Cost Variations in a Differentiated Good Oligopoly*

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Abstract

In the homogenous good case the impact of a marginal cost variation on consumer surplus, industry profits and social surplus was studied in extensive manner. Assuming quantity competition and a standard quadratic utility this paper carries on this analysis in a differentiated good context. In contrast to the homogenous good case consumers are better off in case of a mean preserving cost variation increasing the variance of marginal costs. In the differentiated good context producer surplus as well as consumer surplus increases with the variance of marginal costs.

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1 Introduction

In the homogeneous good case the relationship between the distribution of marginal costs and consumer surplus as well as producer surplus and, therefore, social surplus was studied in extensive manner. Producer surplus increases with the variance of (constant) marginal costs whereas consumer surplus solely depends on average marginal costs. In case of a mean preserving cost variation total costs of production decrease with the variance of marginal costs.[1] Gross revenue solely depends on aggregated output which again depends on average marginal costs. Therefore, producer surplus increases with the variance of marginal costs.[2] Consumer surplus, however, is unchanged in case of a mean preserving cost variation since market price only depends on aggregated output which again depends on average marginal costs.

This paper carries on this analysis to the differentiated good context.[3] In many industries the assumption of homogeneous goods is not appropriate since goods are rather less perfectly substitutable than perfect substitutes. In my analysis I do not focus on vertically differentiated goods but on horizontally differentiated products. Each firm incurs constant but different marginal costs. Firms cost structure is the sole reason for market heterogeneity since products not differ in quality.[4] In contrast to the homogeneous good case consumer surplus is expected to decrease with the variance of marginal costs at first sight. Since goods are differentiated not only industry output but also its distribution on the different firms is crucial for households' utility since there is a diminishing marginal utility of each good. Given a fixed amount of industry output an even distribution on each variety would maximize households' utility. Since equilibrium output and respective marginal costs are inversely proportional, consumer surplus is expected to decrease with the variance of marginal costs. As you can see in the course of the analysis households' gross utility

[1]Salant and Shaffer (1999) useing results from Bergstrom and Varian (1985) show that aggregate costs of production strictly decrease with the variance of marginal costs.

[2]Van Long and Soubeyran (2001) show that industry profits is an increasing function of the variance of the marginal costs if the average of marginal costs is constant.

[3]Corchón and Zudenkova (2009) use a similar model but analyze welfare losses under imperfect Cournot competition. Assuming Dixit-utility they continue the analysis of Corchón (2008).

[4]Quality differences could easily be implemented in my analysis. However, Symeonidis (2003) already showed that consumer surplus as well as producer surplus increase with the variance of quality levels if average quality is constant. Intuitively, quality differences may reinforce or counteract the effects of changes in cost structure. But, the simultaneous analysis of both cost and quality differences yields no additional insight.

actually decrease with the variance of marginal costs. Moreover, consumer surplus should decrease with market heterogeneity since firms have market power to choose prices above the competitive level since goods are less good substitutable. Market power should be profitable for producers but harmful for consumers. But, it can be shown that the exact opposite is true. In case of a differentiated goods consumer surplus as well as producer surplus increase with the variance of marginal costs.

Assuming standard quadratic utility originated by Dixit (1979) I consider an oligopoly consisting of n firms. Each firm produces a variety of a differentiated good and incurs constant but different marginal costs without fixed costs. In the light of the analysis of Kreps and Scheinkman (1983) the assumption of Cournot competition is reasonable in capacity constraint industries.[5] Firms exhibit different but constant marginal costs without fixed costs. Since preferences are quasi-linear social surplus is the measure for Pareto-optimality. In analogy to Février and Linnemer (2004) the impact of an arbitrary marginal costs variation on consumer surplus and producer surplus and, therefore, social surplus is decomposed into an average impact and a heterogeneity impact. The former lets the variance of marginal costs unchanged whereas the latter is a mean preserving cost variation.

In case of homogeneous goods an increase of all marginal costs can be welfare if there is an output shift from inefficient firms to efficient firms. According to Kimmel (1992) this reallocation increases producer surplus and overcompensates diminishing consumer surplus (Seade (1985)) if inverse demand is sufficiently concave and the disparity of output levels is sufficiently high. Since inverse demand is linear in case of Dixit utility the average impact on all surpluses is negative (positive) if all firms are negatively (positively) affected by the cost variation. The results concerning the average impact coincide with the homogeneous good case.

The analysis of the heterogeneity impact, however, yields striking new results. Indeed, households' gross utility decrease with the variance of marginal costs. Comparable to the homogeneous good case aggregated output solely depends on average marginal costs. Since there is a diminishing marginal utility of all goods gross utility decreases with the variance of marginal costs. But, aggregated households expenditures decrease with the variance of marginal costs,

[5]Note that the assumption of quantity competition is not crucial for the results. My results are robust with respect to the type of competition or the utility function. Assuming Shubik (1980) utility I come to same results.

too. Since diminishing aggregated expenditures outweigh declining gross utility consumer surplus increase with the variance of marginal costs in case of a mean preserving cost variation. In contrast to the homogeneous good case gross revenue (equal to aggregated expenditures) decrease with the variance of marginal costs. Nevertheless, producer surplus increase with the variance of marginal costs since diminishing total costs of production overcompensate declining gross revenue.

The intuition behind this result is the following: In case of a cost variation which increases the variance of marginal costs there is an output shift from less efficient to more efficient firms and vice versa. This result coincide with those of Farrell and Shapiro (1990) since they show that an output reduction of a single firm increases social surplus if market concentration increases sufficiently. This happens if a fairly inefficient and, therefore, small firm reduces its output whereas its more efficient competitors increase their output levels. Even though aggregated output declines and, therefore, consumers are worse off, social surplus increases since increasing producer surplus outweighs. In case of a mean preserving cost variation there is a simultaneous increase of the output level of a more efficient firm which reinforces this positive effect on welfare. My results hold true for all marginal costs combinations corresponding with non-negative output levels of all firms. Thus, my analysis generalizes the results of those authors analyzing negative welfare effects of marginal cost reductions of inefficient firms[6] since my results require no threshold values for marginal costs or market shares.

The remainder of the paper is organized as follows: the following section describes the framework of the model. Section 3 presents the central results. Section 4 finally concludes.

[6]For the first time Lahiri and Ono (1988) show that a reduction of the marginal costs of a single firm may reduce welfare if the firm is relatively inefficient. Zhao (2001) continues the analysis of Lahiri and Ono (1988) and derives threshold values for marginal cost and respective market shares such that a cost reduction reduces welfare. Smythe and Zhao (2006) refine the analysis of Zhao (2001) and allow for nonlinear demand and nonlinear costs as well as technological spill-over. Wang and Zhao (2007) extend the analysis of Lahiri and Ono (1988) and Zhao (2001) in the differentiated good context. Assuming a utility originated by Shubik (1980) they derive conditions under which marginal cost reductions reduce welfare in Cournot and Bertrand competition.

