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Problem in Federations**

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Restricted Bailouts and the Commitment Problem in Federations*

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Abstract

Standard models of soft budget constraints consider bailouts as pure monetary transfers. However, in practice often additional obligations or restrictions, such as savings goals, are linked to monetary bailouts. This paper analyzes in a model of a federation if such restrictions change economic outcomes in an soft budget constraint environment and under what circumstances they can increase welfare as compared to pure soft budget and hard budget regimes. We find that restrictions generally harden budget constraints, but not necessarily increase welfare. The evaluation crucially depends on the tax endowment of the central government and on the shape of preferences.

JEL classification: H77, H74, H63; Keywords: bailouts, soft budget constraints, federalism

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1 Introduction

Over the past decade, the soft budget constraint (SBC) phenomenon received a growing attention by public economists. At the heart of this phenomenon there is a lack of commitment of a supporting organization to keep the receiver of the support to an ex-ante agreed budget. In federations this is typically a higher level government such as a central government bailing out lower level governments such as states or municipalities with extraordinary grants to prevent them from the consequences of a fiscal crisis.

One important cause for the lack of commitment are the similar preferences of the supporting and the receiving organization. Consider for example a state in a federation which is not able to provide important public services such as schooling or tap water due to a debt crisis. In such a situation, it would not only benefit the inhabitants of the debt-ridden state to receive a bailout, but also the central government to give a bailout since it cares for all inhabitants, also the ones suffering from shortage in the jurisdiction in fiscal crisis. The motives for such interventions may be of different nature, for instance the maintenance of public services producing positive benefit spillovers to other regions (Wildasin, 1997) or the pursuit of receiving votes (Goodspeed, 2002).

Theoretical papers on the soft budget syndrome usually model bailouts in the form of monetary transfers. For instance, Wildasin (1997) as well as Crivelli and Staal (2006) use matching grants, Quian and Roland (1998) as well as Besfamille and Lockwood (2007) use project refinance and Goodspeed (2002) as well as Breuillé, Madiès and Taugourdeau (2007) use unconditional grants.

However, in practice one can often observe bailouts in federations which are

linked to additional obligations, requirements or restrictions such as savings goals. For example, in response to a series of municipal defaults during the depression in Canada, the Ontario Municipal Board was founded in 1932 and a few years later the Department of Municipal Affairs in 1934 as well as the Windsor Finance Commission in 1935 . These boards restricted the actions of defaulting municipalities by measures like prescribing refunding plans, auditing, inspecting, approving and supervising municipal budgets or even controlling certain expenditures (Bird and Tassonyi, 2003). Another example are adjustment targets which were formulated 1997 in Law 9496 as a condition for debt relief for the Brazilian states. These targets included for instance scheduled declines in debt-revenue ratios, limits on personnel spending or ceilings on investments (Rodden, 2003). Certainly one of the most famous examples is the Emergency Financial Control Board which controlled New York City during the bailout following the 1975 debt crisis. The board could control and reject the city's financial planning, current and capital budgets, negotiated wage contracts as well as local borrowing. If the city had not met the requirements, the EFCB would have had the right to control all municipal accounts and to exercise disciplinary sanctions (Eichhorst and Kaiser, 2006).

In contrast to pure monetary payments that are ex-post beneficial from both the donors and the recipients perspective, additional obligations involve countervailing preferences. It seems reasonable that a central government providing financial assistance might prefer a high tax rate in the recipient region in order to enforce a contribution to the resolution of the crisis, while the region is likely to prefer low taxes and financing the deficit out of central government funds. The countervailing nature of preferences renders such restrictions a credible device for the central government to make bailouts less attractive from the perspective of the region and

may therefore alleviate the soft budget constraint problem.

In our paper, we investigate the effects of restrictions in the form of prescribed tax rates and expenditure savings imposed in conjunction with monetary bailouts on regional incentives, the softness of budget constraints and welfare. In line with the theoretical literature on soft budget constraints (for an literature overview see Kornai, Maskin and Roland, 2003), we find that effort of regions to provide the regional public good is too low in presence of positive bailout expectations. In contrast, regions spend too high effort under a pure hard budget regime, a result, which has recently been derived by Besfamille and Lockwood (2007). The contribution of our paper is to explicitly consider additional obligations which the region has to fulfill in case of accepting a monetary bailout. We consider a model, where regions can raise revenues through two different channels - collecting taxes or saving expenditures. We show if the central government can fully restrict regional actions, i.e. prescribe both, a tax rate and an the amount of expenditure savings, the first best can be implemented. We refer to this setting as *fully restricted bailout regime*. Surprisingly, in a *partly restricted bailout regime*, where restrictions are just allowed in one dimension, e.g. on the tax rate, welfare might be even lower than in an *unrestricted bailout regime*. The intuition for this result is that the regions compensate for the restriction by distorting the unrestricted revenue instrument even more than in case of no restrictions.

The remainder of the paper is organized as follows. Section 2 gives a overview of the related literature. Section 3 presents the basic model set-up and section 4 introduces three benchmark cases: Centralized decision making as a benchmark for the first best (FB) solution, the hard budget constraint (HBC) and an unrestricted bailout (UB) regime. Section 5 presents the results of the fully (FRB) and the

partly restricted bailout (PB) regimes. All cases are evaluated with regard to their welfare in section 6 and section 7 concludes.

2 Related Literature

Our model is related to the literature on the soft budget constraint problem, a phenomenon initially analyzed by Kornai (1979, 1980) in the context of socialist enterprises which got deficits covered by the state. The concept has afterwards been applied to a variety of other fields, for instance banks (most prominently Maskin and Dewatripont, 1995), social insurance institutions in transition economies (e.g. Kornai and Eggleston, 2001) or as in our paper to decentralized countries. A comprehensive overview on the soft budget constraint literature is provided by Kornai, Maskin and Roland (2003).

Within this literature, we are close to recent work dealing with the question of how different institutions and characteristics of federations affect regional choice and the softness of budget constraints? With this in mind, Quian and Roland (1998) as well as Breuillé, Madiès and Taugourdeau (2006) investigate the effects of tax competition, Wildasin (1997) as well as Crivelli and Staal (2006) consider the size of regions, Breuillé, Madiès and Taugourdeau (2007) the type of spending and Akai and Sato (2005) the type of authority which is allocated to local governments. Our paper deals with the role of tax and expenditure restrictions as well as the availability of a transfer system.

In addition, because we investigate three different "second best" regimes (*unrestricted bailout*, *partial bailout* and *hard budget constraints*), we conduct a welfare analysis in a separate section. One part of the results in this section is closely

related to the recent findings of Besfamille and Lockwood (2007), who pioneer the view that not only soft budget constraints but also hard budget constraints involve inefficiencies. Unlike in our model, Besfamille and Lockwood use a framework, based on a project finance approach similar to that of Dewatripont and Maskin (1995). However, similar to our model, they find parameter values, for which either second best regime may dominate or even be efficient.

