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A Dynamic Perspective on Minimum Quality Standards under Cournot Competition*

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Abstract

Imposing a minimum quality standard (MQS) is conventionally regarded as harmful if firms compete in quantities. This, however, ignores its possible dynamic effects. We show that an MQS can hinder collusion, resulting in dynamic welfare gains that reduce and may outweigh the static losses which are caused by regulation's distortive effect on equilibrium qualities. Verdicts on MQS thus depend even more on the market structure at hand than has been acknowledged.

JEL Classification: L41; L51; L15; D43;
Keywords: Minimal quality standard; Cournot competition; collusion

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1 Introduction

Deliberately high or low quality choices are a common strategy used by oligopolistic firms in order to relax competition and thus to raise profits (Shaked and Sutton 1982). As first demonstrated by Ronnen (1991), regulatory authorities can increase welfare in such cases of vertical product differentiation by imposing a *minimum quality standard* (MQS). In particular, a suitably chosen MQS may – but need not – simultaneously reduce hedonic prices and lift average quality.

The circumstances under which an MQS actually raises rather than reduces total surplus have been investigated in a number of studies. Broadly speaking, a moderate MQS is predicted to be socially beneficial if firms compete in prices; it is detrimental if firms compete in quantities. This dichotomy is true, however, only in static environments. The danger of collusion on price or quantity can change the picture because an MQS affects the critical degree of patience which allows anti-competitive behavior to arise in equilibrium. It is known, for example, that an MQS can facilitate collusion in a Bertrand setting (Häckner 1994). This questions an MQS's generally positive effect under price competition.

In this paper, we show that the generally negative effect of an MQS under quantity competition (Valletti 2000) is similarly sensitive to the precise market structure at hand. Namely, imposition of an MQS can destabilize collusion in a Cournot setting. The static costs of an MQS, caused by induced quality distortions, hence need to be traded off against potential dynamic benefits arising from a reduction of collusion incentives. We show that the latter may dominate the former, i.e., an MQS can actually raise total surplus under Cournot competition. Moreover, the anti-collusive effect of an MQS is fairly robust: in contrast to the case of Bertrand competition, it does not depend on whether quality primarily affects fixed or variable costs. It applies to settings with or without side-payments, for myopic or sophisticated quality choices.

We first briefly survey previous investigations of the welfare effects of an MQS in Section 2. Then Section 3 reviews the baseline model of vertical differentiation with an MQS, and Section 4 quantifies the static welfare properties of an MQS. Its role in preventing collusion when quality affects fixed costs is investigated in Section 5; the case when quality affects variable costs and several alternative collusion scenarios are dealt with in Section 6. Section 7 concludes.

2 Related Literature

Ronnen (1991) was the first to demonstrate how an MQS can (a) raise the qualities provided and (b) reduce the gap between them in a Bertrand duopoly with endogenous qualities. Effect (a) counters the tendency of quality under-provision without regulation, where firms cater to their respective marginal customer and not the average one (Spence 1975); whilst (b) curbs excess differentiation intended to alleviate competition. Both increases total surplus.

The reduction in the equilibrium level of differentiation in Ronnen's model is driven by fixed costs of production which are assumed to be convex and increasing in quality. Price changes due to the introduction of an MQS are not caused by changes in marginal costs but better substitutability of the products. In particular, the ratio of price and quality – the so-called *hedonic price* – falls for both products.

As Crampes and Hollander (1995) have highlighted, hedonic prices need not fall in general, e.g., if quality also affects variable costs. It turns out to be crucial for a positive welfare effect of the MQS that it decreases the quality gap between products. If unit costs fail to rise sufficiently more for the high-quality producer than for the low-quality producer as each one increases its respective unregulated quality, then an MQS may actually enlarge the quality gap and reduce total surplus. In this case, gains to the low-quality producer (for whom an MQS creates valuable commitment under Bertrand competition) are outweighed by losses to the high-quality producer and to consumers with relatively low quality preference (namely, greater cost and greater differentiation raise prices by more than the respective willingness to pay for extra quality).[1] However, for a great variety of cost functions, aggregate consumer surplus and with it total surplus increase when a moderate MQS is introduced in a Bertrand oligopoly.

The situation is different under quantity competition. As Valletti (2000) has shown, a binding MQS reduces profits for both producers and moreover decreases market coverage. The extra surplus to consumers with high willingness to pay for quality dominates the loss to those who drop out of the market or keep consuming the low-quality good at a higher hedonic price, i.e., aggregate consumer surplus increases. Still, the net welfare effect of the MQS is negative. The intuitive reason is that firms' need to alleviate competition is relatively small when they compete *à la Cournot*, and hence the unregulated quality gap is not particularly excessive. The intensification of competition identified by Crampes and Hollander (1995) as the key factor behind welfare gains from an MQS is rather subdued; its effect is dominated by the reduction of profits and of the surplus generated with consumers of low or moderate willingness to pay for quality.

Häckner (1994) pointed to another detrimental effect associated with an MQS: it can increase the stability of collusion. In the Bertrand market structure considered by Häckner, notably with exogenous qualities affecting only the fixed costs of production, it is easier to sustain collusion the more similar are firms' products. Intuitively, higher competitive profits which accrue to the high-quality firm for a greater level of differentiation make potential gains from collusion less attractive and give it a greater incentive to deviate. What is beneficial from a static perspective can thus be harmful in a dynamic context.

However, details matter – in particular the cost structure. In contrast to Häckner's study, Ecchia and Lambertini (1997) assume that variable costs rise with quality. The profit advantage to the high-quality producer is then no longer very pronounced.[2] The MQS makes products closer substitutes and thus creates bigger scope for raising profits by a unilateral deviation. Ecchia and Lambertini in summary find that an MQS decreases rather than increases the stability of collusion, i.e., it can be beneficial both from a static and a dynamic perspective for Bertrand competition.

Several authors have generalized Ronnen's original MQS model in different directions. For instance, Jinji and Toshimitsu (2004) introduce an asymmetry in firms' quality costs, which makes

[1]Unlike Ronnen (1991), Crampes and Hollander assume that the market is always fully covered, i.e., consumers either buy the high or the low-quality good. If some do not buy at all, then a higher hedonic price of the low-quality good also diminishes total market coverage and thereby surplus.

[2]See Lehmann-Grube (1997) on the robustness of this high-quality advantage.

the otherwise exogenous assignment to the high and low-quality equilibrium positions endogenous. The effects of an MQS when products are vertically and also horizontally differentiated are analyzed by Garella and Petrakis (2008). In their model, an MQS changes the expectations of imperfectly informed consumers regarding the product qualities; the latter increase after the introduction of an MQS. Kuhn (2007) varies the consumer preferences which are assumed in most models of vertical product differentiation, by allowing for a baseline product benefit which is unrelated to quality. It turns out that an MQS raises surplus in the static Bertrand setting only if the baseline benefit is small relative to the quality-dependent utility component.[3]

Our own analysis investigates the introduction of an MQS in the case of quantity competition, under standard preference and technology assumptions. While there are static losses, as already identified by Valletti (2000), independently of whether quality affects fixed or variable costs, collusion becomes more difficult to sustain with an MQS: the available total collusion profits are reduced by the extra costs induced by the MQS. This – aided by a shift in firms' bargaining positions – turns out to make it relatively more attractive for the high-quality firm to go it alone.

3 Model

We consider a standard vertically differentiated duopoly.[4] Firm $i \in \{1, 2\}$ produces an indivisible good of quality s_i. Without loss of generality we assume $s_1 \geq s_2 > 0$. A unit mass of consumers obtain utility

$$U(p_i, s_i) = \theta \cdot s_i - p_i \tag{1}$$

from buying exactly one unit of quality s_i at price p_i and zero otherwise; θ characterizes the considered consumer's type. It is assumed to be uniformly distributed on $[0, a]$ $(a > 0)$. A consumer with type $\theta = (p_1 - p_2)/(s_1 - s_2)$ is indifferent between both products; one with $\theta = p_2/s_2$ is indifferent between the low-quality product 2 and no purchase at all. This implies the inverse demand functions

$$\begin{aligned} p_1(x_1, x_2, s_1, s_2) &= s_1\,(a - x_1) - s_2\,x_2, \\ p_2(x_1, x_2, s_1, s_2) &= s_2\,(a - x_1 - x_2) \end{aligned} \tag{2}$$

where $x_i \geq 0$ denotes the respective quantity choice.

Firms have access to the same technology. Their production is initially assumed to involve only fixed costs, which increase in quality and are denoted by $C(s_i)$. In line with most of the literature, we consider the simple quadratic form

$$C(s_i) = \gamma s_i^2 \qquad (\gamma > 0). \tag{3}$$

[3]Other recent contributions to the theory of vertical product differentiation include Lambertini and Scarpa (2006) and Marette (2008). Sappington (2005, pp. 132ff) evaluates MQS from the broader perspective of service quality regulation, especially in public utility industries. For empirical investigations of MQS see, e.g., Chitpy and Witte (1997) and Hotz and Xiao (2005). Applied theoretical work on MQS includes Boom (1995) and Bonroy (2003). Also see, e.g., Marette (2007) for the related analysis of minimum safety standards.

[4]See Tirole (1988, Section 7.5), Choi and Shin (1992), Motta (1993) and Wauthy (1996).

The timing of interaction is as follows: First, both firms simultaneously choose their respective quality, which then becomes common knowledge. Second, the firms simultaneously decide on their quantities. Finally, the market is cleared at the prices indicated by (2).[5] This sequence reflects the presumption that quantity decisions entail more flexibility than firms' quality positioning. It is a standard assumption in the literature – if by no means the only economically relevant possibility – and we here adopt a particularly stringent version of it. Namely, in Section 5, we will investigate firms' incentives to collude when quantities are set repeatedly whilst qualities are given by their initial choices.

Firms' equilibrium quantity choices for given qualities $s_1 \geq s_2$ are

$$\hat{x}_1(s_1, s_2) = \frac{a\ (2\, s_1 - s_2)}{4\, s_1 - s_2} \quad \text{and} \quad \hat{x}_2(s_1, s_2) = \frac{a\, s_1}{4\, s_1 - s_2}. \tag{4}$$

They define the *reduced profit functions*

$$\pi_1(s_1, s_2) \quad = \quad \frac{a^2\, s_1\, (2\, s_1 - s_2)^2}{(4\, s_1 - s_2)^2} - \gamma s_1^2, \tag{5}$$

$$\pi_2(s_1, s_2) \quad = \quad \frac{a^2\, {s_1}^2\, s_2}{(4\, s_1 - s_2)^2} - \gamma s_2^2. \tag{6}$$

The first-order conditions characterizing optimal qualities are then

$$\frac{\partial \pi_1(s_1, s_2)}{\partial s_1} \quad = \quad \frac{a^2\ (16\, {s_1}^3 - 12\, {s_1}^2\, s_2 + 4\, s_1\, {s_2}^2 - {s_2}^3)}{(4\, s_1 - s_2)^3} - 2\gamma s_1 = 0, \tag{7}$$

$$\frac{\partial \pi_2(s_1, s_2)}{\partial s_2} \quad = \quad \frac{a^2\, {s_1}^2\ (4\, s_1 + s_2)}{(4\, s_1 - s_2)^3} - 2\gamma s_2 = 0. \tag{8}$$

These conditions define firms' best response functions $R_i(s_j)$ $(i \neq j \in \{1, 2\})$; closed-form solutions exist but are very unwieldy. The resulting *unregulated equilibrium qualities* can be computed as[6]

$$\hat{s}_1 \approx \frac{0.12597\, a^2}{\gamma} \quad \text{and} \quad \hat{s}_2 \approx \frac{0.04511\, a^2}{\gamma}. \tag{9}$$

Now suppose that $\tilde{s}$ is exogenously imposed as an MQS, i.e., firms face the constraint $s_i \geq \tilde{s}$.[7] We will throughout our analysis focus on the case in which the MQS is *not excessive* but *binding*:

[5] In the spirit of Kreps and Scheinkmann (1983), one may think of capacity choices and subsequent price competition. In the context of indefinitely repeated interaction, which we will study below, this interpretation would require, however, that firms can regularly revise their capacities (e.g., with each agricultural season, or with each generation of computer chips).

[6] There exists a quality $s_2 \in (0, s_1)$ which is profitable for firm 2 for any given quality s_1, i.e., there is no monopoly in equilibrium. Second-order conditions are satisfied and neither firm has an incentive to "leapfrog". This remains true after an MQS is imposed (see Appendix A). See Motta (1993) for a detailed comparison of $(\hat{s}_1, \hat{s}_2)$ under price vs. quantity competition and fixed vs. variable quality costs.