2 The model

Consider a market consisting of n firms each of them producing one variety of a differentiated good Q_i with $i = 1, \ldots, n$. Abstracting from fixed cost, each firm incurs constant marginal cost c_i. Let denote q_i the quantity of good i produced by firm i. The quasi-linear preferences of the representative household are described by a quadratic utility according to Dixit (1979). The corresponding inverse demand of good $i = 1, \ldots, n$ is given as follows:

$$p_i = 1 - q_i - \nu Q_{-i} \tag{2.1}$$

Let $Q_{-i} := \sum_{j \neq i} q_j$ denote aggregated output of i's competitors. The parameter of substitution is given by ν. For $\nu > 0$ the goods are substitutes and for $\nu < 0$ the goods are complements. In case of $\nu = 0$ there are n independent goods. To secure that utility is concave the parameter of substitution is assumed to be $\nu \in \left(-\frac{1}{n-1}, 1\right)$. For further insight see appendix A. Each firm maximizes its profit choosing an optimal quantity. Let Q^* denote aggregated output in equilibrium. Summing up the n first order conditions given by $1 - 2q_i^* - \nu Q_{-i}^* - c_i = 0$ and solving for Q^* yields:

$$Q^* = \frac{n(1 - \overline{c})}{2 + \nu(n-1)} \tag{2.2}$$

Let $\overline{c} := \frac{1}{n} \sum_{i=1}^{n} c_i$ denote average marginal cost which is assumed not to exceed 1. Hence, similar to the homogenous good context, aggregated output just depends on average marginal costs. Industry output Q^* is unchanged by a mean preserving cost variation. In contrast to aggregated output the derivation of the equilibrium output q_i^* of firm $i = 1, \ldots, n$ is little more tricky. Therefore the derivation is dedicated to the appendix.

Lemma 2.1 (Equilibrium output of a single firm) *The equilibrium output of firm $i = 1, \ldots, n$ is given as follows:*

$$q_i^* = \frac{(2 - \nu) - [2 + \nu(n-2)]c_i + \nu \sum_{j \neq i} c_j}{(2 - \nu)[2 + \nu(n-1)]}$$

Obviously equilibrium output q_i^* just depends on the sum of competitors marginal cost and not its distribution. This will be crucial for some results as you can see below. As shown in the appendix corresponding equilibrium price p_i^* is given by $p_i^* = q_i^* + c_i$. Analogously to homogenous goods the equilibrium profit $\Pi_i^* := (p_i^* - c_i)q_i^*$ of firm $i = 1, \ldots, n$ is just its squared quantity. Hence:

$$\Pi_i^* = (p_i^* - c_i)q_i^* = (q_i^*)^2 \tag{2.3}$$

To ensure non-negative equilibrium output of all n firms I assume $p_i^* - c_i = q_i^* \geq 0$ for $i = 1, \ldots, n$. Solving $q_i^* \geq 0$ for c_i yields the expression is the following assumption:

Assumption 2.1 (Oligopoly of n firms) *To ensure an oligopoly consisting of n firms, I assume $q_i^* \geq 0$ for $i = 1, \ldots, n$ which is equivalent to the following inequality:*

$$c_i \leq \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j$$

Note that in case of substitutes assumption 2.1 requires that marginal cost must never exceed 1 (equal to the maximum willingness to pay). In case of complements, however, marginal cost may exceed 1 if rivals are sufficiently efficient. This is due to the effect, that in case of complements the willingness to pay for a good increases with the consumption of rivals output which in turn is in reverse proportion to respective marginal cost.

3 Results

In the following the central results concerning producer surplus, consumer surplus and social surplus are presented. In the terminology of Février and Linnemer (2004) the impact of an arbitrary cost variation on the aforementioned variables is decomposed into an average and a heterogeneity impact. The average component comprises cost variations such that the variance is unchanged. The heterogeneity component comprises a change of the variance of marginal costs whereas average marginal costs is unchanged.

Definition 3.1 (Average and heterogeneity impact) *Let denote* AIF *the average impact on the function F and* HIF *the heterogeneity impact on F respective. In my study F is given by producer surplus, consumer surplus and social surplus. The total derivative of F is given by $dF = \sum_{k=1}^{n} \frac{\partial F}{\partial c_k} dc_k$. The average impact is characterized by $dc_1 = \ldots = dc_n = dc$. Without loss of generality the heterogeneity impact is given by a variation of c_k and c_l with $k < l$ such that $dc_k = -dc_l > 0$. The average impact* AIF *and the heterogeneity impact* HIF *on F are given as follows:*

$$\text{AIF} := \sum_{i=1}^{n} \frac{\partial F}{\partial c_i} \qquad \text{HIF} := \frac{\partial F}{\partial c_k} - \frac{\partial F}{\partial c_l}$$

Note that any arbitrary cost variation can be decomposed into these two components. As you can see in the appendix the directional vectors given in definition 3.1 just equal the Eigenvectors given by (B.2). The above defined decomposition just equals a principle axis transformation.

3.1 Producer surplus

In the following the relationship between producer surplus and the distribution of marginal costs is analyzed. Are producers better off in case of more heterogeneous market structures as suspected by antitrust authorities? Does any producer benefit from a cost variation which negatively affects all firms? Producer surplus $\text{PS}^* := \sum_i \Pi_i^*(q_i^*, Q_{-i}^*)$ is just the sum of the equilibrium profits of all firms. The impact of a cost variation on producer surplus is the sum of impacts on each firm.

Proposition 3.1 (Average Impact) *The average impact on the equilibrium profit of each firm and therefore the average impact on producer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation.*

Proof: The average impact on producer surplus is just the sum of the average impact on each single firm profit. $\text{AIPS}^* = \sum_j \text{AIPS}_j^*$ with $\text{AIPS}_j^* = \sum_i \partial_i \left(q_j^*\right)^2$ since $\Pi_i^* = (q_i^*)^2$. It holds:

$$\begin{aligned} \text{AIPS}_j^* &= \sum_i \partial_i \left(q_j^*\right)^2 = 2q_j^* \sum_i \partial_i q_j^* \\ &= 2q_j^* \left(\frac{-[2+\nu(n-2)] + \nu(n-1)}{(2+\nu)[2+\nu(n-1)]} \right) \qquad (3.1) \\ &= \frac{-2q_j^*}{2+\nu(n-1)} < 0 \qquad (3.2) \end{aligned}$$

The average impact on producer surplus is just the sum of all AIPS_j^* given by (3.2).

$$\text{AIPS}^* = \sum_j \frac{-2q_j^*}{2+\nu(n-1)} = \frac{-2Q^*}{2+\nu(n-1)} < 0 \tag{3.3}$$

Since $n \geq 2$ and $\nu \in \left(-\frac{1}{n-1}, 1\right)$ per assumption the average impact on producer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation. □

An increase of all marginal costs has two opposite effects on the equilibrium profit of firm i. On the one hand, making i's competitors less efficient has a positive effect on i's profit since all substitutes of product i are getting more expensive and, therefore, less attractive. This effect is given by $\nu(n-1)$ in (3.1). On the other hand the firm self is disadvantaged by the increase of c_i. This effect is given by $-[2+\nu(n-2)]$ in (3.1). Since the latter effect outweighs the former effect each firm is disadvantaged by an increase of all marginal costs and vice versa.

This result coincides with the homogenous good case since equilibrium profit of each firm and, therefore, producer surplus decrease if all firms are negatively affected in case of linear demand. If inverse demand is sufficiently high and inverse demand is concave enough there is a shift in production from inefficient firms to efficient firms which makes producers better off and possibly overcompensates consumer surplus losses such that social surplus increase. Compare Seade (1985), Kimmel (1992) or Février and Linnemer (2004).

In the following the heterogeneity impact on producer surplus is analyzed. This mean preserving cost variation lets average efficiency of the firms unchanged. A big market share and a highly concentrated market is suspected to promote market power. The exercise of market power is presumed to be profitable for firms. As you can see in the following proposition this presumption is true.