Our model assumptions basically follow standard approaches of modelling soft budget problems in federations. Like most papers, we create the commitment through a two-stage game where regions are allowed to move first. Some authors divide the two stages additionally in two periods (e.g. Goodspeed, 2001) or Breuillé et al., 2006). However, we employ similar to Wildasin (1997) or Crivelli and Staal (2006) just one period. The bailout motive of the central government emerges simply from maximization of joint utility of all regions, which is also standard. Sometimes, unlike in our paper, bailouts are also motivated by benefit spillovers (e.g. in Wildasin, 1997).

Nevertheless, we would like to emphasize two assumptions which differ from standard approaches and are important for our results. First, we do not refinance the bailouts through ex-post central government taxation. We instead allocate a fixed budget ex-ante to the central government, which can in turn decide to spend it ex-post on a national public good or on bailouts. This assumption is a simple way to endow the central government with tax revenue. At the same time it is sufficient to create a commitment problem. Breuillé et. al (2007) explicitly compare the effects of an ex-post manipulatable with an ex-post non-manipulatable head tax and find that in the latter case the central government is still inclined to increase the ex-post transfer above the efficient level, but to a lesser extent than with a

manipulable tax.

The second particularity of our setup is that, regions have two decision variables. The second decision variable allows us to see if additional bailout requirements are still valuable from a welfare perspective if central governments are able to restrict regional actions only in some areas but not in others. This setting is important, because regions in federations are generally endowed with autonomous rights to determine expenditures and revenues, which are as a rule cannot easily be taken away, also in extraordinary situations like debt crises. We consider particularly the situation where the central government is allowed to require a certain tax rate, but leaves expenditure autonomy to the region. But this setting can be interpreted more generally as one restricted and another unrestricted policy field.

3 Model Set-Up

We consider a simple model of a federation with a central government and two regional governments ($i = 1, 2$). Regions are inhabited by representative consumers, deriving utility from private consumption c_i , regional public consumption g_i and national public consumption G according to the additively separable utility function $u(c_i) + h(g_i) + J(G)$ which exhibits standard properties (i.e. $u', h', J' > 0, u'', h'', J'' < 0$, monotonicity, continuous differentiability in all arguments) and Inada conditions $u'(\infty), h'(\infty), J'(\infty) = 0$.

The central government is assumed to receive ex-ante an exogenously fixed amount of tax revenue T , which it can spend either on the national public good or on grants (transfers or bailouts) to regional governments $z_i \geq 0$.

$$T = G + z_i + z_j \quad (1)$$

Representative consumers are endowed (after central government taxation) with identical income $w < \infty$, which can be interpreted as income after revenue equalization. Regional governments may tax this income at a proportional rate $t_i \geq 0$. The income after regional taxation is spent on private consumption.

$$c_i = w(1 - t_i) \quad (2)$$

Regional governments can not only obtain revenue through regional taxation and central government grants but also through exerting effort $a_i \geq 0$ on saving expenditures. Examples for such effort could be the closure of schools losing attendance and distributing the pupils on other schools; or procurement of a software simplifying the administration of a public task, such that less personnel is needed to fulfill it. We assume that the effort is translated linearly into revenue for the regional public good. So, regional consumption is financed through three sources.

$$g_i = wt_i + a_i + z_i \quad (3)$$

The effort spent on efficiency enhancements is assumed to cause convex effort costs $k(a_i)$, $k'(a_i) > 0$, $k''(a_i) > 0 \forall a_i \geq 0$ which diminish the payoff from public and private consumption. These costs can be interpreted as transaction costs, e.g. search costs or administration costs.

In all settings *the timing* is such that regions choose in the first stage all unrestricted variables by maximizing the utility of the representative consumer in their

own region. In the second stage the central government chooses all remaining variables by maximizing the utility of both regions. This sequence of decisions is a standard approach to model non-commitment of the lastly moving agent.

4 Benchmark Cases

We consider three benchmark cases: the first best regime as a benchmark for efficiency, the hard budget regime because it defines the outside option for the region if it denies the bailout and the unrestricted bailout regime as a benchmark for a pure monetary bailout without any further restrictions.

4.1 Centralized Decision Making

The central government solves the following problem¹:

$$\max_{\mathbf{t}, \mathbf{a}, \mathbf{z}} \sum [u(c_i) + h(g_i) + J(G) - k(a_i)] \quad (4)$$

s.t. (1), (2) and (3).

The solution is characterized by conditions (5) – (7).

$$u'(c_i) = h'(g_i) \quad \forall i \quad (5)$$

$$k'(a_i) = h'(g_i) \quad \forall i \quad (6)$$

¹Where \mathbf{t} denotes (t_1, t_2) , \mathbf{a} denotes (a_1, a_2) and \mathbf{z} , (z_1, z_2) .

$$2J'(G) = h'(g_i) \quad \forall i \quad (7)$$

The first two conditions express that in the efficient solution marginal costs of taxation (i.e. forgone private consumption) and of effort are equalized to their marginal benefits, i.e. additional regional public consumption. The last condition is a Samuelson type condition which shows that the benefit from allowing one unit of transfer has to compensate for forgone national public consumption in both regions. Because we are interested in the soft budget constraint problem, we assume throughout that $z_i > 0$ is optimal and that the parameter constellations are such that $a_i > 0$ and $t_i > 0, \forall i$. If to the contrary $2J'(G) > h'(g_i)$ at $z_i = 0$, no bailout is optimal. Finally, the symmetry assumptions imply that $t_i = t_j, a_i = a_j$ and $z_i = z_j, \forall i \neq j$, which holds as well for all cases discussed below.

4.2 Hard Budget Constraint Regime

We define the hard budget constraint (HBC) regime as a regime where transfers are not available. This case is important because it defines the outside option of a region, i.e. the utility a region can obtain by denying any central government assistance and resolving the fiscal crisis on its own. The solution of the HBC problem is characterized by conditions (5) and (6) resulting from problem (4) in the absence of transfers.² Because, we focus on cases where $\forall i : z_i^{FB} > 0$ (FB stands for first best), the HBC regime is inefficient. The following proposition describes the nature of these inefficiencies:

²Please note that the solutions to the regional government problem and the central government problem coincide in the absence of transfers.

Proposition 1 *Provided $z_i^{FB} > 0$, regions choose tax rates and effort levels too high in the HBC regime compared to the efficient benchmark, i.e. $t_i^{HBC} > t_i^{FB}$, $a_i^{HBC} > a_i^{FB}$. Proof: see Appendix.*

The intuition for this result is straightforward. Regions have to compensate for central government bailouts by higher regional revenue contributions to the resolution of the crisis.

4.3 Unrestricted Bailout Regime

The unrestricted bailout (UB) regime is the standard pure SBC case. In this setting, the central government has only the instrument of monetary grants at its disposal and can not commit to an efficient level of grants. Non-commitment is modeled through timing. Regions move first by choosing tax rates and effort levels and the central government moves last by determining transfer policy. We solve the game by backward induction.

