if

$$\frac{\alpha x V}{4} - \frac{\alpha V}{(1+2x)^2} \geq \frac{\alpha V}{(1+x)^2}.$$

This inequality holds for $x \geq 1.18731$.

If S_1 denies the offer in decision point 6, the acquisition game ends in R&D game 8 for $x \geq 1.0333$ and in R&D game 9 for $x < 1.0333$. Thus S_1 accepts every offer $p_{21} \geq p_O$ for $x \geq 1.0333$ and every offer $p_{21} \geq p_N$ for $x < 1.0333$.

If L_2 decides to acquire S_1 in decision point 4, the acquisition game ends in R&D game 4 for $x \geq 1.18731$ and in R&D game 6 for $x < 1.18731$. If L_2 does not acquire S_1 , the acquisition game ends in R&D game 5 for $x \geq 1.0933$, in R&D game 8 for

$1.10933 > x \geq 1.0333$ and in R&D game 9 for $x < 1.0333$. Consider first the case $x \geq 1.18731$: The payoff for L_2 in R&D game 4 is $\frac{\alpha x V}{4} - p_{21}$; in R&D game 5 it is $\frac{\alpha V}{(1+2x)^2}$. The payoff in R&D game 4 is larger for the lowest accepted offer p_{21} . For $x \geq 1.18731$ L_2 therefore bids $p_{21} = p_O$ and acquires S_1 . The next case to consider is $1.18731 > x \geq 1.10933$: The payoff for L_2 if it acquires S_1 is given by R&D game 6 and is thus $\frac{\alpha x^3 V}{(1+x)^2} - p_{21} - p_{22}^B$, where $p_{22}^B = p_O$ and the lowest accepted offer is $p_{21} = p_O$. If L_2 does not acquire, its payoff is given by R&D game 5 and it is $\frac{\alpha V}{(1+2x)^2}$. Thus, L_2 acquires if

$$\frac{\alpha x^3 V}{(1+x)^2} - 2 \cdot \frac{\alpha V}{(1+2x)^2} \geq \frac{\alpha V}{(1+2x)^2}.$$

This inequality holds for every $1.18731 > x \geq 1.10933$. The third case is $1.10933 > x \geq 1.0333$: The payoff for L_2 if it acquires is the same as in the previous case. If L_2 does not acquire, its payoff is given by R&D game 8 and it is $\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - p_{22}^C$ with $p_{22}^C = p_N$. Thus, L_2 acquires if

$$\frac{\alpha x^3 V}{(1+x)^2} - 2 \cdot \frac{\alpha V}{(1+2x)^2} \geq \frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - \frac{\alpha V}{16}.$$

This inequality holds for $x \geq 1.05678$. The last case to consider is $x < 1.0333$. The payoff for L_2 if it acquires is again given by R&D game 6, but L_2 only has to pay $p_{21} = p_N$ to make S_1 accept the offer. If L_2 does not acquire, its payoff is given by R&D game 9 and it is $\frac{\alpha V}{16}$. Thus, L_2 acquires if

$$\frac{\alpha x^3 V}{(1+x)^2} - \frac{\alpha V}{(1+2x)^2} - \frac{\alpha V}{16} \geq \frac{\alpha V}{16}.$$

This inequality holds for every $x < 1.0333$. Thus, in decision point 4 L_2 bids $p_{21} = p_O$ and acquires S_1 for $x \geq 1.05678$, does not acquire S_1 for $1.05678 > x \geq 1.0333$ and bids $p_{21} = p_N$ and acquires S_1 for $x < 1.0333$.

In decision point 10 S_2 accepts every offer that gives it a higher payoff than the payoff as an outsider in R&D game 3. Thus, S_2 accepts every offer $p_{12}^A \geq p_O$.

In decision point 7 L_1 has already acquired S_1 for p_{11} . If L_1 acquires S_2 , L_1 is the merged firm in R&D game 2 and has a payoff $\frac{\alpha x^3 V}{(1+x)^2} - p_{12}^A - p_{11}$, where the lowest accepted offer is $p_{12}^A = p_O$. If it decides not to acquire, its payoff is the one of the merged firm in a triopoly, i.e. $\frac{\alpha x V}{(1+2x)^2} (2x-1)^2 - p_{11}$. Thus, L_1 acquires if

$$\frac{\alpha x^3 V}{(1+x)^2} - \frac{\alpha V}{(1+2x)^2} \geq \frac{\alpha x V}{(1+2x)^2} (2x-1)^2.$$

This inequality always holds and thus L_1 always bids $p_{12}^A = p_O$ and acquires S_2 in decision point 7.

