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Competition, imitation, and R&D productivity in a growth model with sector-specific patent protection*

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Abstract

Recent empirical studies suggest a need for a flexible patent regime responding to industry characteristics. In practice, sector-specific modifications of patent strength already exist but lack theoretical foundation. This paper intends to make up for this neglect by scrutinizing in what direction industry characteristics influence optimal patent strength. It is found that patents ought to be weaker, the more intense competition, the higher R&D productivity, and the more intricate reverse engineering are. Unlike similar step-by-step innovation models of economic growth, the model assumes Cournot competition and introduces an empirically substantiated measure of sector differences in the ability to catch up with the technological leader. It is found that for most empirically plausible cases the familiar inverted-U between patent length and growth carries over to the Cournot set-up.

Keywords: *Competition · imitation · innovation · Schumpeterian growth · sector-specific patent protection · Supplementary Protection Certificates*

JEL-Classifications: *O31 · O34 · O41 · L16 · K20*

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1 Introduction

Do stronger or weaker patent rights foster economic growth? Over the last decade numerous studies have dealt with that very question, but results were rather ambiguous or inconclusive.[1] Recent empirical findings suggest a possible explanation for this inconclusiveness by finding vast industry differences in how patent protection influences innovation and growth.[2] Moser (2005), e.g., finds that countries without patent protection historically specialized on industries for which patents are less important, while innovative activity in countries with patent laws was more diversified. Hence, stronger or weaker patent protection does not necessarily imply higher or lower R&D investments in general. Instead, as changes of a cross-industry uniform patent protection alter the direction of technological change, they induce more innovation in one type of industry at the expense of another. An optimization of aggregate innovation and growth, therefore, implies the necessity to differentiate patent protection according to each industry's characteristics.

In practice, sector-specific modifications of that kind have already been implemented but lack theoretical foundation. In 1992 the European Union introduced *supplementary protection certificates*, which provide an up to five-year additional protection to pharmaceuticals and plant protection products after the corresponding patents expire (Regulation (EEC) No. 1768/92). Their purpose is to compensate for the loss of effective patent term due to regulatory delays in the launch of new products. Recently, the EU issued a regulation according to which pediatric drugs receive a six-month extension to the maximum supplementary protection term (Regulation (EC) No. 1901/2006). In the US, a similar *pediatric exclusivity* came into force with the FDA Modernization Act of 1997 (section 505 (A)) in order to incentivize pediatric studies prior to approval of a new drug. In economic patent literature, however, surprisingly little effort has been made to theoretically develop a flexible patent regime responding to industry needs.

The goal of this paper is to make up for this neglect. Based on empirically and theoretically substantiated measures of industry-specific characteristics, it will be scrutinized in what direction these measures influence optimal patent strength. It is found that patent protection of a sector ought to be weaker, the more intense its product market competition, the higher its R&D productivity, and the more intricate reverse engineering are. Moreover, the model shows that, given a basic specification similar to previous Schumpeterian growth models with Bertrand competition[3], the familiar inverted-U relation between patent length and growth carries over to a Cournot set-up. The basic specification is extended by accounting for an empirically based measure of sector differences in the ability to technologically catch up with the leader. It is found that, except in sectors with fairly easy imitation, most empirically plausible cases still exhibit the inverted-U relation.

The model builds upon similar step-by-step innovation models by Aghion et al. (2001), Acemoglu/Akcigit (2008) and Acemoglu (2009), chapter 14.4., but instead of Bertrand competition, it

[1] See, e.g., Branstetter/Sakakibara (2001), Falvey et al. (2006), Qian (2007), Lerner (2009), Mokyr (2009).

[2] See Arora et al. (2003), Moser (2005), Bessen/Hunt (2007), and Gans et al. (2008). Earlier studies corroborate these industry differences: see, e.g., Taylor/Silberston (1973), Mansfield (1986) and Arundel/Kabla (1998).

[3] See, e.g., Aghion et al. (2001) and Mukoyama (2003).

assumes competition in quantities à la Cournot. In order to ensure comparability, the approach follows Aghion et al. (2001) regarding how to model product market competition, but, as Acemoglu/Akcigit (2008), it introduces flexible patent strength.