The central government maximizes at stage two the utility of the residents in both regions by choosing grants taking tax rate and effort choices of regions as given. The solution is characterized by condition (7), from which we obtain through implicit differentiation the central government response functions.

$$\frac{\partial z_i}{\partial a_i} = -\frac{h''(g_i)+2J''(G)}{h''(g_i)+4J''(G)} < 0 \quad \frac{\partial z_i}{\partial t_i} = -w\frac{h''(g_i)+2J''(G)}{h''(g_i)+4J''(G)} < 0$$

$$\frac{\partial z_j}{\partial a_i} = \frac{2J''(G)}{h''(g_i)+4J''(G)} > 0 \quad \frac{\partial z_j}{\partial t_i} = w\frac{2J''(G)}{h''(g_i)+4J''(G)} > 0$$

As in standard models, the central government responds to tax rate and effort

reductions in region i with an increase of the grant to this region, whereas the grant of the other region is reduced.

Regional governments maximize at stage one just the utility of their own residents s.t. (1) – (3) by taking central government behavior into account.

$$u'(c_i) = \frac{1}{2}h'(g_i) \quad \forall i \quad (8)$$

$$k'(a_i) = \frac{1}{2}h'(g_i) \quad \forall i \quad (9)$$

The solution to this problem, summarized in conditions (8) and (9), clearly differs from the efficient solution in (5) and (6). The reason is that each region does not consider the effects of own tax setting and expenditure behavior on public consumption in other regions. Particularly in the case of two regions only half of the benefits of marginal increases of tax rates and effort levels are considered. How does this affect regional decisions?

Proposition 2 *Provided $z_i^{FB} > 0$, regions choose tax rates and effort levels too low in the UB regime compared to the efficient benchmark, i.e. $t_i^{UB} < t_i^{FB}$, $a_i^{UB} < a_i^{FB}$. Proof: see Appendix.*

We can infer in addition from $\frac{\partial g_i}{\partial a_i} > 0$ and $\frac{\partial g_i}{\partial t_i} > 0^3$, that regional public consumption is inefficiently low $g_i^{UB} < g_i^{FB}$, which holds as well for national public consumption by (7) $G^{UB} < G^{FB}$ and hence bailouts are inefficiently high $z_i^{UB} > z_i^{FB}$.

³ $\frac{\partial g_i}{\partial a_i} = 1 + \frac{\partial z_i}{\partial a_i} = \frac{2J''(G)}{h''(g_i)+4J''(G)}$ and $\frac{\partial g_i}{\partial t_i} = w + w\frac{\partial z_i}{\partial t_i} = w\frac{2J''(G)}{h''(g_i)+4J''(G)}$.

The result shows not only that regions reduce their effort below an efficient level as in standard models of soft budget constraints, but also that ex-ante and ex-post grants differ. The nature of these differences has been extensively discussed in Koethenbuerger (2007) for corrective (or Pigouvian) subsidies. In our case, the difference illustrates that the ex-ante efficient grant z_i^{FB} is not credible, because if regions reduce tax rates and effort to the inefficient levels t_i^{UB} and a_i^{UB} , it is optimal for the central government to increase the grant in turn to the amount z_i^{UB} . In case of commitment the response should be zero. In the next section, we turn to the question if additional bailout restrictions can alleviate the commitment problem.

5 Restricted Bailout Regimes

In this section, we explicitly consider additional obligations or restrictions in conjunction with monetary bailouts. The way we model the obligations resembles a procedure of budgetary approval during a crisis. In practice, regions can typically prepare a budget plan, which is rejected whenever it does not meet the objectives of the monitoring agency. However, from a game theoretic perspective this is the same as if the agency or the central government could choose the restricted variable directly without initial proposal of the region. Therefore we let regions choose only the unrestricted choice variables at stage 1 and the central government all remaining variables at stage 2.

At the same time the central government is also restricted in its actions, because regions could always reject any help and resolve the crisis themselves. In this case, they would obtain the HBC utility. In order to ensure commitment to the

additional bailout restrictions, we assume a constitutional law which endows the central government with rights to prescribe regional tax rates and/or expenditure savings onto the central government whenever monetary grants are paid. This rule is important, because otherwise regions could reject the bailout and we were back in the unrestricted bailout case.⁴

5.1 Fully Restricted Bailout Regime

In this regime, regions receive not only a transfer, but also have to adhere to tax rates and effort levels prescribed by the central government, whenever they accept a bailout. Clearly it is optimal for the central government to offer the first best combination $(t_i^{FB}, a_i^{FB}, z_i^{FB})$ at stage 2. Because there is no leeway for regions to change anything about this offer at stage 1, we leave out this stage and let them directly choose among first best $(t_i^{FB}, a_i^{FB}, z_i^{FB})$ and their (best) outside option $(t_i^{HBC}, a_i^{HBC}, 0)$.⁵ The next proposition holds because the first best offer, although being less attractive from the regional perspective than the allocation in an unrestricted regime, is still more attractive than the HBC allocation and will therefore be accepted.

Proposition 3 *The fully restricted bailout regime implements the first best allocation. Proof: see Appendix.*

⁴This is also the reason why we refer to restrictions and not to conditions. Conditions in the fashion of "When you choose tax rate x and effort level y , then you get a grant of size z " are not credible.

⁵We consider only symmetric equilibria. In principle, asymmetric equilibria (one region accepts and the other rejects) are also possible, but only if we are in solutions where the central government is sufficiently poor. We rule this out by assuming that the central government is rich enough to provide further funds after one region has already accepted the bailout.

Because of the alignment of preferences, the central government would not propose any offer that is harmful to the regions because this would harm as well itself. On the other hand, the restrictions prevent regions from moving to their most preferred allocation, i.e. to the low tax rates and effort levels of an unrestricted regime.

5.2 Partially Restricted Bailout Regime

In the partially restricted bailout (PB) game, regions keep at least some autonomy and can choose effort levels at stage 1, while the central government chooses tax rates and transfers at stage 2, if the bailout is accepted.⁶ Alternatively regions could again reject the bailout. We first solve the two stage game and check afterwards if bailout acceptance is beneficial from the perspective of regions.

The *central government* maximizes at stage 2 the utility of all residents s.t. all budget constraints (1) – (3) and taking regional effort choices as given. Implicit differentiation of conditions (5) and (7), which characterize the tax and transfer policy of the central government yield the central government response functions.

$$\begin{aligned}\frac{dt_i}{da_i} &= \frac{dt_j}{da_i} = -\frac{1}{w} \frac{2h''(g_i)J''(G)}{h''(g_i)u''(c_i)+4h''(g_i)J''(G)+4u''(c_i)J''(G)} < 0 \\ \frac{dz_i}{da_i} &= -\frac{h''(g_i)u''(c_i)+2h''(g_i)J''(G)+2u''(c_i)J''(G)}{h''(g_i)u''(c_i)+4h''(g_i)J''(G)+4u''(c_i)J''(G)} < 0 \\ \frac{dz_j}{da_i} &= \frac{2h''(g_i)J''(G)+2u''(c_i)J''(G)}{h''(g_i)u''(c_i)+4h''(g_i)J''(G)+4u''(c_i)J''(G)} > 0\end{aligned}$$

Similar to the unrestricted bailout case, reduced effort in region i elicits larger transfers to this region, but decreases transfers to the other region. In addition,

⁶The case on partial restrictions on effort, but not on taxes produces similar results, just that effort and taxation are interchanged.

tax rates are raised to compensate for the lower effort. Taking all effects into account, reduced effort of region i , reduces regional public consumption in both regions $\left(\frac{dg_i}{da_i} > 0, \frac{dg_j}{da_i} > 0\right)$.