Proposition 3.2 (Heterogeneity Impact) *In case of a mean preserving cost variation producer surplus increases with the variance of the marginal costs.*

Proof: According to (2.3) equilibrium profit is given by $\Pi_i^* = (q_i^*)^2$. The heterogeneity impact $\text{HIQ}^* := \partial_k q_i^* - \partial_l q_i^*$ on equilibrium output q_i^* is given as follows:

$$\mathrm{HIQ}_i^* = \begin{cases} \frac{-1}{2-\nu}, & \text{for } i = k, \\ \frac{1}{2-\nu}, & \text{for } i = l, \\ 0, & \text{else.} \end{cases} \tag{3.4}$$

The heterogeneity impact on the equilibrium profit of unaffected firms $i \neq k, l$ is zero. Hence, the heterogeneity impact on producer surplus is just the sum of the heterogeneity impact on Π_k^* and Π_l^* which is given as follows:

$$\begin{aligned} \mathrm{HIPS}^* &= \mathrm{HIPS}_k^* + \mathrm{HIPS}_l^* \\ &= \left[\partial_k (q_k^*)^2 - \partial_l (q_k^*)^2\right] + \left[\partial_k (q_l^*)^2 - \partial_l (q_l^*)^2\right] \\ &= 2q_k^* \,\mathrm{HIQ}_k^* + 2q_l^* \,\mathrm{HIQ}_l^* \\ &\overset{(3.4)}{=} 2\,\mathrm{HIQ}_k^* (q_k^* - q_l^*) \\ &= \frac{2}{(2-\nu)^2}(c_k - c_l) \end{aligned} \tag{3.5}$$

Thus, producer surplus increase with the variance of marginal costs. □

The intuition behind this result is the following: A firm which is positively affected by the cost variation benefits due to two effects: Firstly, respective equilibrium output increases according to lemma 2.1. Secondly, its price-cost margin increases, too (equation (2.3)). The reverse holds true for the firm which is negatively affected by the variation. In case of a cost variation which increases the variance the ex ante more efficient and, therefore, bigger firm is getting even larger. Thus, additional profit overcompensates the losses of the firms which is negatively affected by the cost variation.

Even though this result is comparable to the homogeneous good case[7] it is not self-evident in context of differentiated goods since there is an essential effect which counteracts: In the homogeneous good case gross revenue (equal to households' total expenditures) is constant in case of a mean preserving cost variation. Remember that the equilibrium price solely depends on industry output which in turn just depends on average marginal costs. Since total costs of production decrease with the variance of marginal costs producers are better off in case of more heterogeneous market structures. In context of differentiated goods, however, gross revenue declines with the variance of marginal costs.

[7]Compare Bergstrom and Varian (1985) or Février and Linnemer (2004), for instance.

Consequently, the heterogeneity impact on producer surplus is decomposed into its components: gross revenue and total costs. The corresponding results are summarized in the following lemma.

Lemma 3.1 (Total revenue versus total cost) *Both firms' total revenue and total costs decrease with the variance of marginal costs. The effect of total costs outweighs the effect on total revenue. Hence producer surplus increases (decreases) in case of a mean preserving cost variation which increases (decreases) the variance of marginal costs.*

Thus, there is a positive relationship between market heterogeneity and producer surplus as suspected by antitrust authorities. US Merger Guidelines, for instance, presume that firms' market power increase with market concentration. The exercise of market power is presumed to be profitable for firms but harmful for consumers.[8] As you have seen above the first assumption is true. But, as you can see below the second assumption is false.

3.2 Consumer surplus

As you have seen in the previous section producer surplus increases with the variance of marginal costs. Thus, producers are better off in more heterogeneous market structures. The presumption of antitrust authorities that a big market share and a highly concentrated market, respectively, promote market power which is profitable for producers seems to be true. But what about the consumers? The exercise of market power is suspected to be detrimental for competitiveness of the market and, therefore, should be harmful for consumers. Remember that equilibrium profit just equals squared quantity. The price-cost margin equals equilibrium output. Thus, a big equilibrium output corresponds with a big price-cost margin which indicates market power. This market power should be harmful for consumers. Therefore, a homogeneous market structure consisting of equipollent firms should provide more favorable conditions for consumers. As you can see below the exact opposite is true.

Let $\mathrm{CS}^* := U\left(m - \sum_{i=1}^{n} p_i^* n q_i^*, q_1^*, \ldots, q_n^*\right) - U(m, 0, \ldots, 0)$ denote consumer surplus caused by the consumption of the n differentiated products q_i^* with $i = 1, \ldots, n$. The consumption of the numeraire good z is given by $q_0^* = m - \sum_{i=1}^{n} p_i^* q_i^*$. Let m denote the income of the representative household which is assumed to be exogenous.

[8] US Merger Guidelines: 'Other things being equal, market concentration affects the likelihood that one firm, or a small group of firms, could successfully exercise market power.'

Proposition 3.3 (Average Impact) *Consumer surplus increases (decreases) if all firms are positively (negatively) affected by the cost variation.*

Increasing marginal costs of all firms decreases equilibrium quantities of all brands and, therefore, industry output decrease, too. This makes consumers unambiguously worse off. This result coincides with the homogeneous good case. Compare Février and Linnemer (2004), for instance.

Let us now analyze the impact of a mean preserving cost variation on consumer surplus. As aforementioned firms' market power is suspected to increase with respective market share and market concentration, respectively. Price-cost margin is proportional to equilibrium output. In the homogeneous good case consumers are indifferent if average marginal costs is constant. In the differentiated good context products are not perfectly substitutable and, therefore, firms have market power. Moreover, there is a diminishing marginal utility of each product. Hence, given a fixed industry output consumers prefer each brand in same quantity. Nevertheless, it can be shown that the exact opposite is true.

Proposition 3.4 (Heterogeneity Impact) *Consumer surplus increases with the variance of marginal costs if average marginal costs are constant. In case of perfect substitutes (i.e. $\nu = 1$) consumer surplus is unchanged in case of a mean preserving cost variation.*

Proposition 3.4 is the central insight of present paper. Even though there are many counter-arguments consumers are better off in heterogeneous market structures. According to equation (2.2) aggregated output is unchanged by a mean preserving cost variation. But, the disparity of output levels increases with the variance of marginal costs. Normally, consumers prefer each brand in same quantity. For a better understanding the heterogeneity impact on consumer surplus is decomposed into its components: households' utility and aggregated expenditures. Since households expenditures' just equal firms' gross revenue, we can use (D.6) to show that households' expenditures decline with the variance of marginal costs. It remains to analyze the heterogeneity impact on consumers' gross utility. The central results are summarized as follows:

Lemma 3.2 (Expenditures versus utility) *Households' total expenditures (equal to firms' gross revenue) as well as households' gross utility decrease with* *the variance of marginal costs. The heterogeneity impact on households' expenditures, however, outweigh the heterogeneity impact on gross utility. Thus, consumer surplus increases with the variance of marginal costs.*

After a mean preserving cost variation which increases the variance of marginal costs, households consume a more uneven distribution of quantities of the different brands. Intuitively, this effect makes consumers worse off since it decreases households' gross utility. But, the supply of the good which is positively affected by the cost variation increases disproportionately high. The more efficient firm offers a bigger quantity and sell its product at a lower price. This 'aggressive' behavior is profitable for the firm itself (cf. proposition 3.2). But, the increased supply of the firm which is positively affected by the cost variation suppresses the output of the less efficient firm which is negatively affected by the variation. Declining households' expenditures outweigh diminishing gross utility and consumers are better off at the end.