In decision point 5 S_2 accepts every offer $p_{22}^A \geq p_O$.

In decision point 3 L_2 can decide to acquire S_2 and create a symmetric duopoly in R&D game 1 or not to acquire S_2 and become the outsider in the asymmetric duopoly in R&D game 2. If L_2 acquires, its payoff is $\frac{\alpha x V}{4} - p_{22}^A$, where the lowest accepted offer

is $p_{22}^A = p_O$. If L_2 does not acquire, its payoff is $\frac{\alpha V}{(1+x)^2}$. Thus, L_2 acquires if

$$\frac{\alpha x V}{4} - \frac{\alpha V}{(1+2x)^2} \geq \frac{\alpha V}{(1+x)^2}.$$

This inequality holds for $x \geq 1.18731$ and therefore L_2 acquires S_2 for $x \geq 1.18731$ in decision point 3.

If S_1 denies the offer in decision point 2, the acquisition game ends with a payoff of $\frac{\alpha V}{(1+2x)^2}$ for S_1 .¹² Thus, S_1 accepts every offer $p_{11} \geq p_O$.

If L_1 decides to acquire in decision point 1, the acquisition game ends with R&D game 1 for $x \geq 1.18731$ and with R&D game 2 if $x < 1.18731$. If L_1 decides not to acquire, the acquisition game ends with R&D game 4 for $x \geq 1.18731$, with R&D game 6 for $1.18731 > x \geq 1.05678$ or $x < 1.0333$ and with R&D game 8 for $1.05678 > x \geq 1.0333$. Consider first the case $x \geq 1.18731$: L_1 's payoff if it acquires is then given by R&D game 1 minus the price p_{11} paid for S_1 . The lowest price accepted by S_1 is p_O . If L_1 decides not to acquire in decision point 1, the game ends in R&D game 4 where L_1 's expected profit is the same and the price paid to acquire S_2 is p_O . Thus, for $x \geq 1.18731$ L_1 is indifferent between acquiring now or later and due to our assumption given in footnote 6 L_1 acquires S_1 for $p_{11} = p_O$. For all other cases L_1 's payoff if it acquires is given by R&D game 2 and L_1 has to pay $p_{11} = p_{12}^A = p_O$. Thus, L_1 's payoff if it acquires is

$$\frac{\alpha x^3 V}{(1+x)^2} - 2 \frac{\alpha V}{(1+2x)^2}. \quad (2.5)$$

L_1 's payoff if it decides not to acquire depends on x . If $1.18731 > x \geq 1.05678$ or $x < 1.0333$, L_1 's payoff is given by R&D game 6 and it is

$$\frac{\alpha V}{(1+x)^2}.$$

This is larger than the payoff given by (2.5) for $x < 1.22364$. If $1.05678 > x \geq 1.0333$, L_1 's payoff is given by R&D game 8 and it is

$$\frac{\alpha V}{(1+2x)^2}.$$

This is larger than the payoff given by (2.5) for $x < 1.09021$. Thus, in decision point 1 L_1 bids $p_{11} = p_O$ and acquires S_1 for $x \geq 1.18731$ and does not acquire S_1 for $x < 1.18731$. \square

In equilibrium the acquisition game ends thus in a symmetric duopoly in R&D game 1 for $x \geq 1.18731$ with the two firms $\{L_1, S_1\}$ and $\{L_2, S_2\}$, in an asymmetric duopoly in R&D game 6 for $1.18731 > x \geq 1.05678$ or $x < 1.0333$ with the two firms $\{L_1\}$ and $\{L_2, S_1, S_2\}$ or in a triopoly in R&D game 8 for $1.05678 > x \geq 1.0333$ with the three firms $\{L_1\}, \{S_2\}$ and $\{L_2, S_2\}$.

¹²Either acquired by L_2 for $x \geq 1.05678$ or $x < 1.0333$ or as an independent firm in a triopoly for $1.05678 > x \geq 1.0333$.