The idea behind a flexible patent regime in general is that the extent of protection is conditional on parameters influencing the patent office's necessity to compensate innovators. The goal is to minimize over- and under-compensations. Under-compensations undercut incentives to innovate and lower society's innovation rate, while over-compensations lead to more invention profits than socially optimal. Ideally, patent protection would be tailored to each innovation depending on R&D costs and profits accruing without governmental intervention, so that the inventor would exactly break even. However, a differentiation of patent policy to such a high degree seems practically inapplicable, since patent offices would not only need to determine an idea's patentability itself but also the degree of that patentability. The former, which basically constitutes the current system, already comprises considerable implementation costs and, due to information asymmetry between inventor and patent office, uncertainty. An implementation of the latter appears to be illusory.

Do efforts in favor of a flexible patent regime have to end at this point? Not necessarily. But patent authorities are in need of additional information in order to be able to, at least, approximate the optimally differentiated strength of protection. Whether this approximation is related to the ideal case of individual protection for every single invention or whether it needs to be aggregated on a firm, industry or even country level depends on the availability of the additional information. With regard to the nature of this information, two reasons come to mind for why the model presented here focuses on the sector level. Firstly, as the patent office's goal is to optimize the innovator's compensation, the additional knowledge ought to contain information about innovation incentives, i.e., the trade-off between R&D costs and the capability to generate profits in order to break even. Especially the latter comprises market parameters (e.g., the degree of competition) which are inherently obtained on a sectoral level. Secondly, even though some of the parameters, which determine innovation incentives, are considered to be firm or innovation specific (e.g., a firm's capability to reverse engineer), the mentioned empirical studies imply that such information is already available or is comparatively easy to obtain at sectoral level. Therefore, it is appropriate to base theoretical considerations about flexible patent policy upon these kinds of studies, and, to that effect, differentiate the strength of protection sector-specifically.

The model is most closely related to Acemoglu/Akcigit (2008). They build on a similar step-by-step innovation model but use a different variable to determine flexible patent strength: the technological gap between leader and laggard. Due to the so-called *trickle-down effect* they find it optimal to provide stronger exclusive rights to technologically more advanced innovators, since more protection not only provides higher incentives to the leader, but, via the prospect of becoming technological leader himself, it also encourages the laggard to invent.

The model also relates to patent literature, which assumes asymmetric information on cost and value of innovations and suggests a differentiated patent strength to incite inventors to reveal information. Cornelli/Schankerman (1999) propose to use renewal fees to differentiate optimal patent lives according to R&D productivity. Since they consider productivity to be a good approximation for innovation value, they suggest that more productive innovators ought to receive patents with longer lives. Scotchmer (1999) uses a similar model but distinguishes between R&D costs and innovation value. Given that costs increase at least proportionately to the quality of innovation, Scotchmer finds a menu of patents with flexible length to be optimal. Yet in contrast to Cornelli/Schankerman (1999), she also identifies circumstances in which a uniform patent regime can be optimal. Hopenhayn et al. (2006) reach similar conclusions regarding the optimality of a patent menu but, in tradition of the patent design literature[4], suggest flexible patent breadth while holding patent length fixed. As the present model, they analyze optimal patent policy considering the cumulative nature of innovations. However, their approach, as well as the previously mentioned, advocates a differentiation of patent strength on firm or project level. The paper presented here primarily differs from hitherto existing models of flexible patent policy inasmuch as it proposes a sector-specific differentiation. To the best of my knowledge, no other paper suggests flexible patent strength based on industry differences coming to light in numerous empirical studies.

Moreover, while the mentioned models each focus on only one parameter which determines flexible patent strength[5], this approach accounts for three parameters: R&D productivity, reverse engineering capability, and the degree of product market competition. As described above, the first one has been used to differentiate patent strength in previous models of flexible patent policy. The last two proved to be relevant for innovation incentives in empirical studies. Mansfield et al. (1981), e.g., discover sector-specific differences in imitation costs and the ability to reverse engineer, while Prasad (2008) finds that the prediction of an inverted-U relation between competition and innovation by Aghion et al. (2005) varies significantly depending on industry characteristics.