Regional governments, taking this central government policy into account maximize the utility of their own residents s.t. (1) – (3) at stage 1. Optimal effort choices are as in the unrestricted bailout case characterized by condition (9). How does this sequence of decisions affect equilibrium effort and tax rates?

Proposition 4 *Provided $z_i^{FB} > 0$, in the partially restricted bailout regime, effort levels are even lower than in the unrestricted bailout regime and tax rates are higher than in first best, i.e. $a_i^{PB} < a_i^{UB}, t_i^{PB} > t_i^{FB}$. Proof (including optimality of acceptance): See appendix.*

The downward distortion of the effort level is generally driven by the short-sightedness of the region regarding the benefits of spending effort. It neglects the positive effects for the other region. But why is effort even lower than in the unrestricted case? To understand this, suppose the region had chosen a_i^{UB} . This is too low from the perspective of the central government and it therefore increases the regional tax rate above the first best level and hence as well above the level preferred by the region t_i^{UB} . This reduces c.p. the marginal benefit of public consumption g_i and since tax rates and effort are substitute instruments for raising revenue, the region compensates for the higher tax rates by further reducing its effort level.

Corollary 5 : *In comparison to the unrestricted regime, in the partially restricted bailout regime budget constraints are hardened in the sense that less transfers are*

paid ($z_i^{PB} < z_i^{UB}$) and more of the national and regional public goods are provided ($g_i^{PB} > g_i^{UB}, G_i^{PB} > G_i^{UB}$). *Proof: See appendix.*

The reduction of transfers is a result of the increased scope of the central government. Compared to the unrestricted bailout case it has one more instrument at its disposal to resolve the regional crisis. The utilization of the second instrument, i.e. regional taxation, allows the central government to reduce transfers z_i . In effect, the central government can force the region to participate in the resolution of the crisis through taxation and therefore to into the provision of a higher level of public goods before transfers, which entails a higher public good provision after transfers. This is one central result of the paper.

However, it remains an open question, if this regime yields higher welfare than an unrestricted bailout regime? Compared to the latter, in the partially restricted bailout regime regional as well as national public consumption are increased and effort costs are reduced, but private consumption is lower. So it is not possible to make an outright statement. We move on to this issue in the next section.

6 Welfare Analysis

Throughout the paper, we deal with three different kinds of inefficient regimes - the unrestricted bailout regime, the partially restricted bailout regime and the hard budget constraint regime. Although the central government can not credibly commit to a HBC regime in our two stage model, it is nevertheless important to make a welfare statement about this regime since it involves a distinctly different type of inefficiency than the UB and the PB regime. The latter both are inefficient for their soft budget constraints, i.e. the possibility for regional governments to

increase the size of their budgets through their actions, which produces inefficiently low contributions of regions to the resolution of the crisis. In contrast, the HBC regime involves no transfer at all, which renders the contributions of the central government too low and enforces thereby inefficiently high effort of regions.

We show in the sequel, that it is not possible to make general statements about the welfare ranking of the three regimes. Depending on the wealth of the central government and on preferences, either regime may dominate. We proceed in two steps. First, we compare the HBC regime with both SBC regimes (UB, PB) and show that the welfare evaluation depends on the tax endowment of the central government. We show, if the endowment is low, all regimes are efficient. For intermediate values, the UB and PB regimes are dominated by the HBC regime and vice versa for high values of central government endowments. In the second part of the section, we use the example of a logarithmic function to show that either the UB or the PB regime may dominate depending on the shape of preferences. In our example, the PB regime dominates, if the valuation of public consumption is sufficiently high, while the UB regime dominates if effort costs are sufficiently small.

6.1 Efficiency of HBC versus SBC Regimes

The evaluation of hard budget regimes as opposed to soft budget regimes depends on the tax endowment of the central government T relative to the regional wealth endowment w . Because reductions of w and increases of T have qualitatively similar effects, we fix w and analyze changes of T .

To establish our main argument, we first define two threshold values of central

government tax revenue \underline{T} and \bar{T} . \underline{T} is the value where the central government just starts to employ transfers as a financing instrument for regional public goods in first best, i.e. the value of T at which all efficiency conditions (5) – (7) are met for $z_i = 0$. On the other hand $\bar{T} > \underline{T}$ defines the level of tax revenue in the first best regime at which the central government becomes so rich that it prefers to finance regional public consumption completely out of transfers ($g_i = z_i > 0$) and stops to use both regional taxation as well as expenditure savings as a revenue raising instrument, i.e. $u'(w) \geq h'(g_i), k'(0) \geq h'(g_i)$.⁷

Proposition 5 and the figure 1 summarize how welfare evolves in the different regimes across different levels of central government tax revenue.

Proposition 6 *The HBC regime is efficient for all $T \leq \underline{T}$ and inefficient for all $T > \underline{T}$. SBC regimes are efficient for $T < \underline{T}$ as well as $T \geq \bar{T}$ and inefficient for $\underline{T} \leq T < \bar{T}$. Proof: see Appendix.*

⁷The existence of \underline{T} is assured by the Inada conditions on public and private consumption $J'(\infty), h'(\infty), u'(\infty) = 0$, the finiteness of regional wealth $w < \infty$ and hence $u'(w) > 0$ as well as the costliness of the first unit of public effort spent, i.e. $k'(0) > 0$.

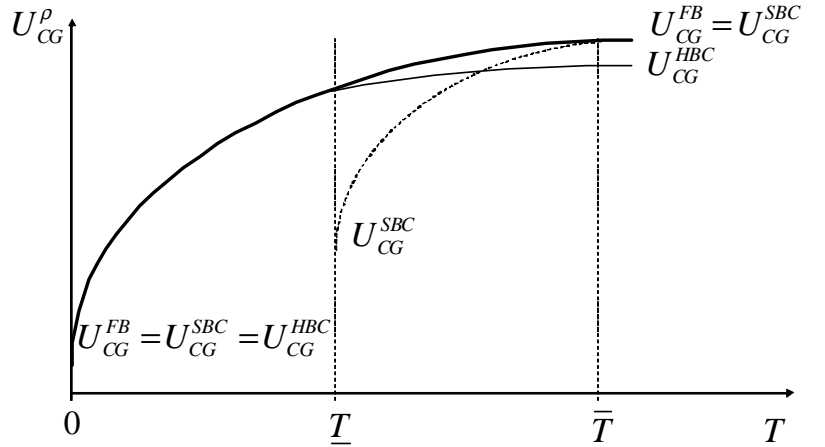


Figure 1: SBC vs. HBC regimes

For $T < \underline{T}$ all regimes are efficient (fat line) because bailout costs (marginal costs of forgoing one unit of national public consumption) costs are very high. In this interval, on the one hand the HBC regime (slim line) entails no welfare losses since a no-bailout policy is even optimal in first best and the availability of transfers adds no value. On the other hand, the SBC regimes (dashed line) are efficient because high bailout costs allow the central government to commit to the no-bailout policy and incentives to raise revenue stay undistorted.