The resulting sub-game perfect equilibria can be characterized as follows. For high efficiencies ($x \geq 1.18731$) large firms bid p_O in every decision point (except for the decision point 14 where L_2 always bids p_N) and the small firms accept every offer greater or equal than this offer. The reason for these equilibrium strategies is that for large efficiencies every merger is beneficial for the merging parties and it would also be beneficial to prevent the other firm from enjoying these efficiencies (off the equilibrium path strategies). For lower efficiencies large firms prefer the respectively other large firm to acquire. The positive externality (reduction of firms in the R&D game) exceeds the negative externality that the merged firm is more efficient. For intermediate efficiencies ($1.18731 > x \geq 1.05678$) L_1 therefore never acquires in decision point 1. L_2 has the disadvantage of making the final offer to S_1 . It is more profitable for L_2 to acquire both firms than to wait and let the acquisition game end up with 3 or 4 independent firms. On the equilibrium path L_2 therefore always bids p_O in decision points 4 and 13 and acquires S_1 and S_2 . For even lower efficiencies ($1.05678 > x \geq 1.0333$) it is too costly for L_2 to acquire both small firms for p_O . However, S_1 is not accepting the lower offer p_N in decision point 6 knowing that if it denies the offer L_2 later acquires S_2 (for p_N) giving S_1 profits p_O . Therefore the game ends in a triopoly for these parameter values. If efficiencies are in the lowest segment ($1.0333 > x$), it is no longer profitable for L_2 to acquire one single small firm, even for p_N . However, it is still profitable for L_2 to acquire both small firms (one for p_N , the other for p_O). Expecting that L_1 will not acquire S_2 if it has not already acquired S_1 , S_1 is accepting the offer p_N in decision point 6 in this case and the game ends in an asymmetric duopoly again. This result is summarized in the following proposition.

Although consumer welfare is not explicitly included in the model, it is possible to compare potential outcomes of the acquisition game by the probability of a successful innovation. This probability for the 4 possible outcomes of the acquisition game is

$$p_{ges}^1 = \alpha \cdot \left(\frac{\frac{\alpha x V}{4}}{2 \frac{\alpha x V}{4}} \right) \cdot 2 = \alpha$$

in the symmetric duopoly,

$$p_{ges}^2 = \left(\frac{\alpha}{x} \cdot \frac{\alpha x V}{(1+x)^2} + \alpha \cdot \frac{\alpha x^2 V}{(1+x)^2} \right) \frac{1}{\frac{\alpha x V}{(1+x)^2}} = \frac{\alpha(1+x^2)}{x(1+x)}$$

in the asymmetric duopoly,

$$\begin{aligned} p_{ges}^3 &= \left(\left(\alpha + \frac{\alpha}{x} \cdot \frac{2\alpha x V}{(1+2x)^2} \right) + \alpha \cdot \frac{2\alpha x V}{(1+2x)^2} (2x-1) \right) \frac{1}{2 \cdot \frac{2\alpha x V}{(1+2x)^2} + \frac{2\alpha x V}{(1+2x)^2} (2x-1)} \\ &= \frac{\alpha(1+2x^2)}{x(1+2x)} \end{aligned}$$

in the triopoly and

$$p_{ges}^4 = 2 \cdot \left(\alpha + \frac{\alpha}{x} \right) \frac{\frac{\alpha \cdot V}{16}}{4 \cdot \frac{\alpha \cdot V}{16}} = \frac{\alpha(1+x)}{2x}$$

in the initial situation with four independent firms.

Proposition 2. *The Probability of a successful innovation is highest in the symmetric duopoly, followed by the triopoly and the asymmetric duopoly. Chances are lowest in the initial situation with four independent firms.*

Proof. The elements of the proposition are shown consecutively:

$$\begin{aligned} p_{ges}^1 - p_{ges}^3 &= \alpha - \frac{\alpha(1+2x^2)}{x(1+2x)} = \frac{\alpha(x-1)}{x(1+2x)} > 0, \\ p_{ges}^3 - p_{ges}^2 &= \frac{\alpha(1+2x^2)}{x(1+2x)} - \frac{\alpha(1+x^2)}{x(1+x)} = \frac{\alpha(x-1)}{1+3x+2x^2} > 0, \\ pp_{ges}^2 - p_{ges}^4 &= \frac{\alpha(1+x^2)}{x(1+x)} - \frac{\alpha(1+x)}{2x} = \frac{\alpha(x-1)^2}{2x(1+x)} > 0. \end{aligned}$$