The benefit of taking three parameters into account instead of one is twofold. Firstly, it enables us to compare their implications on flexible patent protection in an integrating framework. Secondly, since patent policy aims at compensating innovators and more than one parameter affects the necessity and the extent of governmental intervention to do that, the idea is to take on a broader perspective on an inventor's research incentives. O'Donoghue, Scotchmer and Thisse aptly state

> *" [...] that the effectiveness of patent law in supporting research is seriously impeded by the fact that it does not refer to costs or market structure in how patent protection is circumscribed."* [6]

The paper presented here intends to make a contribution to the rectification of this shortcoming. An innovating firm's R&D decision comprises a trade-off between R&D costs and potential benefits from advancing one technology step, both depending on an interplay of the above-mentioned parameters. To give an example, knowledge about an innovation's market power is only useful together with

4 See, e.g., Gilbert/Shapiro (1990), Klemperer (1990), and Denicolo (1996).
5 That is with the exception of Scotchmer (1999), who models productivity and quality of R&D separately.
6 O'Donoghue et al. (1998), p. 25.

information about the length of the market power's existence, because even a monopoly is useless, if it only exists for an infinitesimal short period. The market power's existence, in turn, depends on the rival's ability to reverse engineer and to technologically catch up to the leader. For this reason, the model utilizes the three parameters to answer the following questions regarding the decision to innovate: i) How resource-consuming is the innovation?; ii) how much return can be expected from it?; and iii) how long will the innovation yield profit for?

The model takes advantage of the fact that R&D productivity can be seen as a measure for R&D costs, since it quantifies how productive a unit researcher is. It uses R&D productivity to tackle question i) and to approximate the necessity to compensate innovators using patents. I find that an increment in sector-specific R&D productivity corresponds to weaker optimal patent protection for that sector, because it leads to a stronger increase in incentives to innovate than in incentives to imitate. This, in turn, results from the fact that profits from gaining technological lead always exceed profits from catching up, and a higher productivity simply scales up this difference. Hence, in sectors with a high productivity, the threat of losing the technological lead (relative to innovation profits) as well as the necessity to compensate innovators via patents is relatively low. This result seems intuitive but contradicts previous findings by Cornelli/Schankerman (1999) indicating that more productive inventors should be granted longer protection in order to tilt their R&D effort towards large inventions.

Furthermore, the degree of product market competition is used to evaluate question ii), since it influences an industry's natural ability to compensate innovators. I find that more intense competition corresponds to a weaker optimal patent protection. This seems counterintuitive but is due to the fact that more competition increases incentives to escape it by outperforming the rivals technologically. Since a sector's naturally inherent research incentives are higher once it exhibits less market concentration, the necessity to compensate innovators via patent protection diminishes.

Finally, question iii) will be dealt with by accounting for the laggard's ability to reverse engineer, since an innovator in a sector in which it is comparatively easy to imitate is under higher pressure to break even. Using inter-industry differences in the imitator's R&D productivity, the model suggests that a decline in the sector-specific relation of imitation costs to innovation costs, i.e., more efficient reverse engineering, calls for higher patent protection. Since the threat of losing the technological lead rises with less costly imitation, the necessity to protect innovators using IPR policy increases.

The paper is organized as follows. The basic framework of the underlying step-by-step innovation model is presented in Section 2. This includes a brief discussion of the intuition behind a main driving force of innovation in this kind of models: *incremental profits*. Besides, distinctive features of Cournot competition and the described sector-specificity will be outlined and compared to similar models assuming Bertrand. Section 3 deals with the intuitive analysis and numerical calibration of how variations in sector-specific parameters induce changes in the optimal (i.e., growth-maximizing) amount of patent protection. Thereby, it gives an idea of the potential gains of a flexible patent regime on industry level. Section 4 concludes the paper.