In the following the heterogeneity impact on consumer surplus is analyzed from another point of view. The impact on consumer surplus can be broken down on the heterogeneity impact on the net benefit of each single commodity. Let CS_i denote the net utility caused by the consumption of good $i = 1, \ldots, n$ which is defined as follows:

$$\mathrm{CS}_i := q_i - \frac{1}{2}q_i^2 - \frac{\nu}{2}q_i Q_{-i} - p_i q_i$$

The term $q_i - \frac{1}{2}q_i^2$ reflects the direct utility caused by the consumption of good q_i. The term $\frac{\nu}{2}q_i Q_{-i}$ describes additional utility (or disutility) caused by the simultaneous consumption of the n goods. The expenditures associated with the consumption of commodity i is given by $p_i q_i$. Consumer surplus CS is just aggregated net utility of all n goods. According to (3.4) the heterogeneity impact on the quantities of non-affected firms and thus corresponding equilibrium price is zero. Hence, net utility of the non-affected goods is unchanged in case of mean preserving cost variation since aggregated concurrence output is unchanged. The heterogeneity impact on consumer surplus is the sum of the heterogeneity impacts on the affected goods. The results concerning the net utility of the affected goods are summarized as follows:

Lemma 3.3 (Net utility of a single good) *In case of a mean preserving cost variation which increases the variance of marginal costs additional net* *utility of the good which is positively affected by the cost variation outweighs the loss of net utility of the good which is negatively affected.*

Proof: Consumer surplus can be expressed as follows:

$$\begin{aligned}\text{CS}^* =& q_k^* - \frac{1}{2}(q_k^*)^2 - \frac{\nu}{2} q_k^* Q_{-k}^* - p_k^* q_k^* && (3.6)\\ &+ q_l^* - \frac{1}{2}(q_l^*)^2 - \frac{\nu}{2} q_l^* Q_{-l}^* - p_l^* q_l^* && (3.7)\\ &+ \sum_{j\neq k,l} \left(q_j^* - \frac{1}{2}(q_j^*)^2 - \frac{\nu}{2} q_j^* Q_{-j}^* - p_j^* q_j^* \right)\end{aligned}$$

According to (3.4) the heterogeneity impact on equilibrium output and equilibrium price of the unaffected goods is zero. Since aggregated output just depends on average marginal cost (cf. (2.2)) the heterogeneity impact on aggregated concurrence output Q_{-i}^* is zero, too. Hence, the heterogeneity impact on consumer surplus is just the sum of the heterogeneity impact HICS_k^* and HICS_l^*, respectively. HICS_k^* is given as follows:

$$\begin{aligned}\text{HICS}_k^* =& \partial_k q_k^* \left(1 - q_k^* - \frac{\nu}{2} Q_{-k}^*\right) - \frac{\nu}{2} q_k^* \partial_k Q_{-k}^* - \partial_k p_k^* q_k^* - p_k^* \partial_k q_k^*\\ &- \partial_l q_k^* \left(1 - q_k^* - \frac{\nu}{2} Q_{-k}^*\right) + \frac{\nu}{2} q_k^* \partial_l Q_{-k}^* + \partial_l p_k^* q_k^* + p_k^* \partial_l q_k^*\end{aligned}$$

Equilibrium price p_i^* is given by $p_i^* = 1 - q_i^* - \nu Q_{-i}^*$ for $i = 1, \ldots, n$. Furthermore, it holds: $p_i^* = q_i^* + c_i$ for $i = 1, \ldots, n$. Thus, HICS_k^* is given as follows:

$$\begin{aligned}\text{HICS}_k =& -\frac{\nu}{2} q_k^* \partial_k Q_{-k}^* + \partial_k q_k^* \frac{\nu}{2} Q_{-k}^* - \partial_k p_k^* q_k^*\\ &+ \frac{\nu}{2} q_k^* \partial_l Q_{-k}^* - \partial_l q_k^* \frac{\nu}{2} Q_{-k}^* + \partial_l p_k^* q_k^*\end{aligned}$$

Remember that the heterogeneity impact on q_i^* is denoted by HIQ_i^*. The heterogeneity impact $\text{HIQ}_{-k} := \partial_k Q_{-k}^* - \partial_l Q_{-k}^*$ on aggregated concurrence output is given by $\text{HIQ}_{-k} = \text{HIQ}_l^* = -\text{HIQ}_k^*$. Since the equilibrium price is given by $p_i^* = q_i^* + c_i$ the heterogeneity impact on p_i^* is given by $\text{HIP}_i^* = \text{HIQ}^* + \text{HIC}_i$ whereas HIC_i denotes the 'heterogeneity impact' on the marginal cost of firm $i = 1, \ldots, n$ with $\text{HIC}_k = 1$, $\text{HIC}_l = -1$ and $\text{HIC}_i = 0$ for $i \neq k, l$.

$$\begin{aligned}\mathrm{HICS}_k &= \frac{\nu}{2}Q^*_{-k}\,\mathrm{HIQ}^*_k + \frac{\nu}{2}q^*_k\,\mathrm{HIQ}^*_k - q^*_k\,\mathrm{HIQ}^*_k - q^*_k \\ &= \frac{\nu}{2}Q^*\,\mathrm{HIQ}^*_k - \left(\frac{1-\nu}{2-\nu}\right) q^*_k < 0\end{aligned}$$

Since HIQ^*_k is negative HICS^*_k is also negative irrespective the distribution of marginal costs or the degree of substitutability ν. Similarly CS_l can be derived which is given by $\mathrm{CS}_l = -\frac{\nu}{2}Q^*\,\mathrm{HIQ}^*_k + \left(\frac{1-\nu}{2-\nu}\right) q^*_l > 0$. Summing up HICS^*_k and HICS^*_l yield the heterogeneity impact on consumer surplus given by (F.1). □

In case of a mean preserving variation which increases the variance the additional net utility of the good which is positively affected by the variation outweighs the losses of net utility of the good which is negatively affected even though there is a diminishing marginal utility of each good. This is due to the fact that declining expenditures associated with the consumption of the good which is positively affected overcompensates.

3.3 Social surplus

In view of the results of the previous sections the analysis of social surplus is for the sake of completeness. In case of quasi-linear preferences social surplus is defined as follows:

$$W := U\left(m - \sum_{i=1}^{n} c_i q_i, q_1, \ldots, q_n\right) - U(m, 0, \ldots, 0)$$

Since both producer surplus and consumer surplus increase with the variance of marginal costs, social surplus increase with the variance of marginal costs, too. The following results are a direct implication of the results of the previous sections.

Corollary 3.1 (Average Impact on social surplus) *The average impact on social surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation.*

This result is comparable to the homogeneous good case if inverse demand is linear. As you can see in Février and Linnemer (2004) an increase of all marginal costs can be welfare enhancing if there are at least 3 firms in the market, market heterogeneity is sufficiently high and inverse demand is sufficiently concave. In this case there is an output shift from less efficient firms to more efficient firms. The welfare gains due to a more efficient production overcompensates welfare losses due to less consumer surplus. Since inverse demand is linear in case of Dixit-utility an increase (a reduction) of all marginal costs makes society unambiguously worse off (better off). The heterogeneity impact on social surplus is given as follows:

Corollary 3.2 (Heterogeneity Impact) *Social surplus increases with the variance of marginal costs.*

Even though this result is similar to the homogeneous good case there is a small but significant difference: Households' gross utility decreases with the variance of marginal costs. Diminishing total costs of production, however, outweigh and, therefore, social surplus increases with the dispersion of marginal costs.

4 Conclusion

This paper shows that in contrast to homogeneous good models not only producer surplus but also consumer surplus increase with the variance of marginal costs. Moreover, this paper shows that the positive relationship, especially between consumer surplus and market heterogeneity, not only holds true in vertically differentiated good models but also in horizontally differentiated models with cost asymmetry.[9] Since my analysis also allows for the shutdown of a sufficiently inefficient firms (i.e. $q_i^* = 0$) this analysis illustrates that the existence of fixed costs is not necessary for a shutdown to be welfare enhancing.[10] In this context my results are in line with those authors analyzing possible negative welfare effects of cost reductions.[11] In contrast to this strand of liter-

[9]Symeonidis (2003) analyzes a vertically differentiated good oligopoly. Firms, however, incur incur identical marginal costs. Quality differences are the sole reason for market heterogeneity.