[7] See Argenton (2006) and Lutz et al. (2000) for analysis of an endogenous MQS. Argenton analyzes bilateral bargaining over an MQS by the duopolists. Lutz et al. allow one of them to influence the MQS by a prior quality commitment.

both firms stay in the market,[8] but firm 2 needs to increase its quality in order to comply with regulation, i.e., $\tilde{s} > \hat{s}_2$. The resulting *regulated equilibrium qualities* will be denoted by $s_1^*(\tilde{s})$ and $s_2^*(\tilde{s})$.

Lemma 1 implies that the equilibrium quality gap between both firms decreases in $\tilde{s}$ if firm 2 adopts the mandated quality, i.e., supposing that $s_2^*(\tilde{s}) = \tilde{s}$:

Lemma 1 *Firm 1 responds to any given increase Δs_2 of firm 2's quality by an increase $\Delta s_1 < \Delta s_2$ of its own quality. In particular,*

$$0 < \frac{\partial R_1(s_2)}{\partial s_2} < 1. \tag{10}$$

Proof: Substituting $s_2 \equiv \beta \cdot s_1$ with $\beta \in (0, 1]$ in (7), the first-order condition for firm 1's quality choice can equivalently be written as

$$s_1 = \frac{a^2(\beta^3 - 4\beta^2 + 12\beta - 16)}{2\gamma(\beta - 4)^3}. \tag{11}$$

Moreover, application of the implicit function theorem to equation (7) and afterwards the substitution $s_2 = \beta \cdot s_1$ yield

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{4a^2(\beta - 1)\beta}{4a^2(\beta - 1)\beta^2 - (\beta - 4)^4\gamma s_1}. \tag{12}$$

Using the rearranged first-order condition (11) for s_1, this simplifies to

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{8(1 - \beta)\beta}{\beta^4 - 16\beta^3 + 36\beta^2 - 64\beta + 64}. \tag{13}$$

Now, recalling the fact that $\beta \in (0, 1]$, numerical inspection allows to infer that

$$\frac{\partial R_1(s_2)}{\partial s_2} \in (0, 0.05465]. \tag{14}$$

□

Lemma 2 establishes that indeed $s_2^*(\tilde{s}) = \tilde{s}$:

Lemma 2 *Given an MQS $\tilde{s} > \hat{s}_2$ such that both firms stay in the market, firm 2 selects exactly the mandated quality in equilibrium, i.e.,*

$$s_2^*(\tilde{s}) = \tilde{s}. \tag{15}$$

Proof: Again using the notation $s_2 \equiv \beta \cdot s_1$ with $\beta \in (0, 1]$, the change of firm 2's profit caused by a marginal increase of s_2 can be written as

$$\frac{\partial \pi_2}{\partial s_2} = \frac{a^2(4 + \beta) + 2\beta(\beta - 4)^3\gamma s_1}{(4 - \beta)^3} \tag{16}$$

[8] Firm 2's profit is the smaller one, decreases in $\tilde{s}$, and is zero at $\tilde{s}^c \approx \frac{0.09334\, a^2}{\gamma}$. So "not excessive" means $\tilde{s} \leq \tilde{s}^c$.

Considering a best response by firm 1, i.e., imposing the rearranged first-order condition (11), this becomes

$$\frac{\partial \pi_2}{\partial s_2} = \frac{a^2(\beta^4 - 4\beta^3 + 12\beta^2 - 15\beta + 4)}{(4-\beta)^3}, \tag{17}$$

which is positive (negative) to the left (right) of $\beta = \hat{s}_2/\hat{s}_1$.

Imposition of $\tilde{s}$ means that s_2 rises by $\Delta s \geq \tilde{s} - \hat{s}_2$. By Lemma 1, s_1 rises by less than Δs. A post-MQS equilibrium quality ratio must hence satisfy $\beta > \hat{s}_2/\hat{s}_1$. Thus (17) is negative and firm 2 must select the minimum feasible quality $s_2^*(\tilde{s}) = \tilde{s}$ in equilibrium.

□

Lemmata 1 and 2 jointly imply that the *regulated equilibrium quality ratio*

$$\alpha(\tilde{s}) \equiv \frac{s_2^*(\tilde{s})}{s_1^*(\tilde{s})} = \frac{\tilde{s}}{R_1(\tilde{s})} \in (\hat{s}_2/\hat{s}_1, 1] \tag{18}$$

is a strictly increasing function of $\tilde{s}$. With slight abuse of notation, one can hence directly consider α, shorthand for $\alpha(\tilde{s})$, as being the relevant policy variable.

4 Static Welfare Analysis

The static effects of an MQS on profits, consumer surplus and total surplus in case of fixed quality costs and quantity competition have first been analyzed by Valletti (2000). For the sake of completeness, we here include derivations of his two main findings.[9] In contrast to the case of price competition, *both* producers are made worse off by the MQS:

Proposition 1 *Both firms' profits decrease in the level of the MQS, i.e.,*

$$\frac{d\pi_i(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} < 0 \quad \textit{for} \quad i \in \{1, 2\}. \tag{19}$$

Proof: The marginal profit changes caused by introduction of an MQS are given by

$$\frac{d\pi_1(R_1(\tilde{s}), \tilde{s}))}{d\tilde{s}} = \underbrace{\frac{\partial \pi_1}{\partial s_1}}_{=0} \frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial \pi_1}{\partial s_2} = \frac{\partial \pi_1}{\partial s_2}, \tag{20}$$

$$\frac{d\pi_2(R_1(\tilde{s}), \tilde{s}))}{d\tilde{s}} = \frac{\partial \pi_2}{\partial s_1} \frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial \pi_2}{\partial s_2}. \tag{21}$$

One can compute

$$\frac{\partial \pi_1}{\partial s_2} = \frac{4\, a^2\, {s_1}^2\, (s_2 - 2\, s_1)}{(4\, s_1 - s_2)^3} < 0. \tag{22}$$

[9] Valletti's results apply to more general fixed quality cost functions $C(\cdot)$ with $C'(\cdot), C''(\cdot) > 0$. We investigate the more tedious case of variable quality costs – which is not covered by Valletti – in Section 6.4.

Moreover, we know $\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} > 0$ from Lemma 1 and $\frac{\partial \pi_2}{\partial s_2} < 0$ from the proof of Lemma 2. It thus remains to confirm that

$$\frac{\partial \pi_2}{\partial s_1} = -\frac{2a^2 s_1 s_2^2}{(4\,s_1 - s_2)^3} < 0. \tag{23}$$

□

Now consider the consumer surplus generated by qualities s_1 and s_2,

$$\begin{aligned} S(s_1, s_2) &= \int_{\frac{p_2}{s_2}}^{\frac{p_1-p_2}{s_1-s_2}} (\theta s_2 - p_2)\, d\theta + \int_{\frac{p_1-p_2}{s_1-s_2}}^{a} (\theta s_1 - p_1)\, d\theta && (24)\\ &= \frac{a^2\, s_1\, (4\, s_1{}^2 + s_1\, s_2 - s_2{}^2)}{2\,(4 s_1 - s_2)^2}. && (25) \end{aligned}$$

The change caused by an MQS is given by

$$\frac{dS(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} = \frac{\partial S}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial S}{\partial s_2}, \tag{26}$$

where we know that $\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} > 0$ and, using (25), one can check that $\frac{\partial S}{\partial s_i} > 0$ for $i \in \{1, 2\}$. So consumer surplus rises in $\tilde{s}$.

Its increase is, however, dominated by the decrease of profits:

Proposition 2 *Total surplus decreases in the level of the MQS, i.e.,*

$$\frac{d\big(\pi_1(\cdot) + \pi_2(\cdot) + S(\cdot)\big)}{d\tilde{s}} < 0. \tag{27}$$

Proof: The change in total surplus due to an MQS is equal to

$$\begin{aligned} & \frac{\partial \pi_1}{\partial s_2} + \underbrace{\frac{\partial \pi_1}{\partial s_1}}_{=0} \frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial \pi_2}{\partial s_2} + \frac{\partial \pi_2}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial S}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial S}{\partial s_2} && (28)\\ = \; & \frac{\partial \pi_1}{\partial s_2} + \underbrace{\frac{\partial \pi_2}{\partial s_2}}_{<0} + \underbrace{\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}}}_{\in(0,0.05465]} \underbrace{\frac{a^2\,(4\,s_1{}^2 - 2\,s_1\,s_2 - s_2{}^2)}{2\,(4\,s_1 - s_2)^2}}_{>0} + \frac{\partial S}{\partial s_2} && (29)\\ < \; & \frac{\partial \pi_1}{\partial s_2} + \frac{a^2\,(4\,s_1{}^2 - 2\,s_1\,s_2 - s_2{}^2)}{20\,(4\,s_1 - s_2)^2} + \frac{\partial S}{\partial s_2} && (30)\\ = \; & -\frac{a^2\,(6\,s_1{}^2 + 2\,s_1\,s_2 + s_2{}^2)}{20\,(4\,s_1 - s_2)^2} < 0, && (31) \end{aligned}$$

where $\frac{\partial \pi_2}{\partial s_2} < 0$ because constraint $s_2 \geq \tilde{s}$ binds.

□

The key difference to Ronnen's (1991) Bertrand setting is that in the Cournot case – in view of relatively low competitive pressure even for undifferentiated goods – the quality distortions induced by an MQS does not lead to lower hedonic prices. Hence, consumers with relatively low marginal willingness to pay for quality leave the market or switch from the high to the low quality, weighing down the overall increase in consumer surplus. A second distinction is that the MQS tends to give the low-quality firm valuable commitment power under price competition, i.e., it benefits from the constraint $s_2 \geq \tilde{s}$ in equilibrium. Here, both firms suffer (Proposition 1). These differences jointly reverse Ronnen's baseline finding that an MQS increases welfare.

5 Dynamic Welfare Analysis

By changing firms' static profits, an MQS also affects their incentives to collude under repeated interaction. We will analyze these dynamic effects now. We assume that firms care about their discounted streams of profits

$$\sum_{t=1}^{\infty} \delta^t \pi_{i;t} \tag{32}$$

where $\pi_{i;t}$ denotes firm i's profit in period t. For simplicity we assume that both firms apply the same discount factor $\delta \in (0, 1)$, which may capture pure impatience (determined, e.g., by an interest rate) as well as the likelihood that there is in fact another round of quantity competition between the considered two firms.

Quality, or at least its perception by consumers, often cannot be changed as quickly as output quantity.[10] Producers of a differentiated agricultural good, such as wine or olives, can – and partially must – adjust quantities on a seasonal basis, whilst their baseline quality positioning is a matter of decades; a high-quality incumbent in the European car industry and a new entrant from China can adjust their respective production much more quickly than the documented durability and reliability of their products. Similarly, two franchisors or airlines are likely to treat quality as a top-level, brand-wide, and long-term investment decision but let each season's store orders or seats on a given route be selected by individual franchisees and product managers. We suspect that quality is the "long-term variable" and quantity the "short-term variable" in a variety of industries, ranging from branded beauty products or clothing to watches and whisky, and will focus on this case. In particular, our analysis will presume that quantities are set repeatedly in periods $t = 1, 2, 3, \ldots$, whilst firms' quality choices are made in period $t = 0$ and then become irreversible.

Without loss of generality we assume that the fixed costs associated with the possibility to produce a particular quality are incurred in every period (e.g., for maintenance of physical or human capital, licenses, or advertising).[11] And we suppose initially that firms can transfer profits from one to the other if they decide to collude – e.g., by trading a costless intermediate good

[10] See Eales and Binkley (2003) for an industry study on the perception of quality and vertical differentiation via advertising, which documents this in detail.

[11] A one-off payment can be more natural in some contexts. The distinction is immaterial for the (in)stability of collusion because the corresponding terms drop out in the further computations.

at an inflated price or, as discussed below, trading the final good at a discounted price. A firm can refuse to make side payments at any point in time, but trigger strategies that revert to competitive quantity setting after such a deviation would prevent this from happening in a collusion equilibrium. Situations without the possibility of side payments will be dealt with in Section 6.2.

Following Ecchia and Lambertini (1997), we take collusion to affect only firms' short-term quantity decisions – not their initial choice of quality. This seems plausible for branded goods or services, even though it is conceivable that, say, two airlines coordinate long-term service levels in addition to their route-specific capacities.[12] Regarding an uncoordinated choice of qualities, two scenarios are plausible: the first, perhaps practically more relevant one involves individual profit maximization – say, by the firms' boards, top managers, or owners – under the myopic expectation of competitive quantity setting. This expectation is, however, supposed not to prevent firms from colluding later, from some period $\tau \geq 1$ onwards, as lower-key product managers take control over output, or the respective higher-level decision makers get to know each other and begin to appreciate their collaboration potential. We will first analyze this scenario (cf. Ecchia and Lambertini 1997). The second, game-theoretically more appealing case, in which firms anticipate from the very beginning that they will later collude on quantities and hence already choose qualities that maximize their respective collusion profits, will be studied in Section 6.3.