Thus, $p_{ges}^1 > p_{ges}^3 > p_{ges}^2 > p_{ges}^4$. □

The intuition for this result is that the total probability of a successful innovation is influenced by two effects in the model: First, a higher investment by firms with a high R&D efficiency and second, elimination of firms with low R&D efficiency in the R&D games. A policy that favors at least one acquisitions is therefore beneficial for innovation in this model.

For all asymmetric cases, payoffs are larger for L_1 than for L_2 . In the symmetric case, payoffs are identical. Thus, there is a first-mover advantage in the game for large firms. Small firms' payoffs are identical for $x \geq 1.05678$. If $1.05678 > x > 1.0333$, S_1 can profitably deny a low offer of L_2 and get a higher profit as an independent firm than S_2 that gets acquired in equilibrium. For even lower efficiencies S_1 can no longer profitably deny a low offer and the situation turns around as then the price of S_2 that gets acquired later is higher. Thus, there is no clear first-mover advantage or disadvantage for small firms.

2.2 Robustness

In this section I check for the sensitivity of results to the proposed timing structure in the acquisition game. I first generally discuss the strategic effect of the known timing structure, especially that L_1 firm knows that there is still a bidding decision of L_2 after its second bid while L_2 knows that the acquisition game ends after its second bid. Then I analyze an alternative modeling with simultaneous acquisition decisions.

2.2.1 Timing general

In this section I analyze the impact of the timing structure on the game's results. I discuss the influence of timing on firms' decisions.

In section 2.1.2 I found that payoffs for L_1 and L_2 differ for low efficiencies and there is a first-mover advantage since L_1 can simply wait to let L_2 acquire one or both small firms and benefit from the externalities of the merger. It is crucial for L_1 's strategic decision to wait that it knows that L_2 cannot simply wait as well but will acquire either

one or both small firms since this is more profitable than not acquiring at all and leaving the situation with four independent firms for L_2 . If there was no such final bidding round in the acquisition game, a hold up problem between L_1 and L_2 would occur: The first large firm to acquire would make less expected profits and thus both firms would wait for the other firm to move.

2.2.2 Simultaneous Acquisitions

A way to react to the problem of a first-mover advantage in the acquisition game – which is somehow critical since firms are ex ante symmetric – is to let large firms decide simultaneously whether to acquire one of the small firms or not.¹³ That is described in this section.

If both large firms decide to acquire, the number of firms is reduced from 4 to 2. Both merged firms enjoy the same efficiencies and the resulting situation is symmetric. If only one firm decides to acquire, we have an asymmetric situation with one large, one small and the merged firm. If no large firm decides to acquire, the situation remains unchanged. Investments and expected profits depend on the resulting situation.

Both large firms acquire

In this situation only 2 identical firms remain in the market. Both have small firms' R&D efficiency α_S and large firms' product market efficiency $V_L = x \cdot V_S$. The situation is equivalent to the situation in R&D game 1 described above. Therefore resulting profits are

$$\frac{\alpha x V}{4}.$$

Only one large firm acquires

This is an asymmetric situation with one large one small and the merged firm. The situation is equivalent to the R&D game 3 described above. Resulting profits are thus

$$\frac{\alpha x V}{(1 + 2x)^2} (2x - 1)^2$$

for the merged firm and

$$\frac{\alpha V}{(1 + 2x)^2}$$

for the outsiders.

No large firm acquires

In this case, there are 4 firms in the market, two small firms with higher R&D efficiency and two large firms with superior product market efficiency. The situation is equivalent to R&D game 9 and therefore expected profits are

$$\frac{\alpha V}{16}$$

both for large and small firms.

¹³I abstract from the coordination problem which small firm is acquired by which large firm.