[10]Assuming Dixit-utility and a free-entry equilibrium Koh (2008) shows that the shutdown of a firm is welfare enhancing if cost structure exhibits constant and identical marginal costs since there is excessive entry.

[11]Cf. Lahiri and Ono (1988), Zhao (2001), Smythe and Zhao (2006) or Wang and Zhao (2007).

ature my results apply for the widest set of marginal costs combinations since I require no upper limits for marginal costs and just assume a non-negative equilibrium output of each firm. Therefore, my analysis is helpful for antitrust authorities since it allows a better understanding of the relationship between market heterogeneity and all surpluses, especially consumer surplus. Moreover, this paper helps to reduce prejudices that either a maximum of product diversity is necessarily welfare enhancing or a market consisting of equipollent firms provides favorable conditions for consumers.

Consumers must not worry about a mean preserving cost variation increasing the variance of marginal costs. Consumers as well as producers are better off in more heterogeneous market structures. Firms' profit maximizing behavior is not necessarily at the expense of consumers surplus. Even though big firms have a higher price-cost margin compared to smaller firms (remember that equilibrium quantity equals respective price-cost margin) these firms pass efficiency gains down to consumers by offering a bigger quantity at a lower price. Hence, a big price-cost margin rather indicates firms' efficiency than market power which is harmful for consumers.

Furthermore, my analysis rather supports the Chicago school of antitrust than the Harvard business school since market structure (and thus market concentration) is rather a result of the market process than a bad starting point for competition as suspected by the structure-conduct-performance paradigm.

A Utility

The quadratic utility according to Dixit (1979) is given as follows:

$$U(q_0, q_1, \ldots, q_n) = q_0 + \sum_i q_i - \frac{1}{2}\boldsymbol{q}^T H \boldsymbol{q}$$

whereas q_0 denotes the numeraire good and the matrix of substitution H is given as follows:

$$H = \begin{pmatrix} 1 & \nu & \cdots & \nu \\ \nu & 1 & \cdots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \cdots & 1 \end{pmatrix}$$

Since the corresponding Hessian $\nabla^2 U = -H$ is real and symmetric, it can be decomposed by $P^{-1}DP = -H$ whereas . Let denote D the matrix which contains the Eigenvalues and the matrix P consists of the Eigenvalues of the Hessian. You can prove the correctness by calculating $-HP = PD$. Cf. Jänich (2002), p. 219.

$$D = \begin{pmatrix} -1+\nu & 0 & \cdots & 0 & 0 \\ 0 & -1+\nu & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & -1+\nu & 0 \\ 0 & 0 & \cdots & 0 & [-1-\nu(n-1)] \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 1 \\ -1 & 1 & \cdots & 0 & 0 & 1 \\ 0 & -1 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & 1 & 1 \\ 0 & 0 & \dots & 0 & -1 & 1 \end{pmatrix}$$

Consumers utility is concave if the corresponding Hessian is negative definit which in turns is true, if all Eigenvalues are negative. Cf. Königsberger (1993), p.74. Hence: $-1+\nu < 0 \Leftrightarrow \nu < 1$ and $-1-\nu(n-1) < 0 \Leftrightarrow \nu > -\frac{1}{n-1}$. Thus we assume: $\nu \in \left(-\frac{1}{n-1}, 1\right)$. Utility as given above can be expressed as follows:

$$U(q_0, q_1, q_2, \dots, q_n) = \sum_i q_i - \frac{1}{2}\sum_i (q_i)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i q_j \tag{A.1}$$

B Proof of lemma 2.1

Firm $i = 1, \dots, n$ maximizes its profit $\Pi_i = p(q_i + Q_{-i})q_i - c_i q_i$ choosing an optimal q_i. Note that inverse demand $p(q_i + Q_{-i}) = 1 - q_i - \nu Q_{-i}$ is given by (2.1). The first order condition of firm $i = 1, \dots, n$ is given by $1 - 2q_i - \nu Q_{-i} - c_i = 0$. In matrix form these n first order conditions can be expressed as follows:

$$\begin{pmatrix} 2 & \nu & \cdots & \nu \\ \nu & 2 & \cdots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \cdots & 2 \end{pmatrix} \boldsymbol{q} = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ 1-c_3 \\ \vdots \\ 1-c_n \end{pmatrix}$$

Let denote A the matrix of coefficients of $\boldsymbol{q}$ and $\boldsymbol{c}^T = (1-c_1, \ldots, 1-c_n)$ the vector of constants. Since A is real and symmetric the matrix of coefficients can be decomposed by $A = PDP^{-1}$. Hence $A\mathbf{q} = \mathbf{c}$ can be expressed by $PDP^{-1}\boldsymbol{q} = \boldsymbol{c}$. Let denote P the matrix of Eigenvectors and D the diagonal matrix containing the corresponding Eigenvalues. It is easy to proof that $\lambda_1 = 2 - \nu$ is an $n-1$ fold Eigenvalue of A and $\lambda_2 = 2 + \nu(n-1)$ is the n-th Eigenvalue. Hence the diagonal matrix D is given as follows:

$$D = \begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 \\ 0 & 2-\nu & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 2-\nu & 0 \\ 0 & 0 & \cdots & 0 & [2+\nu(n-1)] \end{pmatrix} \tag{B.1}$$

The matrix P of the corresponding Eigenvectors $\boldsymbol{v}_i$ with $i = 1, \ldots, n$ is given as follows:

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 1 \\ -1 & 1 & \cdots & 0 & 0 & 1 \\ 0 & -1 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & 1 & 1 \\ 0 & 0 & \ldots & 0 & -1 & 1 \end{pmatrix} \tag{B.2}$$

Just prove the accuracy of (B.1) and (B.2) by calculating $AP = PD$. The Cournot-Nash equilibrium q_i^* for $i = 1, \ldots, n$ is determined by solving $PDP^{-1}\boldsymbol{q}^* = \boldsymbol{c}$ in two steps. Firstly we solve $PD\boldsymbol{z}^* = \boldsymbol{c}$ for $\boldsymbol{z}^* := P^{-1}\boldsymbol{q}$. The solution of $\boldsymbol{q}^*$ can now easily be obtained by calculating $\boldsymbol{q}^* = P\boldsymbol{z}^*$. The optimal vector $\boldsymbol{z}^*$ must solve the following system of linear equations $PD\boldsymbol{z}^* = \boldsymbol{c}$:

$$
\begin{pmatrix}
2-\nu & 0 & \cdots & 0 & 0 & [2+\nu(n-1)] \\
-(2-\nu) & 2-\nu & \cdots & 0 & 0 & [2+\nu(n-1)] \\
0 & -(2-\nu) & \cdots & 0 & 0 & [2+\nu(n-1)] \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & -(2-\nu) & 2-\nu & [2+\nu(n-1)] \\
0 & 0 & \cdots & 0 & -(2-\nu) & [2+\nu(n-1)]
\end{pmatrix} \boldsymbol{z}^* = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ \vdots \\ 1-c_n \end{pmatrix}
$$