We will consider the standard measure of *instability of collusion* for indefinitely repeated interaction, namely the maximal discount factor such that, for both firms, the short-run gains from a deviation outweigh anticipated long-run losses from consequent punishment. We refer to it as the *critical discount factor*, denoted by r. In line with Häckner (1994) and Ecchia and Lambertini (1997), punishment is taken to be a reversion to the static Cournot-Nash equilibrium (which corresponds to a subgame-perfect equilibrium involving simple trigger strategies), even though more severe but complex punishments exist.[13]

Comparison of the anticipation of a *collusion profit* π_i^c in every period $t = \tau, \tau+1, \ldots$ and of once receiving the *deviation profit* π_i^d and thereafter the *punishment payoff* π_i^p shows that firm i

[12] The analysis of such "comprehensive collusion" is left for future research: firms would try to maximize their total profit $\pi_\Sigma(s_1, s_2)$ subject to the constraints that quantity collusion later on will be stable for the selected quality levels and, moreover, that the agreed quality levels s_i^c are self-enforcing. Formally, they need to solve

$$\begin{aligned} \max_{s_1 \geq s_2 \geq 0} \pi_\Sigma(s_1, s_2) &= \frac{a^2 s_1}{4} - \gamma {s_1}^2 - \gamma {s_2}^2 \\ \text{s.t.} \quad & r(s_1, s_2) \leq \delta \\ & \pi_1^c(s_1, s_2) \geq \max_{s_1' \geq 0} \pi_1^p(s_1', s_2) \\ & \pi_2^c(s_1, s_2) \geq \max_{s_2' \geq 0} \pi_2^p(s_1, s_2') \end{aligned}$$

where collusion and punishment payoffs π_i^c and π_i^p as well as the critical discount r will be defined below and where it is assumed that a deviation from s_i^c will immediately be punished by a reversion to the Cournot-Nash quantity equilibrium. The three non-linear constraints are very unwieldy to work with, even numerically. See Jehiel (1992) for a related investigation in the context of horizontal product differentiation.

[13] See Abreu (1986).

has an incentive to collude with firm j if and only if

$$\delta > r_i \equiv \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p}. \tag{33}$$

So the critical discount factor is $r = \max\{r_1, r_2\}$.

We first derive π_i^p, π_i^c, and π_i^d for given quality choices s_1 and s_2, and later replace s_i by the respective regulated equilibrium level $s_i^*(\tilde{s})$ in order to analyze the effect of increasing $\tilde{s}$. For Nash reversion equilibria, the punishment payoff $\pi_i^p(s_1, s_2)$ is simply the reduced profit displayed in equation (5) and (6), respectively. Collusion profits $\pi_i^c(s_1, s_2)$ are assumed to result from bargaining over the division of firms' aggregate per period profit π_Σ. Having the possibility of side-payments, firms will "split the difference" between their total competitive and their maximal aggregate profits equally – in line with Nash's (1950) and, in fact, any symmetric and efficient cooperative bargaining solution (e.g., the proportional solution or the one proposed by Kalai and Smorodinsky 1975).[14] We will consider alternatives in Sections 6.1 and 6.2.

Aggregate profit equals

$$p_1(x_1, x_2, s_1, s_2) \cdot x_1 + p_2(x_1, x_2, s_1, s_2) \cdot x_2 - C(s_1) - C(s_2) \tag{34}$$

and is maximized by $x_1^c = \frac{a}{2}$ and $x_2^c = 0$: the firms will eliminate any competition and produce only the quality s_1, which has a higher margin. This finding depends on the assumption of fixed quality costs and scope for side payments: the variations considered in Sections 6.2 and 6.4 involve positive collusive sales of both products.

Note that one possibility to implement $(x_1^c, x_2^c) = (\frac{a}{2}, 0)$ is a mixed retailing agreement, which has firm 2 procure and retail part of firm 1's production. The transfer price can be chosen to implement the collusion payoffs π_1^c and π_2^c. Firm 2, an innocuous retailer of monopolist 1 to competition authorities, is ready to integrate backward into production of quality s_2, but refrains from doing so as long as firm 1 does not deviate from their tacit agreement. This will mean that the introduction of a moderate MQS $\tilde{s} > \hat{s}_2$, which must seem to be inconsequential because it is well below the only visible unregulated quality level $\hat{s}_1$, may paradoxically prompt the introduction of a *low* quality to a market in which only a high quality good was traded before. That even a non-binding MQS may affect a market's performance seems to have received very little attention in the literature; we are aware only of Garella (2006) and Garella and Petrakis (2008) also making this point.

We presume that firm 2 incurs fixed costs $C(s_2)$ even if its current output is set to zero: the firm needs to retain its production capability as a deterrent. The maximal aggregate profit for given qualities is thus

$$\pi_\Sigma(s_1, s_2) \equiv \frac{a^2\, s_1}{4} - \gamma\, {s_1}^2 - \gamma\, {s_2}^2. \tag{35}$$

[14]As demonstrated by Binmore (1987), the Nash solution also approximates non-cooperative alternating-offers bargaining between patient players in single-shot interaction (see Rubinstein 1982). Repeated interaction would support alternative divisions, but our results continue to hold for many other division rules (e.g., proportional to competitive Cournot-Nash profits).

Splitting the difference

$$\pi_\Delta(s_1,s_2) \equiv \pi_\Sigma(s_1,s_2) - \pi_1^p(s_1,s_2) - \pi_2^p(s_1,s_2) = \frac{a^2\, s_1\, s_2\,(4\, s_1 - 3\, s_2)}{4\,(4s_1 - s_2)^2} > 0 \tag{36}$$

between both firms equally then implies the collusion profits

$$\pi_1^c(s_1,s_2) = \pi_1^p(s_1,s_2) + \frac{1}{2}\pi_\Delta(s_1,s_2) = \frac{a^2\, s_1\,\,(8\, s_1 - 5\, s_2)}{8(4\, s_1 - s_2)} - \gamma s_1^2 \tag{37}$$

and

$$\pi_2^c(s_1,s_2) = \pi_2^p(s_1,s_2) + \frac{1}{2}\pi_\Delta(s_1,s_2) = \frac{3\, a^2\, s_1\, s_2}{8(4\, s_1 - s_2)} - \gamma s_2^2. \tag{38}$$

A deviation by firm i involves a best response to firm j's collusive output x_j^c and a refusal to share any part of its profits. While firm 1 is bound by its quantity choice for the current period, we assume that it can immediately react to a deviation of firm 2 by refusing to share profits. This implies that firm 2's incentive to collude is actually independent of discount factor δ:

Lemma 3 *Firm 2 always prefers collusion to a deviation. In particular,*

$$\pi_2^c(s_1,s_2) > \pi_2^d(s_1,s_2). \tag{39}$$

Proof: Given $x_1^c = \frac{a}{2}$, firm 2's profit equals

$$p_2(x_1^c, x_2, s_1, s_2)\cdot x_2 - C(s_2) = s_2\left(\frac{a}{2} - x_2\right) x_2 - \gamma s_2^2 \tag{40}$$

and is maximized by $x_2^d = \frac{a}{4}$. The maximal deviation profit is hence

$$\pi_2^d(s_1,s_2) \quad = \quad \frac{s_2\,\,(a^2 - 16\,\gamma\, s_2)}{16} \tag{41}$$

where, however,

$$\pi_2^d(s_1,s_2) - \pi_2^p(s_1,s_2) \quad = \quad -\frac{a^2 s_2^2\,(8s_1 - s_2)}{16(4s_1 - s_2)^2} < 0. \tag{42}$$

So, in particular, $\pi_2^d(s_1,s_2) < \pi_2^p(s_1,s_2) + \frac{1}{2}\pi_\Delta(s_1,s_2) = \pi_2^c(s_1,s_2)$.

□

So only firm 1's incentive to collude or, respectively, to deviate needs to be considered (and only δ_1 would matter if firm-specific discount factors δ_i were applied). In view of $x_2^c = 0$, the jointly profit-maximizing quantity $x_1^c = \frac{a}{2}$ in fact maximizes firm 1's profit. A deviation by firm 1 thus boils down to refusing to transfer the designated share of profits to firm 2, even though the

$\frac{dr}{d\pi_1^p}$	$= \frac{\pi_1^d+\pi_2^p-\pi_\Sigma}{2(\pi_1^p+\pi_1^d)^2} > 0$	$\frac{\partial \pi_1^p(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \downarrow$
$\frac{dr}{d\pi_2^p}$	$= \frac{1}{2(\pi_1^d-\pi_1^p)} > 0$	$\frac{\partial \pi_2^p(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \downarrow$
$\frac{dr}{d\pi_1^d}$	$= -\frac{\pi_1^p+\pi_2^p-\pi_\Sigma}{2(\pi_1^p+\pi_1^d)^2} > 0$	$\frac{\partial \pi_1^d(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \downarrow$
$\frac{dr}{d\pi_\Sigma}$	$= \frac{1}{2(\pi_1^p-\pi_1^d)} < 0$	$\frac{\partial \pi_\Sigma(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \uparrow$

Table 1: Partial effects of an increase of MQS $\tilde{s}$ on the critical discount factor r

latter cooperated and did not produce any output. Hence firm 1's deviation profit is

$$\pi_1^d(s_1, s_2) = \frac{a^2\, s_1}{4} - \gamma\, {s_1}^2. \tag{43}$$

We are now ready to establish our main result:

Proposition 3 *The critical discount factor increases in the level of the MQS, i.e.,*

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} > 0. \tag{44}$$

Proof: Using Lemma 3 and inserting the respective expressions for π_1^p, π_1^c, and π_1^d (cf. equations (5), (37), and (43)) into (33) yields

$$r(s_1, s_2) = r_1(s_1, s_2) = \frac{12\, s_1 - 3\, s_2}{16\, s_1 - 6\, s_2}. \tag{45}$$

Replacing s_i by the regulated equilibrium quality $s_i^*(\tilde{s})$ and then using the substitution $\alpha(\tilde{s}) = s_2^*(\tilde{s})/s_1^*(\tilde{s})$, we can write the critical discount factor as a function of this regulated equilibrium quality ratio:

$$r(s_1^*(\tilde{s}), s_2^*(\tilde{s})) = \frac{3}{2} \cdot \frac{\alpha(\tilde{s}) - 4}{3\,\alpha(\tilde{s}) - 8} \equiv \rho(\alpha(\tilde{s})). \tag{46}$$

Using that $\alpha(\tilde{s})$ is strictly increasing in $\tilde{s}$ (see Lemmata 1 and 2), one obtains

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} = \frac{d\rho(\alpha(\tilde{s}))}{d\tilde{s}} = \frac{6}{(3\alpha - 8)^2} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} > 0. \tag{47}$$

□

An MQS can hence be an effective policy to prevent collusion between vertically differentiated Cournot duopolists, or to destabilize it. The main reason for the rise of

$$r = \frac{\pi_1^d - \pi_1^c}{\pi_1^d - \pi_1^p} = \frac{\pi_1^d - (\pi_1^p + \frac{1}{2}(\pi_\Sigma - \pi_1^p - \pi_2^p))}{\pi_1^d - \pi_1^p} \tag{48}$$

is the negative impact on firm 1's collusion profit π_1^c caused by a drop of π_Σ. Table 1 decomposes and ranks (by arrow size) the involved partial effects. As we have seen, the MQS induces both firms to produce higher qualities. Therefore *both* incur increased fixed costs, which are partially offset by positive sales of only *one* quality in case of collusion. This reduces the aggregate collusion profit π_Σ which is available for distribution significantly. As a consequence, π_1^c drops and – despite the partially compensating reductions also of π_1^d and π_1^p – the critical discount factor rises on balance. Interestingly, firm 1's relative share of the diminished total collusion profit π_Σ drops under Nash bargaining. The reason is that its competitive profit (and thus its fallback position in bargaining) falls by more than that of firm 2.

The preventive introduction of an MQS involves a trade-off. While competitive quantity decisions create surplus relative to collusive ones, firms' distorted quality choices under an MQS destroy surplus (cf. Proposition 2). There exist a continuum of discount factors for which the MQS's net effect on surplus is positive:

Proposition 4 *Assume that firms collude whenever this is strictly more profitable than a deviation and subsequent reversion to the Cournot-Nash equilibrium (i.e., for $\delta > r$). Then a welfare-enhancing MQS exists if and only if $\delta \in (\underline{\delta}; \bar{\delta})$ for $\underline{\delta} \approx 0.78878$ and $\bar{\delta} \approx 0.82537$.*

Proof: Collusive behavior in the *unregulated* case generates a total surplus of

$$\hat{W}^{col} = \pi_\Sigma(\hat{s}_1, \hat{s}_2) + \int_{\frac{a}{2}}^{a} \big(\theta \hat{s}_1 - p_1(x_1^c, x_2^c, \hat{s}_1, \hat{s}_2)\big)\, d\theta \tag{49}$$

$$= \frac{3a^2}{8}\, \hat{s}_1 - \gamma\, \hat{s}_1^2 - \gamma\, \hat{s}_2^2 \tag{50}$$

per period, which exceeds total surplus under collusion for any regulated equilibrium quality ratio $\alpha > \hat{\alpha}$.