2 The basic framework

2.1 Consumer behavior

Consider a continuous-time economy which is populated by a continuum of infinitely-lived consumers (normalized to 1). The representative household has additively separable intertemporal preferences given by the lifetime utility function

$$U_0 \equiv \int_0^{\infty} e^{-\rho t} \left[ln\, c(t) - L_S(t)\right] dt \;, \tag{1}$$

where ρ is the subjective discount rate, $c(t)$ is consumption at date t and $L_S(t)$ denotes the labor supplied. According to this preference specification, labor supply is infinitely elastic, so that the wage rate $w(t)$ is exogenous and can be normalized to 1 for all t.[7] Consequently, the subjective discount rate ρ equals the interest rate $r(t)$ and expenditure in each sector j at time t can be chosen as the numeraire, so that $p_{Aj}(t)\, x_{Aj}(t) + p_{Bj}(t)\, x_{Bj}(t) = 1$.[8]

Consumer goods are provided by a continuum of industries indexed by $j \in [0, 1]$, so consumption equals aggregate output $Y(t)$ given by

$$ln\, c(t) = ln\, Y(t) = \int_0^1 ln\, X_j(t)\, dj \;, \tag{2}$$

where $X_j(t)$ is the consumer good output of industry j. Each industry exhibits a duopolistic market structure. Industry output $X_j(t)$ consists of two varieties $x_{Aj}(t)$ and $x_{Bj}(t)$ produced by firms A and B. Following Aghion et al. (2001) we have that

$$X_j(t) = \left[x_{Aj}(t)^{\alpha_j} + x_{Bj}(t)^{\alpha_j}\right]^{\frac{1}{\alpha_j}} \;, \tag{3}$$

where $\alpha_j \in (0, 1]$ indicates the substitutability of one variety using the other.[9] The index j refers to the fact that α can differ from sector to sector, which is an important feature for the sector-specific differentiation of patent policy analyzed below.

Due to the logarithm in the utility function, which implies that in equilibrium consumers spend the same amount in each industry, the demand function will take the same form in each industry. So it is sufficient to derive the demand for one industry which is considered exemplary. Given (2)

[7] This follows Aghion et al. (2001) and Mukoyama (2003). For similar models with inelastic labor supply see Aghion et al. (1997) and Acemoglu (2009), ch. 14.4. This assumption does not change the intuition behind the variations of profits w.r.t. competition and technology gap. Yet, when labor supply is inelastic, the endogeneity of wages leads to less drastic reactions of profits to variations in those parameters. E.g., more rivalry leads to a higher output, resulting in a higher demand for workers. Unlike infinitely elastic labor supply, inelastic supply causes wages to increase, which lowers profits and incentives to innovate. Thus, given the results below, inelastic supply would smooth the reaction to parameter variations quantitively, albeit qualitative effects are identical.

[8] To see that, optimize the Hamiltonian $\tilde{\mathcal{H}}(t, c, A, L_S, \nu) = ln\, c(t) - L_S(t) + \nu(t)[w(t)\, L_S(t) + r(t)A(t) - p(t)\, c(t)]$. This yields $\frac{1}{c(t)} = \nu(t)\, p(t)$, $\nu(t)r(t) = \rho\nu(t) - \dot{\nu}$ and $1 = w(t)\, \nu(t)$ as the FOCs. Solving for $\nu(t)$ gives us $\nu(t) = \frac{1}{w(t)} = \frac{1}{p(t)\, c(t)}$, which implies that the numeraire chosen corresponds to $w(t) = 1$. Since $\nu = 1$ always holds, $\dot{\nu}$ becomes zero, resulting in $r(t) = \rho$.

[9] Instead of assuming only one type of good, alternatively, $X_j(t)$ can be interpreted as intermediate goods used to produce one final consumer good $Y(t)$. In that case, (2) would be the Cobb-Douglas production function for this good. Given a normalized final good's price ($p_y(t) = 1$), both approaches yield the same results.

and (3) we maximize (1) subject to the above given budget constraint. This yields

$$p_{Aj}(t) = \frac{x_{Bj}(t)^{\alpha_j - 1}}{x_{Aj}(t)^{\alpha_j} + x_{Bj}(t)^{\alpha_j}} \quad \text{and} \quad p_{Bj}(t) = \frac{x_{Aj}(t)^{\alpha_j - 1}}{x_{Aj}(t)^{\alpha_j} + x_{Bj}(t)^{\alpha_j}} \,, \tag{4}$$

which is the (inverse) demand for consumer goods varieties.