2.2.3 Equilibria with simultaneous acquisitions

Taking optimal decisions in the investment stage as given, expected profits for the 3 situations described above are given in the following matrix. To make results comparable I show the combined profits of one large and one small firm in the non-acquisition cases.¹⁴ Payoffs are summarized in the following matrix. Looking at the payoffs for L_1 if L_2

		L_2	
		Acquire	Not Acquire
L_1	Acquire	$\frac{\alpha x V}{4}$ $\frac{\alpha x V}{4}$	$\frac{\alpha x V}{(1+2x)^2}(2x-1)^2$ $2\frac{\alpha V}{(1+2x)^2}$
	Not Acquire	$2\frac{\alpha V}{(1+2x)^2}$ $\frac{\alpha x V}{(1+2x)^2}(2x-1)^2$	$2\frac{\alpha V}{16}$ $2\frac{\alpha V}{16}$

Table 1: Payoffs for the simultaneous acquisition game

acquires, we see that

$$\frac{\alpha x V}{4} - 2\frac{\alpha V}{(1+2x)^2} = \frac{1}{4}\alpha V \left(x - \frac{8}{(1+2x)^2} \right) > 0$$

for $x > 1$. Thus, it is always optimal to acquire if the other large firm is acquiring. If L_2 does not acquire, we see that

$$\frac{\alpha x V}{(1+2x)^2}(2x-1)^2 - 2\frac{\alpha V}{16} = \alpha V \left(\frac{(2x-1)^2 x}{(1+2x)^2} - \frac{1}{8} \right) > 0$$

for $x > 1.0333$. Thus, for $x > 1.0333$ "Acquire" is a strict dominant strategy for large firms. If $x < 1.0333$, the game has two Nash-Equilibria in pure Strategies (Acquire, Acquire) and (Not Acquire, Not Acquire). The intuition for this result is that with enough efficiencies a merger by itself is profitable and therefore each of the large firms wants to acquire a small firm. The result is therefore comparable to the one with sequential acquisitions. If efficiencies are low, the negative effect that two former independent firms now operate as one overweighs the efficiency effect such that firms prefer not to merge unless the other firms merge as well.

3 Conclusion

This paper analyzes innovation incentives for large and small firms in different strategic contexts. I model a patent contest with 4 firms and a fixed prize for the winner. Prior to the innovation stage large firms play a merger game. In the first setup, large firms sequentially decide on acquiring small firms in order to combine the respective advantages of each type of firm. While the reduction of competitors exhibits a positive effect on the outsiders of that merger, the merged firm is now as least as good as the competitors in one sector and better in the other. I show that if the merger creates large efficiencies firms prefer to merge. For low efficiencies on the other hand large firms prefer to wait

¹⁴This is equivalent to an analysis with a bidding price p_0 if the other large firm acquires and a price p_N if the other large firm does not acquire.

and let the other firm acquire. In the sequential acquisition game there is a first-mover advantage since the second large firm prefers to acquire at least one small firm than to leave the market unchanged with four independent firm. If the acquisition decision is simultaneous, results are comparable.

My model has several limitations. Firstly, I use a contest model in the R&D game. This simplification allows to focus on the different effects a merger has on different types of firms and to get closed form solutions in all analyzed cases. This comes to the expense that timing effects of R&D investments cannot be analyzed.

Secondly, I assume product market profits to be fixed. While this simplification is used in various papers analyzing R&D incentives, it is still a critical assumption. This is particularly relevant as I model acquisitions, i.e. events that change the market structure and are therefore likely to have an effect on the product market and thus on firms' profits.

Finally, acquisitions are modeled as simple as possible. It would be possible to account for inefficiencies in the acquisition and not to give the merged firm the complete advantages of both types of firms in the contest. While these changes would lead to a dramatic increase in the complexity of the model – particularly with regards to the sequential acquisition game – I think that it would not lead to more insights. On the contrary, it could hinder the insights with respect to the effects I want to focus on.

The results of my paper lead to questions concerning the organization of R&D in large and small companies. Why do small and large firms have these advantages and how can the firms strategically act to improve their advantages or to reduce disadvantages? Furthermore, I totally abstracted from reorganization issues connected with the merger. What exactly happens after a merger? Which advantages can be kept, which might be lost and what trade offs does the merged firm face. A theoretical model that answers these questions would be very helpful to improve understanding of different R&D approaches. What is more, it could help to allocate (public) R&D funds to the most promising approaches.

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