Competitive behavior in the presence of an MQS entails smaller profits but greater consumer welfare. The corresponding total per period surplus amounts to

$$\hat{W}^{com} = \frac{a^2\, s_1\, (12 s_1^2 - 5 s_1 s_2 + s_2^2)}{2\,(4 s_1 - s_2)^2} - \gamma s_1^2 - \gamma s_2^2. \tag{51}$$

It is illustrated as a function of the regulated equilibrium quality ratio $\alpha = \frac{s_1^*(\tilde{s})}{s_2^*(\tilde{s})}$ in Figure 1 together with the analogous surplus $W^{col}(\alpha)$ for collusive behavior (with $W^{col}(\hat{\alpha}) \equiv \hat{W}^{col}$).

Any regulated quality ratio exceeding $\bar{\alpha}$ defined by $W^{com}(\bar{\alpha}) = W^{col}(\bar{\alpha})$ lowers welfare independently of the unregulated market conduct. In contrast, a regulated quality ratio $\alpha \in (\hat{\alpha}, \bar{\alpha})$ implies greater surplus *if* it replaces collusive by competitive behavior. By assumption the latter requires the actual discount factor δ to be no greater than the critical one. So letting $\rho(\alpha) = r\big(s_1^*(\tilde{s}), s_2^*(\tilde{s})\big)$ denote the critical discount factor for a regulated quality ratio α (cf. equation (46)), it follows that a welfare-enhancing MQS exists whenever

$$0.78878 \approx \rho(\hat{\alpha}) \equiv \underline{\delta} < \delta < \bar{\delta} \equiv \rho(\bar{\alpha}) \approx 0.82537. \tag{52}$$

□

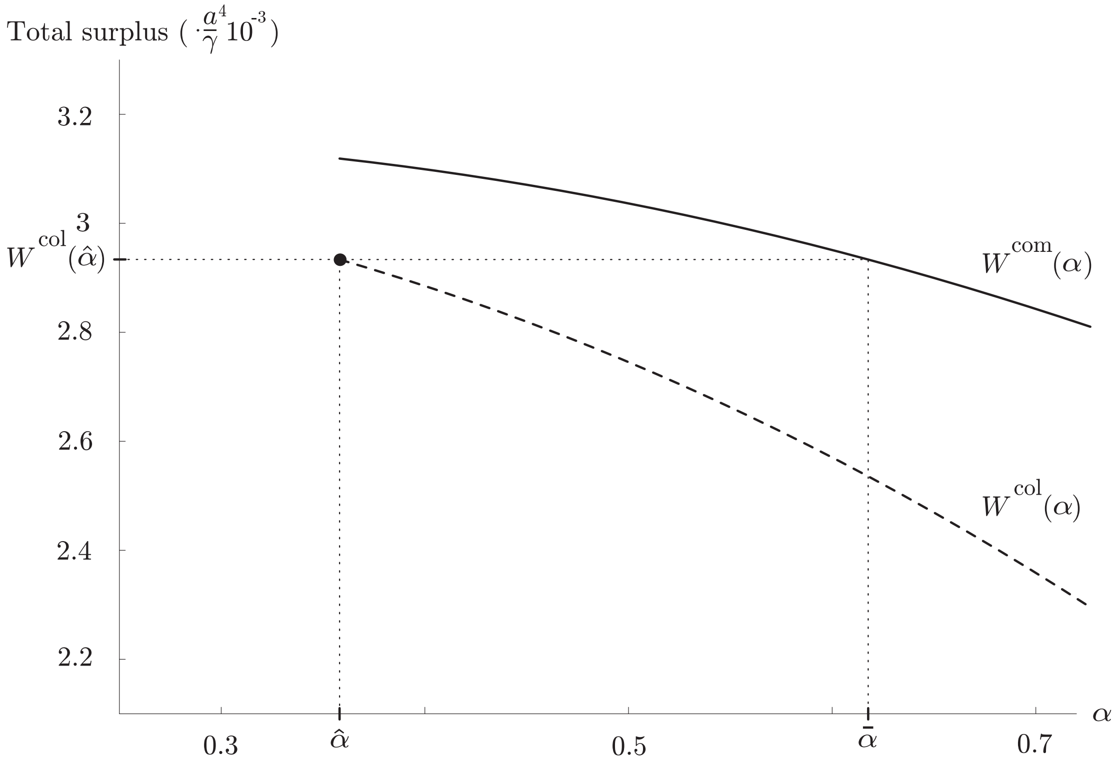

Figure 1: Total surplus under competition and collusion

When interest rates, market continuation probabilities, etc. give rise to a discount factor in the interval characterized by the Proposition and an MQS is introduced, the realized welfare gain depends on the selected MQS. For example, an MQS slightly below the level $\bar{s} \approx \frac{0.07888a^2}{\gamma}$ which corresponds to an regulated equilibrium quality ratio $\bar{\alpha}$ will prevent collusion but entails only a negligible surplus increase relative to an unregulated market. The *optimal* or *surplus-maximizing MQS* for different discount factors δ is illustrated in Figure 2. It realizes the dynamic benefit of collusion prevention at the smallest static cost:

Proposition 5 *The surplus-maximizing MQS is*

$$\tilde{s}_2^*(\delta) = \frac{a^2\,(3-4\,\delta)(44\,\delta^3 - 120\,\delta^2 + 99\,\delta - 27)}{24\,\gamma\,\delta^3\;(2\,\delta - 1)} \tag{53}$$

for $\delta \in (\underline{\delta}, \bar{\delta})$, *and zero or not binding otherwise.*

Proof: Since the static welfare loss of an MQS is increasing continuously in $\tilde{s}$, the optimal choice of $\tilde{s}$ is such that the resulting regulated equilibrium quality ratio α satisfies

$$\rho(\alpha) = \delta \qquad \Longleftrightarrow \qquad \alpha = \frac{4\;(3-4\,\delta)}{3\;(1-2\,\delta)}. \tag{54}$$

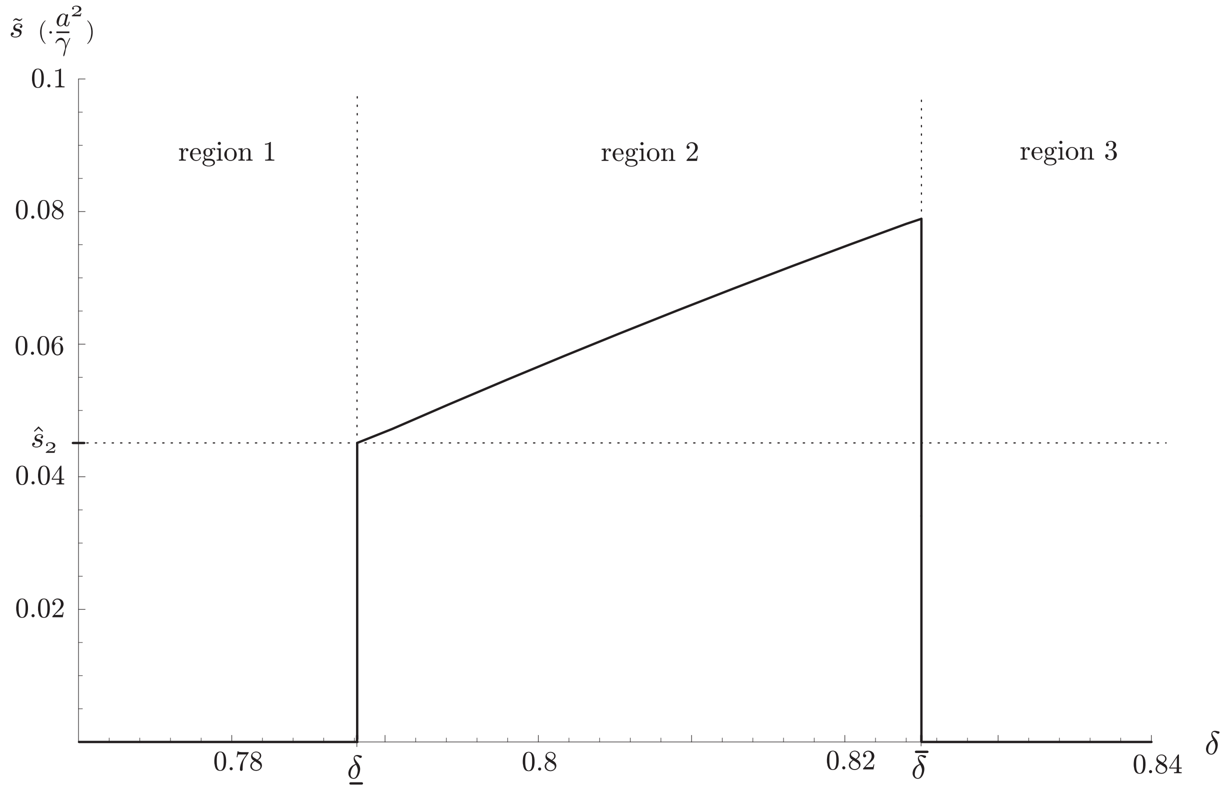

Figure 2: Optimal MQS for given discount factor δ

Profit-maximizing behavior by firm 1 entails

$$s_1(\alpha) = \frac{a^2(\alpha^3 - 4\alpha^2 + 12\alpha - 16)}{2\gamma(\alpha - 4)^3} \tag{55}$$

(substitute $\beta = \alpha$ in equation (11)). Recall, moreover, that $s_2^*(\tilde{s}) = \tilde{s}$ for any non-excessive $\tilde{s}$, i.e., firm 2 maximizes profits by selecting the minimum feasible quality (Lemma 2). So any MQS $\tilde{s}$ with corresponding equilibrium quality ratio α satisfies $\tilde{s} = \alpha \cdot s_1(\alpha)$ and, using (54) in order to substitute for α, this implies

$$\tilde{s}_2^*(\delta) = \frac{a^2\,(3 - 4\,\delta)(44\,\delta^3 - 120\,\delta^2 + 99\,\delta - 27)}{24\,\gamma\,\delta^3\;(2\,\delta - 1)} \tag{56}$$

□

One can distinguish three different regulation regimes: for low discount factors (region 1 in Figure 2), i.e., when firms place great weight on current relative to prospective future profits, collusion would be unstable even without regulation; an MQS could only destroy surplus (Valletti 2000). For intermediate discount factors (region 2), an MQS would prevent collusion and thereby create gains exceeding the static welfare losses first identified by Valletti. Finally, for high discount factors (region 3), the costs of prevention would either exceed the respective benefits or collusion cannot be prevented by an MQS at all.[15]

Before we investigate the robustness of the above findings, a general remark is in order. There are obviously many and potentially better targeted alternatives to an MQS as a policy against collusion, e.g., provision of incentives to whistle-blowers or imposition of high fines for anticompetitive behavior. An anti-collusive MQS therefore seems to make most sense when there are complementing technological reasons for quality regulation, such as spillovers or safety concerns, which are not captured by our model. Note, however, that collusion in the benchmark case considered above has a feature that – even without any externality of quality on other industries or consumers – makes the *ex post* introduction of an MQS, meaning that market outcomes can first be observed for some periods and then any new MQS would "restart" the considered game, particularly appealing. Namely, only a single quality – the high one – will be traded if firms collude.[16]

So an MQS set between the (virtual) low quality and the high quality is 'non-binding' from a sales perspective. It is perfectly innocuous, involves no direct welfare loss, and should be opposed neither by consumers nor firms *if* the observed market structure with a seemingly monopolistic firm 1 and a potential second producer, firm 2, currently content to play a retailing role, involves no collusion. The combination of firm 2's production possibility (i.e., a latent threat of backward integration) and a generous retailing agreement with firm 1 may subtly hint at anti-competitive

[15]The latter is the case for $\delta > \delta^c \approx 0.84366$: the implied MQS would be excessive.

[16]Sorenson (1997), in a Bertrand model without quality costs, shows that firms will also offer identical products if they collude on prices. Similarly, collusion is easier to sustain under Bertrand competition if firms offer more similar qualities (Sorenson 1998).

behavior, but are unlikely to stand up as evidence of collusion in any court. It has some elegance then to regulate the traded quality in a superficially non-intrusive way: the MQS causes no harm if suppliers' relations have been misjudged; otherwise it uses their self-interest to bring down collusion.

6 Extensions

In the baseline collusion scenario, the low quality producer receives half of the difference between total competitive profits and the maximal aggregate profit; it always prefers collusion to a deviation. With this in mind, consider a situation in which the critical discount factor r is slightly above firms' discount factor δ, i.e., collusion as investigated in Section 5 would not be stable. It is at least conceivable – and may be quite likely – that the low quality producer offers part of its designated equal split to the high quality producer, i.e., the firms settle for a lower side payment, raise firm 1's collusion payoff, and push the critical discount factor below δ. Both firms would thus be better off relative to otherwise unavoidable competition.