Summing up the first and the second row yields the new second row. The new second row is added to the third row which again yields the new third row and so on. The resulting row echelon form is given as follows:

$$
\begin{pmatrix}
2-\nu & 0 & \cdots & 0 & 0 & [2+\nu(n-1)] \\
0 & 2-\nu & \cdots & 0 & 0 & 2[2+\nu(n-1)] \\
0 & 0 & \cdots & 0 & 0 & 3[2+\nu(n-1)] \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 2-\nu & (n-1)[2+\nu(n-1)] \\
0 & 0 & \cdots & 0 & 0 & n[2+\nu(n-1)]
\end{pmatrix} \boldsymbol{z}^* = \begin{pmatrix} 1-c_1 \\ 2-c_1-c_2 \\ \vdots \\ (n-1)-\sum_{i=1}^{n-1} c_i \\ n-\sum_{i=1}^{n} c_i \end{pmatrix}
$$

Solving the last row for z_n^* yields:

$$
z_n^* = \frac{n-\sum_{i=1}^{n} c_i}{n[2+\nu(n-1)]} \tag{B.3}
$$

Inserting z_n^* given by (B.3) in the row before last is given by

$$
(2-\nu)z_{n-1}^* + (n-1)[2+\nu(n-1)]z_n^* = (n-1) - \sum_{i=1}^{n-1} c_i
$$

yield the solution for z_{n-1}^* which is given as follows:

$$
\begin{aligned}
z^*_{n-1} &= \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1)[2+\nu(n-1)]z^*_n\right) \\
&\overset{(B.3)}{=} \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1)[2+\nu(n-1)]\left(\frac{n-\sum_{i=1}^n c_i}{n[2+\nu(n-1)]}\right)\right) \\
&= \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1) + \frac{n-1}{n}\sum_{i=1}^{n} c_i\right) \\
&= \frac{1}{2-\nu}\left(\frac{n-1}{n}\sum_{i=1}^{n} c_i - \sum_{i=1}^{n-1} c_i\right) \qquad \text{(B.4)}
\end{aligned}
$$

Since the equilibrium quantities q_i^* are given by $\boldsymbol{q}^* = P\boldsymbol{z}^*$, the solution for q_n^* is given by $q_n^* = -z^*_{n-1} + z^*_n$ with z^*_{n-1} and z^*_n given by (B.3) and (B.4) respective. Hence:

$$
\begin{aligned}
q_n^* &= \frac{-1}{2-\nu}\left(\frac{n-1}{n}\sum_{i=1}^{n} c_i - \sum_{i=1}^{n-1} c_i\right) + \frac{n-\sum_{i=1}^n c_i}{n[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - \frac{2-\nu}{n}\sum_{i=1}^n c_i + [2+(n-1)\nu]\left(\sum_{i=1}^{n-1} c_i - \frac{n-1}{n}\sum_{i=1}^n c_i\right)}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - \frac{2-\nu}{n}\sum_{i=1}^n c_i + 2\sum_{i=1}^{n-1} c_i + (n-1)\nu\sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&\quad + \frac{-2\frac{n-1}{n}\sum_{i=1}^n c_i - \frac{(n-1)^2}{n}\nu\sum_{j=1}^n c_j}{(2-\nu)[2+(n-1)\nu]}
\end{aligned}
$$

Rearranging the terms by collecting the coefficients of c_n and c_i for $i \neq n$ yields:

$$
\begin{aligned}
q_n^* &= \frac{(2-\nu) + \left[-\frac{2-\nu}{n} - 2\frac{n-1}{n} - \frac{n-1}{n}(n-1)\nu\right] c_n}{(2-\nu)[2+(n-1)\nu]} \\
&\quad + \frac{\left[-\frac{2-\nu}{n} + 2 + (n-1)\nu - 2\frac{n-1}{n} - \frac{(n-1)^2}{n}\nu\right] \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) + \left[-2 + \nu\left(\frac{1}{n} - \frac{(n-1)^2}{n}\right)\right] c_n + \left[\nu\left(\frac{1}{n} + (n-1) - \frac{(n-1)^2}{2}\right)\right] \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - [2+\nu(n-2)]c_n + \nu \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]}
\end{aligned}
\tag{B.5}
$$

Analogously the equilibrium outputs q_i^* for $i = 1, \ldots, n-1$ can be derived. Hence q_i^* for $n = 1, \ldots, n$ is given as follows:

$$
q_i^* = \frac{(2-\nu) - [2+\nu(n-2)]c_i + \nu \sum_{j \neq i} c_i}{(2-\nu)[2+\nu(n-1)]} \qquad \square
$$

C Proof of equation 2.3

In the following I show that equilibrium profit Π_i^* of firm $i = 1, \ldots, n$ just equals its squared quantity q_i^*. For this purpose I show that $p_i^* - c_i = q_i^*$ is true. Equilibrium price p_i^* can be obtained by inserting equilibrium quantities given by (B.5) in the inverse demand. It holds $p_i^* - c_i = 1 - q_i^* - \nu Q_{-i}^* - c_i$ whereas aggregated concurrence output $Q_{-i}^* = \sum_{j \neq i} q_j^*$ in equilibrium is given as follows:

$$
Q_{-i}^* = \frac{(n-1)(2-\nu) - [2+\nu(n-2)] \sum_{j \neq i} c_j + \nu(n-1)c_i + \nu(n-2) \sum_{j \neq i} c_j}{(2-\nu)[2+\nu(n-1)]}
$$

It remains to show that $p_i^* - c_i = q_i^*$:

$$
\begin{aligned}
p_i^* - c_i =& - q_i^* + \frac{(2-\nu)[2+\nu(n-1)] - \nu(n-1)(2-\nu)}{(2-\nu)[2+\nu(n-1)]} \\
&+ \frac{-\nu^2(n-1) - (2-\nu)[2+\nu(n-1)]}{(2-\nu)[2+\nu(n-1)]} c_i \\
&+ \frac{\nu[2+\nu(n-2)] - \nu^2(n-2)]}{(2-\nu)[2+\nu(n-1)]} \sum_{j\neq i} c_j \\
=& - q_i^* + \frac{2(2-\nu) + [-4 + (2-n)2\nu]c_i + 2\nu \sum_{j\neq i} c_j}{(2-\nu)[2+\nu(n-1)]} \\
=& - q_i^* + 2q_i^* \\
=& q_i^* \quad \square
\end{aligned}
$$

D Proof of lemma 3.1

The heterogeneity impact $\text{HIR}^* := \partial_k R^* - \partial_l R^*$ on total revenue $R^* := \sum_i p_i^* q_i^*$ is just the sum of the heterogeneity impacts on each firms revenue.

$$
\begin{aligned}
\text{HIR}^* :=& \partial_k R^* - \partial_l R^* \\
=& \partial_k \sum_i R_i^* - \partial_l^* \sum_i R_i^* \\
=& \sum_i \Big(\partial_k R_i^* - \partial_l R_i^* \Big) \\
=& \sum_i \text{HIR}_i^*
\end{aligned}
$$

According to (3.4) the heterogeneity impact on the output of the unaffected firms is zero. Since $p_i^* = q_i^* + c_i$ the heterogeneity impact on the equilibrium price of the unaffected firms is zero. Thus, the heterogeneity impact on total revenue is given as follows:

$$
\text{HIR}^* = \text{HIR}_k^* + \text{HIR}_l^* \tag{D.1}
$$

The heterogeneity impact on revenue of firm i HIR_i^* is given as follows:

$$
\begin{aligned}
\mathrm{HIR}_i^* &:= \partial_k R_i^* - \partial_l R_i^* \\
&= \partial_k(p_i^* q_i^*) - \partial_l(p_i^* q_i^*) \\
&= \partial_k p_i^* q_i^* + p_i^* \partial_k q_i^* - (\partial_l p_i^* q_i^* + p_i^* \partial_l q_i^*) \\
&= (\partial_k p_i^* - \partial_l p_i^*) q_i^* + (\partial_k q_i^* - \partial_l q_i^*) p_i^* \\
&= \mathrm{HIP}_i^* \, q_i^* + \mathrm{HIQ}_i^* \, p_i^*
\end{aligned} \tag{D.2}
$$