At $T = \underline{T}$ the SBC regimes become inefficient because marginal deviations from first best policy start to pay off and regions switch into the SBC equilibrium. On the other hand the inefficiency of the HBC regime appears as well, but increases only gradually as T rises above \underline{T} . This is due to a gradual replacement of regional taxation and effort by transfers in first best, whereas tax rates and effort are kept constant in the HBC regime.

At $T = \bar{T}$ regional taxation and effort are completely phased out as revenue

raising instruments in first best, and hence also in the SBC regimes. Given that inefficiencies from too low taxation or effort cannot occur anymore, the SBC regimes coincide with first best for all $T \geq \bar{T}$.

Besfamille and Lockwood (2007) derive similar results in a project finance framework depending on effort costs for increasing the benefit of a project and refinancing (or bailout) costs. They find, when the effort costs are very low, both the HBC and the SBC regime are efficient. In contrast, the HBC regime dominates with intermediate effort and refinancing (or bailout) costs, while at high effort costs and sufficiently low refinancing costs this is true for the SBC regime.

6.2 Efficiency of PB vs. UB regimes

In this section, we show that contrary to our initial expectation, the partially restricted bailout regime does not generally dominate the unrestricted bailout regime. This finding is surprising because in the PB regime the central government has an additional instrument at its disposal and budget constraints are hardened compared to the unrestricted regime. We employ a Cobb-Douglas type logarithmic function of the following form to establish our assertion.

$$U_{CG}^{\rho}(c_i, g_i, a_i, G) = \sum_i (\alpha \ln c_i + \beta \ln g_i + \gamma \ln (L - a_i) + \delta \ln G) \quad \rho \in (PB, UB) \quad (10)$$

Where $\alpha, \beta, \gamma, \delta > 0$ are weighting factors adding up to one: $\alpha + \beta + \gamma + \delta = 1$ and L is a constant. To facilitate understanding, one could interpret L as time endowment, a_i as time spent on searching and implementing measures for leaning public administration and $L - a_i$ as leisure time.

Solving for the PB and UB regimes s.t. (2) – (1) yields the following utility difference:⁸

$$\Delta(\alpha, \beta, \gamma, \delta) = U_{CG}^{PB} - U_{CG}^{UB} = \ln \frac{2\alpha + \beta + 2\gamma + \delta}{\alpha + \beta + 2\gamma + \delta} - \alpha \ln 2 \quad (11)$$

This difference is independent of wealth, tax and time endowments (w, T, L) . We investigate for which combinations of α, β, γ and δ either the UB or the PB regime dominates. We first state our result and explain the intuition afterwards.

Proposition 7 *If the joint valuation of public goods $(\beta + \delta)$ is sufficiently high, i.e. $\beta + \delta \geq (2 - \frac{1}{\ln 2})$, the PB regime at least weakly dominates the UB regime. On the other hand, the UB regime at least weakly dominates the PB Regime if the joint valuation of public and private consumption $(\alpha + \beta + \delta)$ is sufficiently small, i.e. $(\alpha + \beta + \delta) \leq (2 - \frac{1}{\ln 2})$. Proof: see appendix.*

The interesting finding of this proposition is that the PB regime might be inferior to the UB regime although the central government has a second instrument at its disposal. Because of the multiplicity of parameters and utility components, the welfare evaluation of the results is complex. Two important sources are driving the explanation. Firstly, a shift of weight from one to another parameter causes a "weighting effect", meaning that a given distortion enters more heavily into the welfare function. Since in the PB regime effort choices are more strongly distorted than in the UB regime, an increase of γ deteriorates the welfare of the PB relative to UB the regime. In the same manner, an increase of the weighting factors for regional and national public consumption (β, δ) , that are more heavily distorted

⁸A summary of results for this utility function can be found in the appendix.

in the UB regime, relatively improve the evaluation of PB. The effects of α are ambiguous and depend on the remaining parameters.

The second element of the welfare evaluation is an "incentive effect", determining the size of distortions in the PB relative to the UB regime. In the UB regime the distortion from first best is largest when the joint weight on forgone public consumption ($\beta + \delta$) takes intermediate values, while for very low and very high values the distortion is small. The intuition for this result is that a low importance of public consumption entails low provision of public consumption in first best. However, marginal reductions from initially high amounts are extremely costly due to sharply increasing marginal costs of public consumption. On the other hand, for high values of ($\beta + \delta$) marginal costs of downward deviations are virtually constant, while the benefits of additional consumption and leisure, being already low in first best for low ($\alpha + \gamma$), are sharply decreasing. Only for intermediate values it pays off to significantly deviate from first best choices. Similarly, in the PB regime, incentives to deviate from FB are maximized when γ and ($\alpha + \beta + \delta$), respectively take intermediate values. This explains the result.

For low values of γ , both the "incentive effect" to distort in the PB regime and the "weighting effect" of this distortion is low. Given γ is low, a sufficiently high value of ($\beta + \delta$) entails a high "incentive effect" multiplied by a high "weighting effect" for the UB regime, yielding a clear domination of PB. As γ increases, ($\beta + \delta$) do necessarily decrease as all weights sum up to one. At the critical value of $\gamma \geq \frac{1}{\ln 2}$, the distortions in the PB regime become dominant. They decrease as γ rises further because the "incentive effect" stops to grow and begins to fall, while the continuing rise of the "weighting effect" dominates and maintains the dominance of the UB regime until γ approaches one.

7 Conclusion

The main question of this paper is, if the soft budget constraint problem can be alleviated if additional obligations or restrictions are available? Our analysis shows, this is not necessarily the case. Constraints, even if they are credible, are not always welfare enhancing. If the scope of the central government is limited and it can require certain actions from regions just in one policy area but not in others, the outcome may be even worse than in an unrestricted regime. The intuition for this finding is that a region which is regulated in one area, e.g. by minimum tax requirements, may substitute very strongly by reduced employment of other revenue collection instruments, such as effort on cutting expenditures. In contrast, a fully restricted regime implements the first best outcome in our model.

Thus, from a policy perspective the paper suggests that a comprehensive approach is a more promising path to reconcile the aims of helping a debt ridden jurisdiction and enforcing a sufficiently high contribution of the region to the reduction of debt. Single empirical cases support this hypothesis. For example, the quick resolution of the 1975 New York fiscal crisis, under the strict surveillance of the cities budgetary performance by the EFCB and the contrasting experience with the German city of Bremen, which received a total bailout of 8.5 bn € over the period 1994-2004, virtually without further obligations and nevertheless increased its debt from 8.8 bn € to 11.4 bn €. However, it is an important task for future research to test the hypotheses derived in the paper on a sound empirical basis.