This questions the rather standard application of a fixed division rule such as Nash bargaining to the static collusion rent in our model. Its replacement by the (present value of the) dynamic stream of rents is, however, also problematic: it presupposes the very stability of collusion which is being investigated. We therefore suggest to consider the *minimal* critical discount factor which is achievable for *any* conceivable division rule as an alternative indicator of collusion stability. We consider its reaction to an MQS in the next subsection. Afterwards, attention will be turned to possible collusion without side payments, individual quality choices that already anticipate quantity collusion, and finally the case of variable quality costs.

Obviously, many other extensions or variations are possible. For example, consumers' preferences could be modified in analogy to Kuhn (2007), the number of active firms might be increased as in Scarpa (1998), or the timing of the quality choices could be varied as in Constantatos and Perrakis (1998). We conjecture that such modifications would qualify some statements (e.g., regarding the change of consumer surplus), but not reverse the basic anti-collusive effect of the MQS.

6.1 Minimal critical discount factor

When side payments are possible, collusion profits in general amount to

$$\pi_1^c(s_1, s_2) \quad = \quad \pi_1^p(s_1, s_2) + q \cdot \pi_\Delta(s_1, s_2) \tag{57}$$

and

$$\pi_2^c(s_1, s_2) \quad = \quad \pi_2^p(s_1, s_2) + (1 - q) \cdot \pi_\Delta(s_1, s_2) \tag{58}$$

for some $q \in [0,1]$ and the given rent $\pi_\Delta(s_1, s_2)$. The critical discount factor of the high quality producer can then be written as

$$\rho_1(\alpha, q) \quad = \quad \frac{8 - 4\,q - 3\,\alpha + 3\,q\,\alpha}{8 - 3\,\alpha} \tag{59}$$

for any regulated quality ratio α. Now note that $\frac{\partial \rho_1(\alpha,q)}{\partial q} = \frac{4-3\,\alpha}{3\,\alpha-8} < 0$, i.e., raising firm 1's share of aggregate profits facilitates and stabilizes collusion. The critical discount factor is smallest (and collusion easiest to maintain) if q is chosen to be maximal under the constraint that firm 2's incentive to collude remains unchanged, i.e.,

$$\sum_{t=1}^{\infty} \delta^t \big(\pi_2^p(\alpha) + (1-q)\cdot \pi_\Delta(\alpha)\big) \geq \delta \pi_2^d(\alpha) + \sum_{t=2}^{\infty} \delta^t \pi_2^p(\alpha). \tag{60}$$

Inequality (60) happens to be satisfied for *all* $q \in [0,1]$. The *minimal critical discount factor* such that collusion can be maintained by Nash reversion strategies under *any* solution to the bargaining problem between both firms hence evaluates to

$$\rho^*(\alpha) = \rho_1(\alpha, 1) = \frac{4}{8 - 3\,\alpha}. \tag{61}$$

It follows that the minimal critical discount factor increases when a moderate MQS is imposed ($\frac{\partial \rho^*(\alpha)}{\partial \alpha} > 0$). The anti-collusive effect of an MQS therefore does not hinge on the – in our dynamic context perhaps slightly dissatisfying – assumption of Nash bargaining; it is very robust with respect to the division of collusion rents.[17]

6.2 Collusion without side payments

We have so far assumed that firms can collude on quantities rather explicitly: they pick the total profit-maximizing production plan and then organize side payments.[18] The latter might be problematic if something like the suggested retailing camouflage for firm 2 is infeasible. Actual payments from firm 1 to firm 2 would provide antitrust authorities with accessible hard evidence in legal proceedings. This might make the use of second-best policies against collusion, such as the investigated use of an MQS, unnecessary. It is therefore of interest that an MQS can also prevent tacit collusion *without* the possibility of side payments.

We maintain the assumption that firms' try to maximize total profits

$$p_1(x_1, x_2, s_1, s_2) \cdot x_1 + p_2(x_1, x_2, s_1, s_2) \cdot x_2 - C(s_1) - C(s_2) \tag{62}$$

but suppose that side payments are replaced by coordinated quantity choices. The collusive

[17] In fact, $\frac{\partial \rho_1(\alpha,q)}{\partial \alpha} > 0$ holds for every $q \in (0,1]$.

[18] See Jehiel (1992) on the effect of side payments on the equilibrium degree of endogenous horizontal product differentiation. Firms that later collude on prices will ex ante pick two differentiated products if side payments are possible; they offer identical products if the antitrust enforcement system prevents monetary transfers.

quantity choices x_1^c and x_2^c must result in profits which exceed the respective competitive profit $\pi_i^p(s_1, s_2)$ for either firm. As in the previous subsection, we consider the surplus shares that yield the *minimal* critical discount factor.[19]

In particular, we maximize (62) subject to the constraint

$$\pi_2^c(x_1, x_2, s_1, s_2) \equiv p_2(x_1, x_2, s_1, s_2) \cdot x_2 - C(s_2) = z \cdot \pi_2^p(s_1, s_2) \tag{63}$$

for a given $z \geq 1$. The unwieldy solution determines collusion profits $\pi_i^c(z)$ and – computing the respective best responses – deviation profits $\pi_i^d(z)$. For a given imposed collusion profitability z for firm 2, we thus obtain expressions for $\rho_1(\alpha, z)$ and $\rho_2(\alpha, z)$ such that

$$\rho(\alpha, z) \equiv \max\{\rho_1(\alpha, z), \rho_2(\alpha, z)\} \tag{64}$$

is the critical discount factor. Minimization of $\rho(\alpha, z)$ with respect to $z \geq 1$ then yields the minimal critical discount factor such that collusion can be maintained by Nash reversion equilibria. It is worth noting that the respective minimizer z^* is strictly greater than 1: firm 1's collusion quantity is much smaller than $x_1^c = \frac{a}{2}$ from Section 5; firm 2 would deviate to a bigger quantity if it were kept at its competitive profit. In contrast to the case *with* side payments, both differentiated products have positive sales.

The minimal critical discount factor for given quality ratio, $\rho(\alpha)$, is illustrated in Figure 3. We find again that the imposition of an MQS (corresponding to $\alpha > \hat{\alpha}$) makes collusion harder. This confirms that the indicated beneficial dynamic effect of an MQS is no artifact of a particular form of collusion and remains effective when first-best policies – namely, enforcement of a legal ban – are difficult to implement.

6.3 Anticipation of collusion

In Section 5, we have presumed that the irreversible quality choices in $t = 0$ are made under the anticipation of subsequent quantity *competition*. This feature of the model, also found in Ecchia and Lambertini (1997), seems quite plausible if collusive behavior is decided on or emerges at a different management level inside the firms (e.g., product, region or key account managers) than their brand-wide and more long-term quality positioning (owner, board, CEO).

However, the assumption is unappealing from a game-theoretic perspective: even if one assumes that different instances of firms 1 and 2 (say, top and middle management) are in charge of quality and quantity decisions, the first-moving agents can only be said to maximize profits by solving equations (7) and (8) if collusive outcomes later on take them by surprise. This section therefore follows the related analysis of horizontal differentiation (Jehiel 1992; Friedman and Thisse 1993)

[19] This time we have technical reasons for doing so, namely the Pareto frontier's non-linear shape does not admit a closed-form solution for the Nash bargaining outcome. Admittedly, the minimal critical discount factor entails an analytical bias in favor of collusive market outcomes. But note that this does not necessarily maximize the odds for an MQS having an anti-collusive effect. In particular, numerical inspection of $\rho(\alpha, z)$ (defined below) reveals the same qualitative behavior for various fixed levels of z.

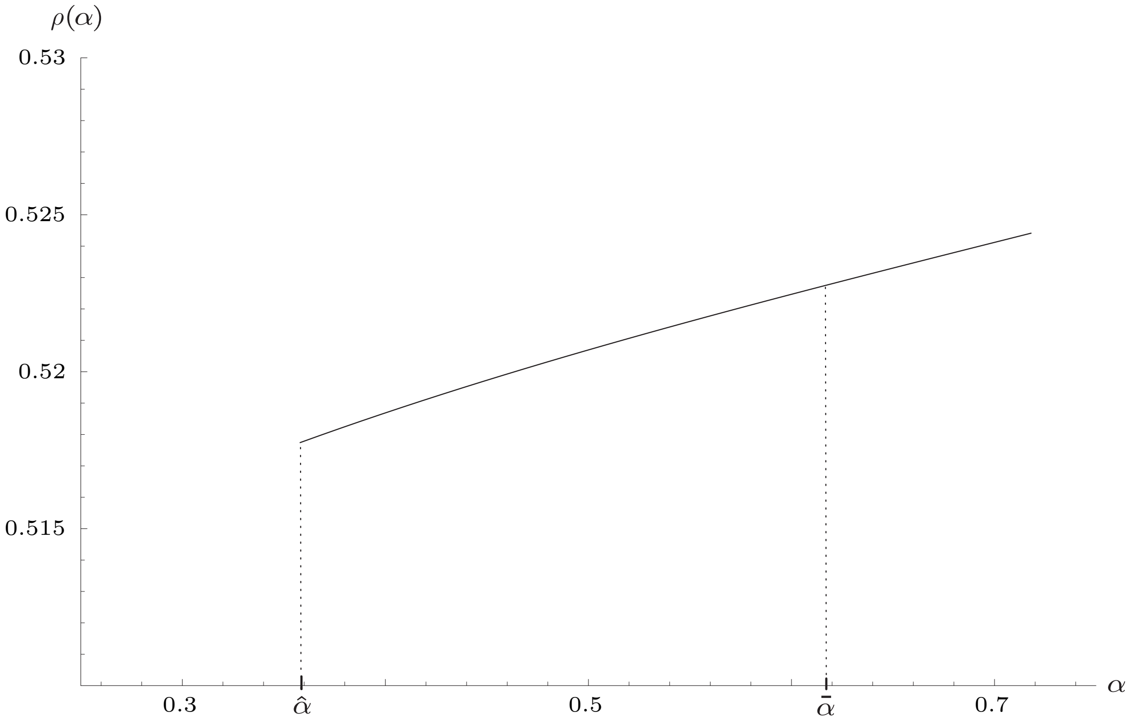

Figure 3: Minmal critical discount factor without side payments

and considers a scenario in which qualities are still chosen competitively, but in correct anticipation of subsequent price collusion.[20]

We return to the analytically simpler case in which side payments are feasible. Recall our earlier finding that the critical discount factor is a strictly increasing function of the quality ratio $\alpha(\tilde{s})$ (see equations (45) and (47)). It hence remains to check that this quality ratio still strictly increases in the MQS $\tilde{s}$ if the two firms maximize their respective collusion profits π_i^c, given by equation (37) or (38), instead of their competitive profits $\pi_i \equiv \pi_i^p$ in period $t = 0$.

The first-order conditions for optimal quality choices

$$\frac{\partial \pi_1^c(s_1, s_2)}{\partial s_1} = \frac{5a^2}{8} + s_1 \left(\frac{3a^2\,(s_2 - 2s_1)}{(s_2 - 4s_1)^{\,2}} - 2\gamma \right) = 0 \qquad (65)$$

$$\frac{\partial \pi_2^c(s_1, s_2)}{\partial s_2} = \frac{3a^2 s_1^2}{2\,(s_2 - 4s_1)^{\,2}} - 2\gamma s_2 = 0 \qquad (66)$$

replace (7) and (8) if firms anticipate quantity collusion at the quality setting stage. The resulting *unregulated equilibrium qualities for anticipated collusion*,[21]

$$\hat{s}_1 \approx \frac{0.12830a^2}{\gamma} \quad \text{and} \quad \hat{s}_2 \approx \frac{0.06015a^2}{\gamma}, \qquad (67)$$

[20] Friedman and Thisse (1993) argue that it is a realistic feature that quality choices are made non-cooperatively despite later price or quantity collusion. For instance, firms may need to select their qualities before even knowing their competitors (e.g., because of a patent race between many potential entrants).