Let HIP_i^* denote the heterogeneity impact on the equilibrium price of firm i. Since the equilibrium price $p_i^* \overset{(2.3)}{=} q_i^* + c_i$ the heterogeneity impact on market price p_i^* is given as follows:

$$
\begin{aligned}
\mathrm{HIP}_i^* &= \partial_k(q_i^* + c_i) - \partial_l(q_i^* + c_i) \\
&= \mathrm{HIQ}_i^* + \mathrm{HIC}_i
\end{aligned} \tag{D.3}
$$

Let $\mathrm{HIC}_i := \partial_k c_i - \partial_l c_i$ denote the 'heterogeneity impact' on the marginal cost of firm $i = 1, \dots, n$ with

$$
\mathrm{HIC}_i = \begin{cases} 1, \text{ for } i = k, \\ -1, \text{ for } i = l, \\ 0, \text{ else.} \end{cases} \tag{D.4}
$$

Hence, HIR_i^* is given as follows:

$$
\begin{aligned}
\mathrm{HIR}_i^* &\overset{(D.2)}{=} \mathrm{HIP}_i^* \, q_i^* + \mathrm{HIQ}_i^* \, p_i^* \\
&\overset{(D.3)}{=} (\mathrm{HIQ}_i^* + \mathrm{HIC}_i) q_i^* + \mathrm{HIQ}_i^* \, p_i^* \\
&= \mathrm{HIQ}_i^* (q_i^* + p_i^*) + \mathrm{HIC}_i \, q_i^* \\
&\overset{(2.3)}{=} \mathrm{HIQ}_i^* (2q_i^* + c_i) + \mathrm{HIC}_i \, q_i^*
\end{aligned}
$$

Since the heterogeneity impact on the quantity and the marginal cost of the unaffected firms $j \neq k, l$ is zero and $\mathrm{HIQ}_k = -\,\mathrm{HIQ}_l = \frac{-1}{2-\nu}$, the heterogeneity impact on revenue is given as follows:

$$
\mathrm{HIR}_i^* = \begin{cases} \frac{-1}{2-\nu}(2q_k^* + c_k) + q_k^*, \text{ for } i = k, \\ \frac{1}{2-\nu}(2q_l^* + c_l) - q_l^*, \text{ for } i = l, \\ 0, \text{ else.} \end{cases}
$$

Since $2\,\mathrm{HIQ}_k + 1 = \frac{-\nu}{2-\nu}$ the heterogeneity impact on the revenue of firm i is given as follows:

$$\mathrm{HIR}_i^* = \begin{cases} \left(\frac{-\nu}{2-\nu}\right) q_k^* + \frac{-1}{2-\nu} c_k, & \text{for } i = k, \\ \left(\frac{\nu}{2-\nu}\right) q_l^* - \frac{-1}{2-\nu} c_l, & \text{for } i = l, \\ = 0, & \text{else.} \end{cases} \tag{D.5}$$

In case of substitutes (i.e. $\nu \geq 0$) the heterogeneity impact on revenue k is negative and the heterogeneity impact on revenue l is positive. Note that in case of complements this is not true in general. Hence the heterogeneity impact on total revenue is given as follows:

$$\begin{aligned} \mathrm{HIR}^* &\overset{(D.1)}{=} \mathrm{HIR}_k^* + \mathrm{HIR}_l^* \\ &\overset{(D.5)}{=} \left[\left(\frac{-\nu}{2-\nu}\right) q_k^* + \frac{-1}{2-\nu} c_k\right] + \left[\left(\frac{\nu}{2-\nu}\right) q_l^* - \frac{-1}{2-\nu} c_l\right] \\ &= \left(\frac{-\nu}{2-\nu}\right)(q_k^* - q_l^*) + \frac{-1}{2-\nu}(c_k - c_l) \\ &= \frac{-\nu}{2-\nu}\left[\frac{-(c_k - c_l)}{(2-\nu)}\right] + \left(\frac{-1}{2-\nu}\right)(c_k - c_l) \\ &= -2\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \begin{cases} < 0, & \text{for } c_k > c_l, \\ = 0, & \text{for } c_k = c_l, \\ > 0, & \text{for } c_k < c_l. \end{cases} \end{aligned} \tag{D.6}$$

Total revenue increases with the variance of marginal costs. This result is true in case of substitutes and complements even though the heterogeneity impact on revenue R_k must not be negative in case of complements (cf. (D.5)). In the following the heterogeneity impact on total costs $C^* := \sum_i c_i q_i^*$ is investigated.

$$
\begin{aligned}
\partial_k C^* - \partial_l C^* &= \partial_k \sum_i c_i q_i^* - \partial_l \sum_i c_i q_i^* \\
&= \sum_i \Big(\partial_k (c_i q_i^*) - \partial_l (c_i q_i^*) \Big) \\
&= \sum_i \Big(\partial_k c_i q_i^* + c_i \partial_k q_i^* - \partial_l c_i q_i^* - c_i \partial_l q_i^* \Big) \\
&= \sum_i \Big(\mathrm{HIC}_i \, q_i^* + \mathrm{HIQ}_i^* \, c_i \Big) \\
&\overset{(D.4)}{=} q_k^* - q_l^* + \mathrm{HIQ}_k^* \, c_k + \mathrm{HIQ}_l^* \, c_l \\
&= \frac{-2(c_k - c_l)}{(2-\nu)} \begin{cases} > 0, & \text{for } c_k < c_l, \\ = 0, & \text{for } c_k = c_l, \\ < 0, & \text{for } c_k > c_l. \end{cases}
\end{aligned} \tag{D.7}
$$

The heterogeneity impact on total costs is negative (positive) if the more (less) efficient firm is getting more efficient. Obviously the heterogeneity impact on total revenue outweighs the heterogeneity impact on total costs for $c_k > c_l$:

$$
\begin{aligned}
\partial_k C^* - \partial_l C^* \overset{(D.7)}{=} \frac{-2}{2-\nu}(c_k - c_l) &< \frac{-2(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{(D.6)}{=} \partial_k R^* - \partial_l R^* \\
\Leftrightarrow 1 &> \frac{1-\nu}{2-\nu}
\end{aligned}
$$

For $c_k > c_l$ diminishing total costs of production outweigh diminishing gross revenue. For $c_k < c_l$ the exact opposite holds true. Thus, the heterogeneity impact on producer surplus is positive (negative) if the more (less) efficient firm is getting more efficient. Note that the heterogeneity impact on producer surplus is just the difference between the heterogeneity impact on revenue and total costs.