From a theoretical perspective the analysis shows that not each policy, which hardens budget constraints, in the sense that less transfers are paid, is welfare

enhancing. This holds not only for the partial bailout regime, but also for the hard budget constraint regime, which just addresses the problem of preventing regions from deviating downwards, while completely ignoring the benefits of helping the region out of the crisis.

In addition, the welfare analysis identifies the budget size of the central government as a determinant of the evaluation of different regimes. This finding might explain different attitudes of federal governments towards soft budget constraints. The model predicts that governments of more decentralized countries with a small role of the federal level should be in favor of hard budget regimes, whereas governments of more centralized countries with a larger federal budgets should be more likely to prefer soft budget regimes. Similarly, developments of revenues over the business-cycle increasing or reducing the relative size of the federal budget may influence the judgement of the central government. But this is again an interesting empirical question.

Finally, it should be pointed out that restrictions during a bailout can of course not make up for fiscally imprudent behavior before the bailout takes place. In this sense, the paper discusses the optimal policy once the crisis is at hand and adds to the literature on budget rules by providing theoretical arguments on using such rules during fiscal crises.

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8 Appendix

8.1 Proof of Proposition 1

We next show that if $z_i^{FB} > 0$, then assumptions $h'(g_i) > 0, h''(g_i) < 0$ imply that $t_i^{HBC} > t_i^{FB}$ and $a_i^{HBC} > a_i^{FB}$.

Proof: Suppose t_i^{FB}, a_i^{FB} is optimal in the hard budget regime.

Then $wt_i^{FB} + a_i^{FB} = g_i^{HBC} < g_i^{FB} = wt_i^{FB} + a_i^{FB} + z_i^{FB}$ imply

$$\frac{\partial U_i^{HBC}}{\partial t_i} = -wu'(c_i) + wh'(g_i) > 0 \text{ and } \frac{\partial U_i^{HBC}}{\partial a_i} = h'(g_i) - k'(a_i) > 0, \text{ meaning}$$

that it pays to deviate upwards in both the tax and the effort dimensions, which provides the contradiction.

In addition equalization of marginal benefits of consumption and marginal effort costs ($u'(c_i) = k'(a_i)$) in both the HBC and the FB regimes insures that upwards deviation occurs in both dimensions.

8.2 Proof of Proposition 2

Next we show $t_i^{UB} < t_i^{FB}, a_i^{UB} < a_i^{FB}$ whenever $z_i^{FB} > 0$.

Proof: Suppose $t_i^{UB} = t_i^{FB}, a_i^{UB} = a_i^{FB}$, then $u'(c_i^{FB}) = h'(g_i^{FB})$ and $k'(a_i^{FB}) = h'(g_i^{FB})$ imply

$$\frac{\partial U_i^{HBC}}{\partial t_i} = -wu'(c_i) + w\frac{1}{2}h'(g_i) < 0 \quad \frac{\partial U_i^{HBC}}{\partial a_i} = \frac{1}{2}h'(g_i) - k'(a_i) < 0, \text{ i.e.}$$

downward deviation increases regional utility. Again the equality of marginal effort costs and marginal utility from private consumption ($u'(c_i) = k'(a_i)$), both in the first best and in the soft budget regimes excludes solutions where one variable deviates upwards, while the other deviates downwards.

8.3 Proof of Proposition 3

We show that given $z_i^{FB} > 0 \forall i$, the central government offer is more valuable to the region than the maximal utility which the region can obtain under a hard budget regime: $U_i(\mathbf{a}^{FB}, \mathbf{t}^{FB}, \mathbf{z}^{FB}) > U_i(\mathbf{a}^{HBC}, \mathbf{t}^{HBC}, \mathbf{0})$, where \mathbf{a} denotes the vector (a_1, a_2) , $\mathbf{t} = (t_1, t_2)$ and $\mathbf{z} = (z_1, z_2)$. We proceed in two steps:

(1) Part I of the proof shows, if transfers are paid in first best $z_i^{FB} > 0$, then it is as well optimal to pay transfers at the higher tax rate t_i^{HBC} and the higher effort level a_i^{HBC} : $z_i^{HBC} \equiv z_i^*(\mathbf{t}^{HBC}, \mathbf{a}^{HBC}) > 0$, where the $*$ indicates the optimal central government policy for a given vector of regional choices. To see this, suppose to the contrary $z_i^*(\mathbf{t}^{HBC}, \mathbf{a}^{HBC}) = 0$. Then the HBC solution would

be characterized by the following conditions: $h'(g_i^{HBC}) = u'(c_i^{HBC})$, $h'(g_i^{HBC}) = k'(a_i^{HBC})$, $h'(g_i^{HBC}) \leq 2J'(w_G)$. But if this was true, then $z_i^{FB} = 0$. The central government condition for transfers: $h'(g_i) > 2J'(G)$ assures in addition that regional utility would be raised if the region could obtain the grant z_i^{HBC} .

(2) Part II of the proof shows that if the region could choose freely, it would prefer $(a_i^{FB}, t_i^{FB}, z_i^{FB})$ over $(a_i^{HBC}, t_i^{HBC}, z_i^{HBC}(\mathbf{t}^{HBC}, \mathbf{a}^{HBC}))$. The first and second order conditions⁹ of the SBC problem imply $\forall t_i > t_i^{UB} \wedge a_i > a_i^{UB}$:

$$\frac{\partial U_i}{\partial t_i} = -wu'(c_i) + w\frac{1}{2}h'(g_i) < 0 \text{ and } \frac{\partial U_i}{\partial a_i} = \frac{1}{2}h'(g_i) - k'(a_i) < 0.$$

Therefore it is profitable to deviate downwards from (t_i^{HBC}, a_i^{HBC}) to (t_i^{FB}, a_i^{FB}) .

8.4 Proof of Proposition 4

Proposition We prove that $t_i^{PB} > t_i^{FB}$, $a_i^{PB} < a_i^{UB}$. We proof the proposition in two steps and in a third step that acceptance is optimal. Define $t_i^*(\mathbf{a})$, $z_i^*(\mathbf{a})$ as optimal central government policy for a given set of effort choices (a_1, a_2) in the PB regime.

$$(1) \quad t_i^{PB} > t_i^{FB}, a_i^{PB} < a_i^{FB} :$$

Suppose to the contrary $a_i^{PB} = a_i^{FB}$ and hence $t_i^*(\mathbf{a}^{FB}) = t_i^{FB}$ and $z_i^*(\mathbf{a}^{FB}) = z_i^{FB}$. The central government optimality conditions imply $\frac{\partial U_i^{PB}(\mathbf{a}^{FB})}{\partial a_i} = -\frac{1}{2}h'(g_i^{FB}) < 0$, i.e. downward deviations increase regional utility. Therefore $a_i^{PB} < a_i^{FB}$, and $t_i^{PB} > t_i^{FB}$ by $\frac{\partial t_i^*}{\partial a_i} < 0$.