[21] The second-order conditions are satisfied.

define the new benchmark ratio $\hat{s}_2/\hat{s}_1$ for regulation. We can show that the *regulated equilibrium quality ratio for anticipated collusion*

$$\alpha(\tilde{s}) \equiv \frac{s_2(\tilde{s})}{s_1(\tilde{s})} = \frac{\tilde{s}}{R_1(\tilde{s})} \in (\hat{s}_2/\hat{s}_1, 1] \tag{68}$$

is a strictly increasing function of MQS $\tilde{s}$ in perfect analogy to Section 3 (where $R_1(s_2)$ captures the new best response by firm 1 to firm 2's quality choice s_2). First, Lemma 4 establishes that the equilibrium quality gap between both firms decreases in $\tilde{s}$ if firm 2 adopts the mandated quality, i.e., supposing that $s_2^*(\tilde{s}) = \tilde{s}$:

Lemma 4 *When firms already anticipate later quantity collusion at the quality setting stage ($t = 0$), firm 1 responds to any given increase Δs_2 of firm 2's quality by an increase $\Delta s_1 < \Delta s_2$ of its own quality. In particular,*

$$0 < \frac{\partial R_1(s_2)}{\partial s_2} < 1.$$

Proof: Substituting $s_2 \equiv \beta \cdot s_1$ with $\beta \in (0, 1]$ in (65), the first-order condition for firm 1's quality choice can equivalently be written as

$$s_1 = \frac{a^2\left(5\beta^2 - 16\beta + 32\right)}{16(\beta - 4)^2\gamma}. \tag{69}$$

Moreover, application of the implicit function theorem to equation (65) and afterwards the substitution $s_2 = \beta \cdot s_1$ yield

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{\beta(8\beta - 8)}{\beta^2(8\beta - 8) - (\beta - 4)^4 s_1}. \tag{70}$$

Using the rearranged first-order condition (69) for s_1, this simplifies to

$$\frac{\partial R_1(s_2)}{\partial s_2} = -\frac{64(\beta - 1)\beta}{5\beta^4 - 120\beta^3 + 304\beta^2 - 512\beta + 512}. \tag{71}$$

Now, recalling the fact that $\beta \in (0, 1]$, numerical inspection allows to infer that

$$\frac{\partial R_1(s_2)}{\partial s_2} \in (0, 0.05319]. \tag{72}$$

□

Second, Lemma 5 establishes that indeed $s_2^*(\tilde{s}) = \tilde{s}$ also if firms anticipate quantity collusion when they set their qualities:

Lemma 5 *Given an MQS $\tilde{s} > \hat{s}_2$ such that both firms stay in the market when they already anticipate later quantity collusion at the quality setting stage ($t = 0$), firm 2 selects exactly the mandated quality in equilibrium, i.e.,*

$$s_2(\tilde{s}) = \tilde{s}.$$

Proof: Again using the notation $s_2 \equiv \beta \cdot s_1$ with $\beta \in (0, 1]$, the change of firm 2's profit caused by a marginal increase of s_2 can be written as

$$\frac{\partial \pi_2^c}{\partial s_2} = \frac{3a^2 - 4(\beta - 4)^2 \beta \gamma s_1}{2(\beta - 4)^2}. \tag{73}$$

Considering a best response by firm 1, i.e., imposing the rearranged first-order condition (69), this becomes

$$\frac{\partial \pi_2^c}{\partial s_2} = \frac{a^2 \left(-5\beta^3 + 16\beta^2 - 32\beta + 12\right)}{8(\beta - 4)^2}, \tag{74}$$

which is positive (negative) to the left (right) of $\beta = \hat{s}_2/\hat{s}_1$.

Imposition of $\tilde{s}$ means that s_2 rises by $\Delta s \geq \tilde{s} - \hat{s}_2$. By Lemma 4, s_1 rises by less than Δs. A post-MQS equilibrium quality ratio must hence satisfy $\beta > \hat{s}_2/\hat{s}_1$. Thus (74) is negative and firm 2 must select the minimum feasible quality $s_2^*(\tilde{s}) = \tilde{s}$ in equilibrium.

□

So, in summary, an MQS $\tilde{s}$ continues to push firms' equilibrium quality choices closer together if they correctly anticipate later quantity collusion, i.e., $\partial\alpha(\tilde{s})/\partial\tilde{s} > 0$ remains true. Given that the critical discount factor strictly increases in $\alpha(\tilde{s})$, the introduction of a moderate MQS thus continues to make collusion less profitable, and to prevent it (depending on firms' actual discount factor).

It is again straightforward to show that a collusion-preventing MQS can enhance welfare. The only difference to the case where firms myopically expect later quantity competition is that *any* MQS which avoids collusion also increases social welfare.

Proposition 6 *A welfare-enhancing MQS exists if* $\delta \in (\underline{\delta}; \bar{\delta})$ *for* $\underline{\delta} \approx 0.80332$ *and* $\bar{\delta} \approx 0.83622$.

Proof: The high-quality producer obtains a higher profit than the low-quality producer. The competitive profit of the low quality producer is given by (6). Substituting $s_2 = \alpha s_1$ and using (65), we obtain

$$\pi_2^p(\alpha) = \frac{a^4 \alpha \left(-25\alpha^5 + 160\alpha^4 - 606\alpha^3 + 1240\alpha^2 - 1600\alpha + 768\right)}{256(\alpha - 4)^4 \gamma}, \tag{75}$$

which is positive as long as $\alpha < \alpha^c \approx 0.68386$. Any MQS which implies a higher regulated equilibrium quality ratio pushes firm 2 out of the market.[22]

In the unregulated market the equilibrium quality ratio is equal to $\hat{\alpha} \equiv \frac{\hat{s}_2}{\hat{s}_1} \approx 0.46878$ which implies a social surplus of $\hat{W}^{col} \approx \frac{0.02803a^4}{\gamma}$ in case of collusion.

Suppose that an MQS which avoids collusion is introduced. The change in social welfare due to this policy is then given by (substituting $s_2 = \alpha\, s_1$ and using (65))

$$W^{com}(\alpha) - \hat{W}^{col} = -\frac{a^4 \left(5\alpha^2 - 16\alpha + 32\right) \left(5\alpha^4 - 16\alpha^3 + 29\alpha^2 + 24\alpha - 64\right)}{256(\alpha - 4)^4 \gamma} - \hat{W}^{col} \tag{76}$$

[22] This would be the case if $\tilde{s} > \tilde{s}^c \approx \frac{0.09093a^2}{\gamma}$.

Numerical inspection reveals that this is positive for all $\alpha \in (\hat{\alpha}, \alpha^c)$. Hence an MQS always increases social welfare if it prevents collusion. The latter is the case if the discount factor δ of the high quality producer satisfies

$$0.80332 \approx \rho(\hat{\alpha}) \equiv \underline{\delta} < \delta < \bar{\delta} \equiv \rho(\alpha^c) \approx 0.83622. \tag{77}$$

□

6.4 Variable quality costs

We finally consider the case in which quality affects variable costs – for instance, because greater quality requires more expensive raw materials, specialized labor, more time, etc. Appendix B contains formal proofs of the reported results.

In line with the related literature we assume quality-dependent unit costs

$$c(s_i) = \gamma s_i^2 \tag{78}$$

without fixed costs. For given qualities $s_1 \geq s_2$, the equilibrium quantities are

$$\hat{x}_1(s_1, s_2) = \frac{2\,a\,s_1 - 2\,\gamma\,{s_1}^2 - a\,s_2 + \gamma\,{s_2}^2}{4\,s_1 - s_2}, \tag{79}$$

$$\hat{x}_2(s_1, s_2) = \frac{s_1\,(a + \gamma\,s_1 - 2\,\gamma\,s_2)}{4\,s_1 - s_2} \tag{80}$$

and result in the reduced profits

$$\pi_1(s_1, s_2) = \frac{s_1\,(2\,s_1\,(\gamma\,s_1 - 2\,\alpha) + s_2\,(a - \gamma\,s_2))^2}{(s_2 - 4\,s_1)^2}, \tag{81}$$

$$\pi_2(s_1, s_2) = \frac{{s_1}^2\,s_2\,(a + \gamma\,s_1 - 2\,\gamma\,s_2)^2}{(s_2 - 4\,s_1)^2}. \tag{82}$$

The implied *unregulated equilibrium qualities* can be computed as

$$\hat{s}_1 \approx \frac{0.36905\,a}{\gamma} \text{ and } \hat{s}_2 \approx \frac{0.29279\,a}{\gamma}. \tag{83}$$

As in the case of fixed quality costs, firm 2 chooses $s_2^*(\tilde{s}) = \tilde{s}$ if the constraint $s_i \geq \tilde{s}$ is imposed, and again the *regulated equilibrium quality ratio*

$$\alpha(\tilde{s}) \equiv \frac{s_2^*(\tilde{s})}{s_1^*(\tilde{s})} = \frac{\tilde{s}}{R_1(\tilde{s})} \in (\hat{s}_2/\hat{s}_1, 1] \tag{84}$$

is strictly increasing in $\tilde{s}$. Firms' profits can be shown to satisfy

$$\frac{d\pi_1(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} < 0, \tag{85}$$

$$\frac{d\pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} \begin{cases} > 0; & \tilde{s} < \tilde{s}^l \\ < 0; & \tilde{s} > \tilde{s}^l \end{cases} \tag{86}$$

with $\tilde{s}^l \approx \frac{0.29443\,a}{\gamma}$. In contrast to the case of fixed quality costs, the low quality producer may now benefit from an MQS: the latter may create a valuable commitment to offering a relatively high quality, with greater margins dominating the implied quantity reduction (as would be typical for Bertrand competition).

The effect of an MQS on total consumer surplus can now be negative, namely

$$\frac{\partial S(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{\partial \tilde{s}} \begin{cases} > 0; & \tilde{s} < \tilde{s}^m \\ < 0; & \tilde{s} > \tilde{s}^m \end{cases} \tag{87}$$

with $\tilde{s}^m \approx \frac{0.29887\,a}{\gamma}$. Whilst the MQS had only an indirect demand effect on prices in the fixed cost case, higher qualities also raise variable costs here and thus potentially reduce total consumer surplus. It is hence not surprising that, as before, total surplus decreases in the level of the MQS, i.e.,

$$\frac{d(\pi_1(\cdot) + \pi_2(\cdot) + S(\cdot))}{d\tilde{s}} < 0. \tag{88}$$

So the static net effect of an MQS on surplus is again disadvantageous. We are, however, interested mainly in the potential long-run effects of quality regulation. Retaining the Nash bargaining assumption of Section 5, one can derive

$$\begin{aligned} \pi_1^c(s_1, s_2) \;=\; & \frac{s_1\left(8\gamma^2 s_1{}^3 + \gamma s_1{}^2\left(-16a + 3\gamma s_2\right)\right)}{32 s_1 - 8 s_2} \\ & + \frac{s_1\left(s_1\left(8a^2 + 2a\gamma s_2 - 5\gamma^2 s_2{}^2\right) + s_2\left(-5a^2 + 8a\gamma s_2 - 3\gamma^2 s_2{}^2\right)\right)}{32 s_1 - 8 s_2} \end{aligned} \tag{89}$$

and

$$\pi_2^c(s_1, s_2) = \frac{s_1\, s_2\, \left(3a^2 + 3\gamma^2 s_1{}^2 - 8a\gamma s_2 + 5\gamma^2 s_2{}^2 + \gamma s_1\,(2a - 5\gamma s_2)\right)}{32 s_1 - 8 s_2} \tag{90}$$

with, in contrast to the baseline scenario of Section 5, *both* products on offer. It also turns out that both firms face a short-term temptation to cheat. The corresponding deviation profits are given by

$$\pi_1^d(s_1, s_2) = \frac{s_1\left(-2a + 2\gamma s_1 + \gamma s_2\right)^2}{16}, \tag{91}$$

$$\pi_2^d(s_1, s_2) = \frac{s_2\left(a + \gamma s_1 - \gamma s_2\right)^2}{16}. \tag{92}$$

We show in Appendix B that the critical discount factor associated with Nash bargaining initially

	Fixed quality costs		Variable quality costs	
	static	**dynamic**	**static**	**dynamic**
Bertrand case	MQS raises welfare ... (Ronnen 1991; Jinji and Toshimitsu 2004)	... but facilitates collusion (Häckner 1994)	MQS typically raises welfare (if it reduces the quality gap) ... (Crampes and Hollander 1995; Kuhn 2007)	... and hinders collusion, too (Ecchia and Lambertini 1997)
	static	**dynamic**	**static**	**dynamic**
Cournot case	MQS reduces welfare ... (Valletti 2000; Jinji and Toshimitsu 2004)	... but hinders collusion and can thus raise welfare	MQS reduces welfare ...	... but hinders collusion and can thus raise welfare

Table 2: Effects of an MQS on welfare for different market structures

decreases in the level of the MQS, but then increases for sufficiently high $\tilde{s}$:

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{ds} \begin{cases} < 0; & \tilde{s} < \tilde{s}^b \\ > 0; & \tilde{s} > \tilde{s}^b \end{cases} \tag{93}$$

with $\tilde{s}^b \approx \frac{0.29868\,a}{\gamma} > \tilde{s}$.

Again a trade off between static costs and dynamic benefits of an MQS exists: on the one hand, the MQS destroys surplus by distorting firms' unregulated quality choices, but on the other hand it can prevent or destabilize collusion. In contrast to the benchmark case with fixed quality costs, any MQS which prevents collusion automatically raises total welfare (i.e., whenever $r(\tilde{s}) > \delta > r(0)$). The corresponding interval of discount factors δ is therefore larger; namely, a welfare-enhancing MQS exists for the case of variable quality costs if $\delta \in (\underline{\delta}; \bar{\delta})$ for $\underline{\delta} \approx 0.54336$ and $\bar{\delta} \approx 0.69523$. The optimal MQS is again the respective lowest one which prevents collusion.