E Proof of proposition 3.3

According to (A.1) the Dixit-utility is given as follows:

$$U(q_1^*, q_2^*, \ldots, q_n^*) = \sum_i q_i^* - \frac{1}{2}\sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_i^* q_j^*$$

The average impact $\text{AICS}^* := \sum_{i=1}^{n} \partial_i \, \text{CS}^*$ on consumer surplus in equilibrium is given as follows:

$$\begin{aligned}
\text{AICS}^* &= \sum_k \partial_k \left(\sum_i q_i^* - \frac{1}{2}\sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_i^* q_j^* - \sum_i p_i^* q_i^* \right) \\
&= \sum_k \Bigg\{ \sum_i \partial_k q_i^* - \sum_i q_i^* \partial_k q_i^* - \nu \left(\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} (\partial_k q_i^* q_j^* + q_i^* \partial_k q_j^*) \right) \\
&\qquad - \sum_i (\partial_k p_i^* q_i^* + p_i^* \partial_k q_i^*) \Bigg\}
\end{aligned}$$

Since market price p_i^* is given by $p_i^* = q_i^* + c_i$ the average impact is given as follows:

$$\begin{aligned}
\text{AICS}^* = \sum_k \Bigg\{ &\sum_i \partial_k q_i^* - \sum_i \partial_k q_i^* q_i^* - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_j^* \partial_k q_i^* - \nu \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} q_i^* \partial_k q_j^* \\
&- \sum_i q_i^* \partial_k (q_i^* + c_i) - \sum_i (q_i^* + c_i) \partial_k q_i^* \Bigg\}
\end{aligned}$$

Rearranging the terms deftly allows to factor out $p_i^* = 1 - q_i^* - \nu Q_{-i}^*$.

$$\begin{aligned}
\text{AICS}^* &= \sum_k \left\{ \sum_i \partial_k q_i^* \left(1 - q_i^* - \nu Q_{-i}^* - c_i\right) - 2\sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \\
&= \sum_k \left\{ \sum_i \partial_k q_i^* \left(p_i^* - c_i\right) - 2\sum_i q_i^* \partial_k q_i^* - q_k^* \right\}
\end{aligned}$$

Since $p_i^* - c_i = q_i^*$ the average impact is given as follows:

$$\begin{aligned}
\text{AICS}^* &= \sum_k \left\{ \sum_i q_i^* \partial_k q_i^* - 2 \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \\
&= \sum_k \left\{ - \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \qquad \text{(E.1)} \\
&= - \sum_i q_i^* \sum_k \partial_k q_i^* - Q^*
\end{aligned}$$

Note that the average impact $\text{AIQ}_i^* := \sum_k \partial_k q_i^*$ on the equilibrium output of firm $i = 1, \ldots, n$ is given by $\text{AIQ}_i^* = \frac{-1}{2+\nu(n-1)}$.

$$\begin{aligned}
\text{AICS}^* &= - \sum_i q_i^* \, \text{AIQ}_i - Q^* \\
&= - \text{AIQ}_i \, Q^* - Q^* \\
&= -(\text{AIQ}_i + 1) Q^* \\
&= - \frac{1 + \nu(n-1)}{2 + \nu(n-1)} Q^* \qquad \text{(E.2)}
\end{aligned}$$

Since $-\frac{1+\nu(n-1)}{2+\nu(n-1)} \leq 0$ for $\nu \in \left(-\frac{1}{n-1}, 1\right)$ and $Q^* > 0$ the average impact on consumer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation. □

F Proof of proposition 3.4

In the following the heterogeneity impact on consumer surplus $\text{HICS}^* := \partial_k \text{CS}^* - \partial_l \text{CS}^*$ is derived. The partial derivatives $\partial_k \text{CS}^*$ and $\partial_l \text{CS}^*$ are given as follows:

$$\begin{aligned}
\partial_k \text{CS}^* &= \partial_k \left(\sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* - \sum_i p_i^* q_i^* \right) \\
&\overset{(E.1)}{=} - \sum_i q_i^* \partial_k q_i^* - q_k^* \\
\partial_l \text{CS}^* &\overset{(E.1)}{=} - \sum_i q_i^* \partial_l q_i^* - q_l^*
\end{aligned}$$

The heterogeneity impact on consumer surplus $\mathrm{HICS}^* := \partial_k \mathrm{CS} - \partial_l \mathrm{CS}$ is given as follows:

$$\begin{aligned}
\mathrm{HICS}^* &= -\sum_i q_i^* \partial_k q_i^* - q_k^* - \left(-\sum_i q_i^* \partial_l q_i^* - q_l^* \right) \\
&= -\sum_i q_i^* \left(\partial_k q_i^* - \partial_l q_i^* \right) - (q_k^* - q_l^*) \\
&= -\sum_i q_i^* \,\mathrm{HIQ}_i - (q_k^* - q_l^*)
\end{aligned}$$

According to (3.4) the heterogeneity impact on the equilibrium output of the unaffected firms is zero.

$$\begin{aligned}
\mathrm{HICS}^* &= -q_k^* \,\mathrm{HIQ}_k - q_l^* \,\mathrm{HIQ}_l - (q_k^* - q_l^*) \\
&= -(q_k^* - q_l^*) \,\mathrm{HIQ}_k - (q_k^* - q_l^*) \\
&= -(\mathrm{HIQ}_k + 1)(q_k^* - q_l^*) \\
&= -\left(\frac{1-\nu}{2-\nu} \right) (q_k^* - q_l^*) \\
&= \frac{1-\nu}{(2-\nu)^2} (c_k - c_l)
\end{aligned} \tag{F.1}$$

The heterogeneity impact on consumer surplus is positive (negative) if the more inefficient (efficient) firm is getting more efficient. □

G Proof of lemma 3.2

Note that households expenditures just equal to firms total revenue which was analyzed already in appendix D. Hence the heterogeneity impact on households expenditures is given by (D.6). Thus it remains analyzing the heterogeneity impact on consumers utility $U(q_0^*, q_1^*, \ldots, q_n^*)$ given by $\partial_k U^* - \partial_l U^*$.

$$\begin{aligned}
\partial_k U^* &= \partial_k \left(\sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* \right) \\
&= \sum_i \partial_k q_i^* - \sum_i \partial_k q_i^* q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_j^* \partial_k q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* \partial_k q_j^* \\
&= \sum_i \partial_k q_i^* \left(1 - q_i^* - \nu Q_{-i}^*\right) \\
&\overset{(2.1)}{=} \sum_i \partial_k q_i^* p_i^*
\end{aligned}$$

The heterogeneity impact on consumers utility is given as follows:

$$\begin{aligned}
\partial_k U^* - \partial_l U^* &= \sum_i \left(\partial_k q_i^* - \partial_l q_i^*\right) p_i^* \\
&= \sum_i \mathrm{HIQ}_i^* \, p_i^*
\end{aligned}$$

According to (3.4) the heterogeneity impact on the output of the unaffected firms is zero. Since $\mathrm{HIQ}_k = -\,\mathrm{HIQ}_l$ it holds:

$$\begin{aligned}
\partial_k U^* - \partial_l U^* &= \mathrm{HIQ}_k \, p_k^* + \mathrm{HIQ}_l \, p_l^* \\
&= \mathrm{HIQ}_k (p_k^* - p_l^*) \\
&\overset{(2.3)}{=} \mathrm{HIQ}_k [q_k^* + c_k - (q_l^* + c_l)] \\
&= \mathrm{HIQ}_k (q_k^* - q_l^*) + \mathrm{HIQ}_k (c_k - c_l) \\
&= \frac{-1}{2-\nu} \left[\frac{-1}{2-\nu} (c_k - c_l) \right] + \frac{-1}{2-\nu} (c_k - c_l) \\
&= -\frac{(1-\nu)}{(2-\nu)^2} (c_k - c_l)
\end{aligned} \tag{G.1}$$

The heterogeneity impact on consumer surplus is negative (positive) if the more (less) efficient firm is positively affected by the cost variation. It is easy to check that the heterogeneity impact on consumer expenditures outweighs the heterogeneity impact on consumer utility.

$$\partial_k U^* - \partial_l U^* \overset{(G.1)}{=} -\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{c_k > c_l}{>} -2\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{(D.6)}{=} \partial_k R^* - \partial_l R^*$$

Since the heterogeneity impact on consumers expenditures outweighs the heterogeneity impact on consumers utility the heterogeneity impact on consumer surplus is positive (negative) if the more (less) efficient firm is getting more efficient. □

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