$$(2) \quad \text{Now we show in addition: } a_i^{PB} < a_i^{UB} < a_i^{FB}.$$

Suppose to the contrary: $a_i^{FB} > a_i^{PB} \geq a_i^{UB}$.

$$\frac{\partial^2 U_i}{\partial t_i^2} = \frac{\partial(-wu'(c_i) + w\frac{1}{2}h'(g_i))}{\partial t_i} = w^2(u''(c_i) + \frac{1}{2}h''(g_i)) < 0, \frac{\partial^2 U_i}{\partial a_i \partial t_i} = \frac{\partial(-wu'(c_i) + w\frac{1}{2}h'(g_i))}{\partial a_i} = \frac{1}{2}wh''(g_i) < 0, \frac{\partial^2 U_i}{\partial a_i^2} = \frac{\partial(\frac{1}{2}h'(g_i) - k'(a_i))}{\partial a_i} = \frac{1}{2}h''(g_i) - k''(a_i) < 0 \text{ and hence } D > 0.$$

$$t_i^*(\mathbf{a}^{FB}) = t^{FB} \text{ and } \frac{\partial t_i^*}{\partial a_i} < 0 \Rightarrow t_i^*(\mathbf{a}^{PB}) > t_i^*(\mathbf{a}^{FB}) \underset{\text{Proposition 2}}{>} t_i^{UB}$$

It follows from $a_i^{PB} \geq a_i^{UB}$ and $t_i^*(\mathbf{a}^{PB}) > t_i^{UB}$, that $g_i^{PB} > g_i^{UB}$ implying as well $h'(g^{PB}) < h'(g^{UB})$ and from condition (9), $k'(a^{PB}) < k'(a^{UB})$, which in turn implies $a_i^{PB} < a_i^{UB}$, contradicting our initial statement.

(3) Acceptance of the partially restricted bailout is more valuable from the perspective of the region than resolving the crisis alone.

We have already shown in the proof to proposition 3 that $z_i^*(\mathbf{a}^{HBC}, \mathbf{t}^{HBC}) > 0$, i.e. the central government would give a transfer if the regions chose $(\mathbf{a}^{HBC}, \mathbf{t}^{HBC})$ in the unrestricted regime and the acceptance of this grant would increase the utility of the regions $U_i(a_i^{HBC}, t_i^{HBC}, z_i^*(\mathbf{a}^{HBC}, \mathbf{t}^{HBC})) > U_i(a_i^{HBC}, t_i^{HBC}, 0)$. Given, the central government could choose t_i and z_i freely, it would prefer by the central government response functions $\left(\frac{\partial t_i}{\partial a_i} < 0, \frac{\partial z_i}{\partial t_i} < 0\right)$, $t_i^*(\mathbf{a}^{HBC}) < t_i^{HBC}$ and $z_i(\mathbf{t}^*(\mathbf{a}^{HBC}), \mathbf{a}^{HBC}) > z_i(\mathbf{t}^{HBC}, \mathbf{a}^{HBC})$. A move to this (z_i, t_i) combination would be again utility enhancing from the perspective of the regions, because the central government would reduce t_i only as long as $u'(c_i) \geq h'(g_i)$ and increase z_i only as long as $h'(g_i) \geq 2J'(G)$. Therefore $U_i(a_i^{HBC}, t_i^*(\mathbf{a}^{HBC}), z_i(\mathbf{t}^*(\mathbf{a}^{HBC}), \mathbf{a}^{HBC})) > U_i(a_i^{HBC}, t_i^{HBC}, z_i(\mathbf{t}^{HBC}, \mathbf{a}^{HBC}))$ and by revealed preference:

$$U_i(a_i^{PB}, t_i^*(\mathbf{a}^{PB}), z_i^*(\mathbf{t}^*(\mathbf{a}^{PB}), \mathbf{a}^{PB})) > U_i(a_i^{HBC}, t_i^*(\mathbf{a}^{HBC}), z_i(\mathbf{t}^*(\mathbf{a}^{HBC}), \mathbf{a}^{HBC})),$$

which establishes the result that acceptance is indeed optimal.

8.5 Proof of Corollary 5

$$z_i^{PB} < z_i^{UB}, g_i^{PB} > g_i^{UB} :$$

$$\text{From } a_i^{PB} < a_i^{UB} \stackrel{(9)}{\Rightarrow} h'(g_i^{PB}) < h'(g_i^{UB}) \Rightarrow g_i^{PB} > g_i^{UB}$$

$$\stackrel{(7)}{\Rightarrow} G^{PB} > G^{UB} \stackrel{(1)}{\Rightarrow} z_i^{PB} < z_i^{UB}.$$

8.6 Proof of Proposition 6

We denote in the following $U_{CG}^\rho(\mathbf{z}, \mathbf{t}, \mathbf{a})$ as the utility of the central government in regime $\rho \in (FB, UB, PB, HBC)$. We proceed in four steps.

(1) For $T \leq \underline{T}$, the first order conditions of the HBC regime coincide with the FB regime.

(2) Differentiation of the utility difference between the first best and the HBC regime for $T > \underline{T}$, shows that the HBC regime constantly deteriorates compared to the efficient solution as T increases. This can be illustrated by the derivative of the utility difference:

$$\frac{dU_{CG}^{FB}}{dT} - \frac{dU_{CG}^{HBC}}{dT} = 2J'(G^{FB}) - 2J'(G^{HBC}) > 0 \text{ (note that } G^{HBC} = T \text{ and } G^{FB} = T - z_i^{FB} - z_j^{FB} < T).$$

(3) The SBC regimes (UB,PB) are efficient for $T < \underline{T}$ because over this range $\frac{dz_i}{dt_i} = \frac{da_i}{da_i} = 0$ and marginal deviations from the first best policy do not pay off for regions.¹⁰ At $T = \underline{T}$ central government responses $\left(\frac{dz_i}{dt_i} \text{ and/or } \frac{da_i}{da_i}\right)$ change from zero to positive values and marginal deviations from first best become beneficial for regions. Furthermore, because regions distort their choices until their optimality conditions ((8) and/or (9)) are met, there is a discrete utility difference between SBC and FB at $T = \underline{T}$.

(4) As T rises, in the HBC regime tax rates and effort levels stay constant, i.e. $\frac{dt_i}{dT} = \frac{da_i}{dT} = 0$, whereas they depend negatively on T in all the remaining regimes (see table), i.e. $\frac{dt_i^\rho}{dT}, \frac{da_i^\rho}{dT} < 0$, entailing $\frac{dz_i^\rho}{dT} > 0, \rho \in (UB, PB, FB)$.

¹⁰We abstract from possible SBC equilibria below \underline{T} because they just add complexity but qualitatively do not change results.