7 Concluding remarks

Our main findings are summarized in Table 2 together with the most closely related literature. In particular, we have shown that an MQS can be welfare-increasing also if firms compete in quantities. Whilst the distortion of the unregulated equilibrium qualities, which is induced by the MQS, lowers the generated total surplus in a static single-shot environment, it also reduces the attractiveness of collusion relative to competitive behavior when firms interact repeatedly. The MQS can thereby prevent or destabilize collusion, and has dynamic benefits. Under the right circumstances these benefits outweigh the static costs of an MQS, i.e., the MQS raises total surplus. Interestingly, this may be achieved by a superficially non-binding MQS in the benchmark case (where side payments are feasible): the MQS changes firms' competitive fallback positions, and this can make it impossible to sustain collusion. Implementation of such an MQS is unlikely to face resistance because the involved firms would indirectly have to admit to colluding. Moreover, the introduction of an MQS based on a misinterpretation or misjudgment of firms' conduct would have no negative side effects, since the introduced MQS would be non-binding *de facto*, rather than only superficially so.

The anti-collusive impact of a binding non-excessive MQS is surprisingly robust. In particular, we have considered side payments as well as pure quantity coordination, moved from fixed quality costs to variable quality costs, investigated both the standard measure of collusion stability as well as an alternative that allows for more flexibility in bargaining. And we have shown that our key results regarding the prevention of collusion and its welfare effects continue to hold if firms already anticipate a later collusive agreement at the quality-setting stage. Many other variations, such as different rent sharing rules, are unlikely to make a significant difference. For example, it seems that our results could be extended to markets with more than two active firms, as in Scarpa (1998): the available range of non-excessive MQS is bound to shrink, but detrimental static effects of the MQS still need to be traded off against beneficial dynamic effects, and may be dominated by them. We have confirmed this for examples with three firms.

Of course, one cannot expect our results to hold for *all* reasonable specifications of firms' costs or consumers' utility (see Kuhn 2007). And, critically, other pro-competitive policy measures exist, which may be much more effective and economical than an MQS in preventing collusive behavior. Legislation offers authorities a number of alternatives to the distorting introduction of a new MQS or the tightening of an existing one – e.g., lower evidence requirements in antitrust cases, stiffer penalties, greater investment in detection technology, or leniency programs for whistle blowers. Deterrence of collusion by such means will in many contexts be strictly preferable to the less direct MQS-based policies studied above. But, first, it should not be taken for granted that no or smaller economic distortions are induced by these (lobbying, corruption, efforts of concealment and deception). Second, there may be other good reasons for the introduction of an MQS – for instance environmental and technological spillovers in industries such as transportation or telecommunication, public health or consumer safety concerns, strategic trade policy, etc. When the related benefits of an MQS are compared to the static welfare losses studied by Valletti (2000), the potential dynamic gains identified here should be taken into account. They might tip the balance in a number of "marginal" market environments.

The conventional wisdom concerning the merits of an MQS has repeatedly needed updating in the past. The early investigations emphasized that firms' cost structure and the intensity of their competition matter; several more recent studies indicate how the baseline results depend on consumers' preferences, the number of active firms, or the timing of their decisions. Our investigation has highlighted the role of the time horizon and the associated market conduct. In particular, quality regulation under quantity competition can make sense from a dynamic perspective; tempting dichotomous verdicts deserve further qualification.

Appendix A

The second-order condition for firm 1's quality $s_1^*(\tilde{s})$ in the baseline model

$$\frac{\partial^2 \pi_1(s_1, s_2)}{\partial s_1^2} = \frac{8\, a^2\, {s_2}^2\, (s_2 - s_1)}{(4\, s_1 - s_2)^4} - 2\,\gamma < 0 \tag{94}$$

is satisfied for all $s_1 > s_2$. Further conditions for the boundary point maximum $s_2^*(\tilde{s}) = \tilde{s}$ need not be checked.

The low quality producer has no incentive to leapfrog, i.e., it cannot gain by a deviation $s_2' \geq s_1^*(\tilde{s})$: if the low quality producer chooses $s_2' \geq s_1^*(\tilde{s})$, its profit is $\pi_2^L(s_1^*(\tilde{s}), s_2')) \equiv \pi_1(s_2', s_1^*(\tilde{s}))$, and decreases in the now lower quality $s_1^*(\tilde{s})$; namely

$$\frac{\partial \pi_1(s_1, s_2)}{\partial s_2} = \frac{4\, a^2\, {s_1}^2\, (s_2 - 2\, s_1)}{(4\, s_1 - s_2)^3} < 0. \tag{95}$$

By Lemma 1, $s_1^*(\tilde{s}) \geq \hat{s}_1$ and therefore

$$\pi_2^L(s_1^*(\tilde{s}), s_2')) \leq \pi_1(s_2', \hat{s}_1) \approx \frac{s_2' \left(0.01587\, a^6 - 0.51975\, a^4\, \gamma\, s_2' + 5.00777\, a^2\, \gamma^2\, {s_2'}^2 - 16\, \gamma^3\, {s_2'}^3\right)}{\left(4\, \gamma\, s_2' - 0.12597\, a^2\right)^2}. \tag{96}$$

The latter term is maximal at $s_2' = \frac{0.12961\, a^2}{\gamma}$ and bounded above by $\frac{-0.00186\, a^4}{\gamma}$. So firm 2 cannot attain a positive profit by leapfrogging. Firm 1 cannot leapfrog firm 2 because $s_2^*(\tilde{s}) = \tilde{s}$.

Appendix B

This appendix considers the case of *variable quality costs* (without fixed costs), namely unit costs are $c(s_i) = \gamma s_i^2$. The reduced profit functions (81) and (82) yield the first-order conditions

$$\begin{aligned} \frac{\partial \pi_1(s_1, s_2)}{\partial s_1} &= \frac{s_1 \left(-2\, a\, s_1 + 2\,\gamma\, {s_1}^2 + s_2\, (a - \gamma\, s_2)\right)^2}{(s_2 - 4\, s_1)^2} = 0 & (97) \\ \frac{\partial \pi_2(s_1, s_2)}{\partial s_2} &= \frac{{s_1}^2\, s_2\, (a + \gamma\, s_1 - 2\,\gamma\, s_2)^2}{(s_2 - 4\, s_1)^2} = 0, & (98) \end{aligned}$$

from which the indicated unregulated qualities $(\hat{s}_1, \hat{s}_2)$ can be deduced.

As in the baseline case of fixed quality costs, the quality gap decreases in $\tilde{s}$:

Lemma 6 $\frac{\partial R_1(s_2)}{\partial s_2} < 1$.

Proof: Substituting $s_2 \equiv \beta \cdot s_1$ with $\beta \in (0, 1]$ in (97), the first-order condition for firm 1's quality choice can equivalently be written as

$$s_1 = \frac{a\, (\beta^2 - 2\,\beta + 8)}{(\beta^3 + 4\,\beta^2 - 10\,\beta + 24)\, \gamma}. \tag{99}$$

The second-order condition is satisfied: using the rearranged first-order condition (99) and $s_2 \equiv \beta \cdot s_1$ with $\beta \in (0,1]$, one obtains

$$\frac{\partial^2 \pi_1}{\partial s_1^2} = -\frac{24\,a\,(\beta^5 - 4\,\beta^4 + 8\,\beta^3 - 16\,\beta^2 + 20\,\beta - 16)\,\gamma}{(\beta^3 - 6\,\beta^2 + 16\,\beta - 32)\,(\beta^3 + 4\,\beta^2 - 10\,\beta + 24)} < 0. \tag{100}$$

The implicit function theorem applied to (97) and (99) yields

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{\beta^4 - 4\,\beta^3 + 26\,\beta^2 + 16\,\beta - 32}{6\beta^4 - 36\,\beta^3 + 60\,\beta^2 - 96\,\beta + 192}. \tag{101}$$

Numerical inspection then allows to infer

$$\frac{\partial R_1(s_2)}{\partial s_2} \in \left[-\frac{1}{16}, \frac{1}{6}\right). \tag{102}$$

□

Lemma 7 *$s_2^*(\tilde{s}) = \tilde{s}$.*

Proof: Using $s_2 = \beta \cdot s_1$ and (99), we have

$$\frac{\partial \pi_2}{\partial s_2} = \frac{a^2\,(3\,\beta^5 - 22\,\beta^4 + 103\,\beta^3 - 308\,\beta^2 + 512\,\beta - 256)}{(\beta - 4)\,(\beta^3 + 4\,\beta^2 - 10\,\beta + 24)^2}, \tag{103}$$

which is positive (negative) to the left (right) of $\beta = \hat{s}_2/\hat{s}_1$. By Lemma 6, we must have $\beta > \hat{s}_2/\hat{s}_1$ in equilibrium, i.e., (103) is negative and $s_2^*(\tilde{s}) = \tilde{s}$ becomes a boundary point maximum.

Firm 2 has no incentive to leapfrog, i.e., to choose $s_2' \geq s_1^*(\tilde{s})$. Its profit would then be $\pi_2^L(s_1^*(\tilde{s}), s_2') \equiv \pi_1(s_2', s_1^*(\tilde{s}))$. This decreases in the now lower quality $s_1^*(\tilde{s})$; namely, with (99) and $\beta \in [0,1]$ we obtain

$$\left.\frac{\partial \pi_1}{\partial s_2}\right|_{s_1 = R_1(s_2)} = -\frac{8\,a^2\,(\beta^5 - 6\,\beta^4 + 28\,\beta^3 - 60\,\beta^2 + 68\,\beta - 32)}{(\beta - 4)\,(\beta^3 + 4\,\beta^2 - 10\,\beta + 24)^2} < 0. \tag{104}$$

$s_1^*(\tilde{s})$ satisfies (99); hence $s_1^*(\tilde{s}) \geq \frac{7a}{19\gamma} \equiv s_1^{min}$ and

$$\pi_2^L(s_1^*(\tilde{s}), s_2') \leq \pi_1(s_2', s_1^{min}) = \frac{4\,s_2'\left(42\,a^2 - 361\,a\,\gamma\,s_2' + 361\,\gamma^2\,{s_2'}^2\right)^2}{361\,(7\,a - 76\,\gamma\,s_2')^2}. \tag{105}$$

The latter term is *maximized* by $s_2' = \frac{7\,a}{19\,\gamma} = s_1^{min}$, corresponding to a quality ratio $\alpha = 1$. In contrast, the reference profit $\pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))$ (see (82)), written as a function of the equilibrium quality ratio $\alpha \in (\hat{s}_2/\hat{s}_1, 1]$

$$\Pi_2(\alpha) \quad = \quad \frac{a^3\,\alpha\,(\alpha^2 - 5\,\alpha + 8)\,(\alpha^4 - 7\,\alpha^3 + 26\,\alpha^2 - 56\,\alpha + 64)}{(\alpha^3 + 4\,\alpha^2 - 10\,\alpha + 24)^3\,\gamma}, \tag{106}$$

is *minimized* by $\alpha = 1$ and then equal to $\pi_2^L(s_1^{min}, s_1^{min})$. So $\pi_2^L(s_1^*(\tilde{s}), s_2') \leq \pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))$.

□

Proposition 7

$$\frac{d\pi_1(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} < 0 \quad and \quad \frac{d\pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} \begin{cases} > 0; & \tilde{s} \leq \tilde{s}^l \approx \frac{0.29442\, a}{\gamma} \\ < 0; & \tilde{s} > \tilde{s}^l. \end{cases} \tag{107}$$

Proof: Firm 1's profits (see (81)) can be written as a function of the regulated equilibrium quality ratio, using (99), as follows:

$$\Pi_1(\alpha) = \frac{16\, a^3\, (\alpha^2 - 2\,\alpha + 2)\, (\alpha^4 - 4\,\alpha^3 + 14\,\alpha^2 - 20\,\alpha + 16)}{(\alpha^3 + 4\,\alpha^2 - 10\,\alpha + 24)^3\, \gamma}. \tag{108}$$

Firm 2's profit is shown in (106). Changes due to the MQS are

$$\frac{d\Pi_1(\alpha)}{d\tilde{s}} = \frac{\partial \Pi_1(\alpha)}{\partial \alpha} \cdot \underbrace{\frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}}}_{>0} < 0, \tag{109}$$

with

$$\text{sign}\left(\frac{\partial \Pi_1(\alpha(\tilde{s}))}{\partial \alpha}\right) = \text{sign}(\alpha^3 - 4\,\alpha^2 + 18\,\alpha - 16) < 0 \tag{110}$$

using $\frac{\hat{s}_2}{\hat{s}_1} \leq \alpha \leq 1$, and

$$\frac{d\Pi_2(\alpha)}{d\tilde{s}} = \frac{\partial \Pi_2(\alpha)}{\partial \alpha} \cdot \underbrace{\frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}}}_{>0} \tag{111}$$

with

$$\text{sign}\left(\frac{\partial \Pi_2(\alpha(\tilde{s}))}{\partial \alpha}\right) = \text{sign}(-\alpha^7 + 15\,\alpha^6 - 75\,\alpha^5 + 236\,\alpha^4 - 462\,\alpha^3 + +528\,\alpha^2 \tag{112}$$

$$-1184\,\alpha + 768) \tag{113}$$

where the latter is positive for $\alpha < \alpha^l$ and negative for $\alpha \geq \alpha^l$ with $\alpha^l \approx 0.79769$. Finally, $\alpha = \alpha^l$ is equivalent to $\tilde{s} = \tilde{s}^l$.

□

The consumer surplus for given qualities s_1 and s_2 is

$$S(s_1, s_2) = \int_{\frac{p_2}{s_2}}^{\frac{p_1 - p_2}{s_1 - s_2}} (\theta s_2 - p_2)\, d\theta + \int_{\frac{p_1 - p_2}{s_1 - s_2}}^{a} (\theta s_1 - p_1)\, d\theta \tag{114}$$

which becomes

$$\Sigma(\alpha) = \frac{a^3\left(9\,\alpha^7-68\,\alpha^6+309\,\alpha^5-834\,\alpha^4+1480\,\alpha^3-1344\,\alpha^2+384\,\alpha+512\right)}{2\,(24-10\,\alpha+4\,\alpha^2+\alpha^3)^3\,\gamma}. \tag{115}$$

expressed in terms of the regulated equilibrium quality ratio. We then have

$$\frac{d\Sigma(\alpha)}{d\tilde{s}} = \frac{\partial\Sigma(\alpha)}{\partial\alpha}\cdot\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}} \tag{116}$$

with

$$\begin{aligned}\text{sign}\left(\frac{\partial\Sigma(\alpha)}{\partial\alpha}\right) &= \text{sign}(-3\,\alpha^9+40\,\alpha^8-266\,\alpha^7+1081\,\alpha^6-3030\,\alpha^5+6178\,\alpha^4 \\ &\quad -10272\,\alpha^3+13472\,\alpha^2-11520\,\alpha+4096).\end{aligned} \tag{117}$$

Numerical inspection shows that $\frac{\partial\Sigma(\alpha)}{\partial\alpha}$ is positive for $\alpha < \alpha^m$ and negative for $\alpha > \alpha^m$ with $\alpha^m \approx 0.80944$, where the latter corresponds to $\tilde{s}^m \approx \frac{0.29887\,a}{\gamma}$.

Proposition 8

$$\frac{d(\pi_1(\cdot)+\pi_2(\cdot)+S(\cdot))}{d\tilde{s}} < 0. \tag{118}$$

Proof: Total surplus expressed in terms of quality ratio α is

$$\Gamma(\alpha) \equiv \Pi_1(\alpha)+\Pi_2(\alpha)+\Sigma(\alpha) \tag{119}$$

$$= \frac{a^3\left(11\,\alpha^7-60\,\alpha^6+255\,\alpha^5-550\,\alpha^4+792\,\alpha^3-192\,\alpha^2-896\,\alpha+1536\right)}{2\,(24-10\,\alpha+4\,\alpha^2+\alpha^3)^3\,\gamma}. \tag{120}$$

The change in total surplus is equal to

$$\frac{d\Gamma(\alpha(\tilde{s}))}{d\tilde{s}} = \frac{\partial\Gamma(\alpha(\tilde{s}))}{\partial\alpha}\cdot\underbrace{\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}}}_{>0} < 0 \tag{121}$$

with

$$\begin{aligned}\text{sign}\left(\frac{\partial\Gamma(\alpha(\tilde{s}))}{\partial\alpha}\right) &= \text{sign}(-11\,\alpha^9+112\,\alpha^8-730\,\alpha^7+2689\,\alpha^6-7046\,\alpha^5+13970\,\alpha^4 \\ &\quad -21280\,\alpha^3+29600\,\alpha^2-32000\,\alpha+12288).\end{aligned} \tag{122}$$

Numerical inspection shows that $\frac{\partial\Gamma(\alpha(\tilde{s}))}{\partial\alpha} < 0$ for all $\alpha \in \left(\frac{\hat{s}_2}{\hat{s}_1}, 1\right]$.

□

Total profit $(p_1(x_1,x_2,s_1,s_2)-c(s_1))\cdot x_1 + (p_2(x_1,x_2,s_1,s_2)-c(s_2))\cdot x_2$ is maximized by

$$x_1^c(s_1,s_2) = \frac{a-\gamma s_1-\gamma s_2}{2} \quad \text{and} \quad x_2^c(s_1,s_2) = \frac{\gamma s_1}{2}. \tag{123}$$

Nash bargaining over aggregate collusion profits then yields (89) and (90).

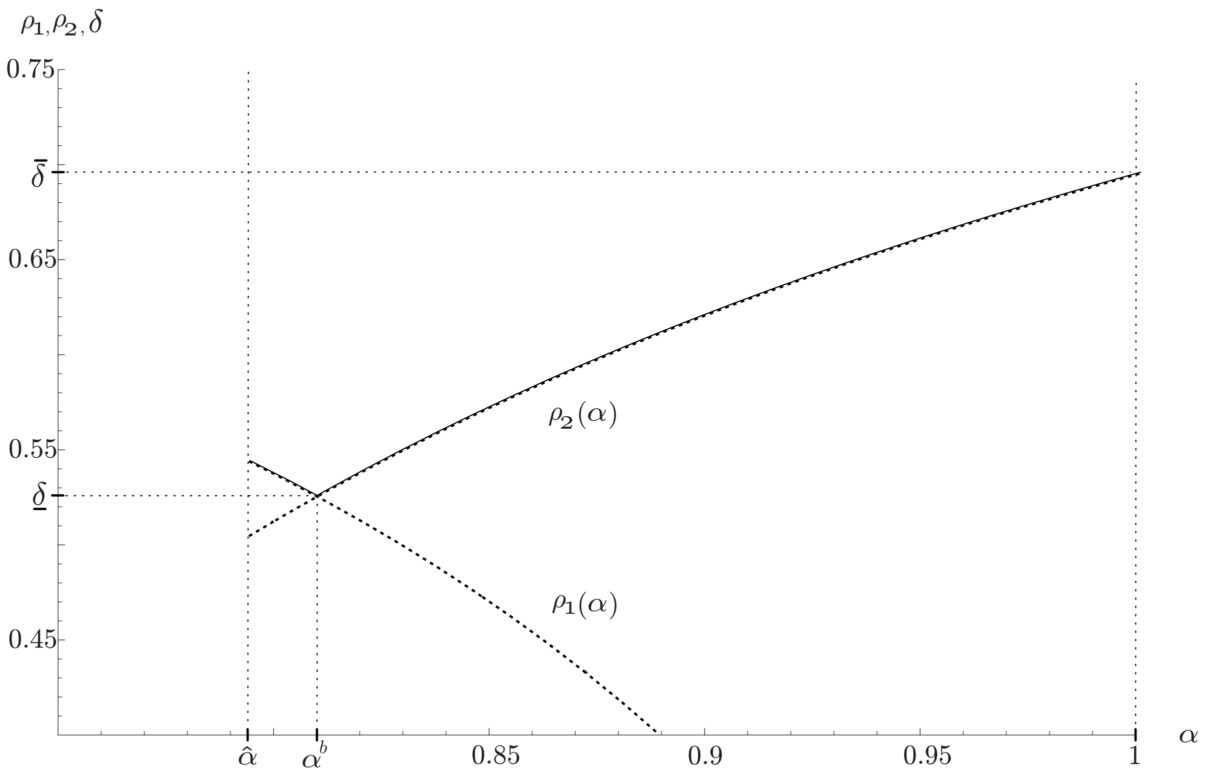

Figure 4: Critical discount factor for given equilibrium quality ratio

An optimal deviation from (x_1^c, x_2^c) for given qualities (s_1, s_2) respectively amounts to

$$x_1^d(s_1, s_2) = \frac{2\,a - 2\,\gamma\,s_1 - \gamma\,s_2}{4} \quad \text{and} \quad x_2^d(s_1, s_2) = \frac{a + \gamma\,s_1 - \gamma\,s_2}{4}, \tag{124}$$

and implies the profits in (91) and (92). Punishment profits $\pi_i^p(s_1, s_2)$ are given by equations (81) and (82).

Proposition 9

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{ds} \begin{cases} < 0; & \tilde{s} < \tilde{s}^b \approx \frac{0.29869\,a}{\gamma} \\ > 0; & \tilde{s} > \tilde{s}^b. \end{cases} \tag{125}$$

Proof: Inserting the respective expressions for π_i^p, π_i^c and π_i^d into $r_i \equiv \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p}$ and using (99), $r_1(s_1, s_2)$ and $r_2(s_1, s_2)$ can be written as functions of the regulated equilibrium quality ratio α:

$$\rho_1(\alpha) \;\; \equiv \;\; \frac{\alpha^5 + 2\,\alpha^4 - 130\,\alpha^3 + 448\,\alpha^2 - 704\,\alpha + 384}{\alpha^5 + 16\,\alpha^4 - 240\,\alpha^3 + 704\,\alpha^2 - 960\,\alpha + 512}, \tag{126}$$

$$\rho_2(\alpha) \;\; \equiv \;\; \frac{19\,\alpha^4 - 10\,\alpha^3 - 64\,\alpha^2 + 256\,\alpha - 128}{3\,\alpha^2\,(11\,\alpha^2 - 40\,\alpha + 64)}. \tag{127}$$

As illustrated in Figure 4, the functions intersect at $\alpha = \alpha^b \approx 0.80896$, which corresponds to MQS $\tilde{s}^b \approx \frac{0.29869\,a}{\gamma}$. If $\alpha \leq \alpha^b$ ($\alpha > \alpha^b$) firm 1's (firm 2's) temptation to deviate is critical. It is easy to see that $\frac{\partial \rho_1(\alpha)}{\partial \alpha} < 0$ and $\frac{\partial \rho_2(\alpha)}{\partial \alpha} > 0$. Using that $\alpha(\tilde{s})$ is strictly increasing in $\tilde{s}$ (see Lemmata 6 and 7), one obtains

$$\frac{d\rho(\alpha(\tilde{s}))}{d\tilde{s}} = \begin{cases} \frac{d\rho_1(\alpha))}{d\tilde{s}} = \underbrace{\frac{\partial\rho_1(\alpha(\tilde{s}))}{\partial\alpha}}_{<0} \cdot \underbrace{\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}}}_{>0} < 0; \quad \tilde{s} < \tilde{s}^b \approx \frac{0.29869\,a}{\gamma} \\ \frac{d\rho_2(\alpha)}{d\tilde{s}} = \underbrace{\frac{\partial\rho_2(\alpha(\tilde{s}))}{\partial\alpha}}_{>0} \cdot \underbrace{\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}}}_{>0} > 0; \quad \tilde{s} > \tilde{s}^b. \end{cases} \tag{128}$$

□

Proposition 10 *A welfare-enhancing MQS exists if* $\delta \in (\underline{\delta}; \overline{\delta})$ *for* $\underline{\delta} \approx 0.54337$ *and* $\overline{\delta} \approx 0.69524$*.*

Proof:

For given a $\delta > r(\hat{s}_1, \hat{s}_2)$ which makes collusion sustainable in an unregulated equilibrium, an MQS $\tilde{s}$ induces competitive behavior whenever it implies an equilibrium quality ratio α such that $\rho(\alpha) > \delta$. Such an MQS therefore exists for any δ satisfying

$$\underline{\delta} \equiv \min_{\alpha \in [\hat{s}_2/\hat{s}_1, 1]} \rho(\alpha) < \delta < \max_{\alpha \in [\hat{s}_2/\hat{s}_1, 1]} \rho(\alpha) \equiv \overline{\delta}, \tag{129}$$

where one obtains $\underline{\delta} \approx 0.54337$ and $\overline{\delta} \approx 0.69524$.

Total surplus rises relative to collusion with unregulated qualities for all regulated equilibrium levels $\alpha(\tilde{s})$: In analogy to the fixed cost case, we obtain

$$W^{com}(\alpha) - W^{col}(\hat{\alpha}) = \frac{a^3 \left(\alpha^2 - 2\,\alpha + 8\right) \left(11\,\alpha^5 - 38\,\alpha^4 + 91\,\alpha^3 - 64\,\alpha^2 - 64\,\alpha + 192\right)}{2\left(\alpha^3 + 4\,\alpha^2 - 10\,\alpha + 24\right)^3 \gamma} - b \tag{130}$$

with $b \approx 0.05818$. Numerical inspection reveals that this is always positive.

□

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