ρ	FB	PB	UB
$\frac{dz_i^\rho}{dT}$	$\frac{2hjk-2hju+2jku}{4hjk-4hju+hku+4jku}$	$\frac{2hjk-hju+2jku}{4hjk-2hju+hku+4jku}$	$\frac{hjk-hju+2jku}{2hjk-2hju+hku+4jku}$
$\frac{dt_i^\rho}{dT}$	$\frac{1}{w} \frac{-2hjk}{4hjk-4hju+hku+4jku}$	$\frac{1}{w} \frac{-2hjk}{4hjk-2hju+hku+4jku}$	$\frac{1}{w} \frac{-hjk}{2hjk-2hju+hku+4jku}$
$\frac{da_i^\rho}{dT}$	$\frac{2hju}{4hjk-4hju+hku+4jku}$	$\frac{hju}{4hjk-2hju+hku+4jku}$	$\frac{hju}{2hjk-2hju+hku+4jku}$

Notation: $u \equiv u''(c_i^\rho)$, $h \equiv h''(g_i^\rho)$, $k \equiv k''(a_i^\rho)$, $j \equiv J''(G^\rho)$, $\rho \in (FB, PB, UB)$.

By proposition 2 it follows from $a_i^{FB} = 0$ and $t_i^{FB} = 0$, that $a_i^{UB} = 0$ and $t_i^{UB} = 0$ and by proposition 4 $a_i^{PB} = 0$, which proofs that PB and UB coincide with FB if $T \geq \bar{T}$.

8.7 Summary of results for the logarithmic function

ρ	FB	UB	PB
c_i^ρ	$\frac{\alpha}{2} \frac{2L+2w+T}{\alpha+\beta+\gamma+\delta}$	$\alpha \frac{2L+2w+T}{2\alpha+\beta+2\gamma+\delta}$	$\frac{\alpha}{2} \frac{2L+2w+T}{(\alpha+\beta+2\gamma+\delta)}$
g_i^ρ	$\frac{\beta}{2} \frac{2L+2w+T}{\alpha+\beta+\gamma+\delta}$	$\frac{\beta}{2} \frac{2L+2w+T}{2\alpha+\beta+2\gamma+\delta}$	$\frac{\beta}{2} \frac{2L+2w+T}{(\alpha+\beta+2\gamma+\delta)}$
l_i^ρ	$\frac{\gamma}{2} \frac{2L+2w+T}{\alpha+\beta+\gamma+\delta}$	$\gamma \frac{2L+2w+T}{2\alpha+\beta+2\gamma+\delta}$	$\gamma \frac{2L+2w+T}{(\alpha+\beta+2\gamma+\delta)}$
G^ρ	$\delta \frac{2L+2w+T}{\alpha+\beta+\gamma+\delta}$	$\delta \frac{2L+2w+T}{2\alpha+\beta+2\gamma+\delta}$	$\delta \frac{2L+2w+T}{(\alpha+\beta+2\gamma+\delta)}$

8.8 Proof of Proposition 7

Consider the utility difference: $\Delta(\alpha, \beta, \gamma, \delta) = U_{CG}^{PB} - U_{CG}^{UB} = \ln \frac{2\alpha+\beta+2\gamma+\delta}{\alpha+\beta+2\gamma+\delta} - \alpha \ln 2$.

1. We substitute $(\beta + \delta)$ by $x = (\beta + \delta)$. In order to cover all possible cases, we use two specifications of $\Delta(\alpha, \beta, \gamma, \delta)$.

2. First we replace x , using the restriction of $\alpha + \gamma + x = 1$, by $1 - (\alpha + \gamma)$ to obtain a utility difference, which only depends on α and γ :

$$\Delta(\alpha, \gamma) = U_{CG}^{PB} - U_{CG}^{UB} = \ln \frac{1+\alpha+\gamma}{1+\gamma} - \alpha \ln 2. \text{ When is } \Delta(\alpha, \gamma) \text{ negative, i.e. the}$$

UB regime dominates?

$$\ln \frac{(1+\alpha+\gamma)}{1+\gamma} - \alpha \ln 2 < 0 \quad \Leftrightarrow \quad \gamma > \frac{1+\alpha-2^\alpha}{(2^\alpha-1)}$$

$$\rightarrow \max \left(\frac{1+\alpha-2^\alpha}{(2^\alpha-1)} \right) = \lim_{\alpha \rightarrow 0} \left(\frac{1+\alpha-2^\alpha}{(2^\alpha-1)} \right) = \lim_{\alpha \rightarrow 0} \left(\frac{1-(\ln 2)2^\alpha}{(\ln 2)2^\alpha} \right) = \frac{1-\ln 2}{\ln 2} \approx 0.44.$$

Hence if $\gamma > \frac{1-\ln 2}{\ln 2}$, the UB regime always dominates.

3. Second, we use again the restriction $\alpha + \gamma + x = 1$ to replace γ and to obtain another specification of the utility difference, only depending on a and x :

$\Delta(\alpha, x) = \ln \frac{2-x}{2-x-\alpha} - \alpha \ln 2$. When is $\Delta(\alpha, x)$ positive, i.e. PB dominates?

$$\ln \frac{2-x}{2-x-\alpha} - \alpha \ln 2 > 0 \quad \Leftrightarrow \quad x > \frac{(2-\alpha)2^\alpha-2}{(2^\alpha-1)}$$

$$\max \left(\frac{(2-\alpha)2^\alpha-2}{(2^\alpha-1)} \right) = \lim_{\alpha \rightarrow 0} \left(2 - \frac{\alpha 2^\alpha}{(2^\alpha-1)} \right) = \lim_{\alpha \rightarrow 0} \left(2 - \frac{2^\alpha(1+\alpha(\ln 2))}{(\ln 2)2^\alpha} \right) = 2 - \frac{1}{\ln 2} \approx 0.56.$$

Hence the PB regime dominates for all parameter constellations fulfilling $(\beta + \delta) > 2 - \frac{1}{\ln 2}$.

4. The last possible specification of the utility difference, i.e. $\Delta(x, \gamma)$ does not yield any additional insights and is therefore not considered.

5. To demonstrate, that there are solutions for both cases fulfilling the parameter constraints, we calculate two examples. As endowment parameters we choose:

$$L = 0.5 \quad w = 0.1 \quad w_G = 0.4.$$

Example 1 $\gamma > 0.44$ (dominance of UB regime):

$$\alpha = 0.13 \quad \beta = 0.2 \quad \gamma = 0.47 \quad \delta = 0.2$$

$$U_{CG}^{PB} - U_{CG}^{UB} \approx -5.4 \times 10^{-3} < 0 \quad t_i^{UB} \approx 0.11 > 0 \quad a_i^{PB} \approx 0.15 > 0.$$

Example 2 $\beta + \delta > 0.56$ (dominance of PB regime):

$$\alpha = 0.1 \quad \beta = 0.3 \quad \gamma = 0.3 \quad \delta = 0.3$$

$$U_{CG}^{PB} - U_{CG}^{UB} \approx 4.8 \times 10^{-3} > 0 \quad t_i^{UB} \approx 0.21 > 0 \quad a_i^{PB} \approx 0.25 > 0.$$