2.2 Producer behavior

Unlike previous models of this type, which assume Bertrand competition, in this model firms compete in quantities à la Cournot. Which type of oligopolistic competition is more realistic depends on whether price or quantity is the decisive action parameter. Cournot is more appropriate in the case of more protracted manufacturing processes, which imply a bigger time-lag between decision on production volumes and delivery of the goods. Besides, Kreps/Scheinkman (1983) show in a two stage model that Bertrand competition yields Cournot results, if preceded by simultaneous decisions about production capacities in the first stage.

Each firm i in industry j produces its variety of the consumption good at time t using labor $L_{ij}(t)$ and the firm-specific production knowledge $\mathcal{A}_{ij}(t)$, so that

$$x_{ij}(t) = \mathcal{A}_{ij}(t)\, L_{ij}(t) \,, \tag{5}$$

where $i \in \{A, B\}$. The two suppliers differ regarding their endogenously determined technology level, which pins down the productivity of a unit labor employed in production. Both firms can invest in R&D, which stochastically leads to innovations. Each innovation manifests itself by raising the investing firm's technology level by one discrete standardized step $\gamma > 1$. Technology of a firm i, whose R&D efforts succeded k_{ij} times, therefore, can be written as $\mathcal{A}_{ij}(t) = \gamma^{k_{ij}(t)}$. This can be interpreted as the amount of consumer goods that one unit of labor can produce, or, put differently, $\gamma^{-k_{ij}(t)}$ units of labor are required for production of one unit output. Hence, marginal cost of producing consumer good j for firm i at time t is $m_{ij}(t) = \gamma^{-k_{ij}(t)}\, w(t)$. The technological leader in each industry will be denoted by i and the laggard by $-i$, so that $m_{ij}(t) \leq m_{-ij}(t)$.

Since in a duopolistic market a firm's profit maximizing quantities not only depend on its own marginal costs but also on those of the rival, it is useful to simplify the ratio of marginal costs to $\frac{m_{ij}(t)}{m_{-ij}(t)} = \frac{\gamma^{-k_{ij}(t)}\, w(t)}{\gamma^{-k_{-ij}(t)}\, w(t)} = \gamma^{-n_j(t)}$, where $i, -i \in \{A, B\}$ and $i \neq -i$. $n_j(t) = k_{ij}(t) - k_{-ij}(t)$ is the technology gap between leader and laggard, measured by the number of steps γ the leader is ahead. Hence, if there is a technology gap in industry j, the ratio of marginal costs is always smaller than 1 and the leader has a cost advantage, whose size positively correlates with the gap. In case of $n_j(t) = 0$, the industry exhibits neck-to-neck competition, and the ratio equals one.

In spite of uncertainty in the R&D-process, each firm's objective is to maximize expected profits, because every consumer holds a balanced portfolio of shares of all firms. Using the demand functions above, the profit function to be maximized is $\pi_{ij}(t) = \frac{x_{ij}(t)^{\alpha_j}}{x_{ij}(t)^{\alpha_j} + x_{-ij}(t)^{\alpha_j}} - m_{ij}(t)\, x_{ij}(t)$, where $i, -i \in$

$\{A, B\}$ and $i \neq -i$. The first-order conditions are

$$m_{ij}(t) = \frac{\alpha_j x_{ij}(t)^{\alpha_j - 1} x_{-ij}(t)^{\alpha_j}}{(x_{ij}(t)^{\alpha_j} + x_{-ij}(t)^{\alpha_j})^2} \quad \text{and} \quad m_{-ij}(t) = \frac{\alpha_j x_{-ij}(t)^{\alpha_j - 1} x_{ij}(t)^{\alpha_j}}{(x_{ij}(t)^{\alpha_j} + x_{-ij}(t)^{\alpha_j})^2} \,. \tag{6}$$

Using this expression, we can write the duopolists' reaction function as $\frac{m_{ij}(t)}{m_{-ij}(t)} = \frac{x_{-ij}(t)}{x_{ij}(t)}$. Combining this with (6) yields the profit maximizing outputs

$$x_{ij}(t) = \frac{\alpha_j}{m_{ij}(t)} \frac{\left(\frac{m_{ij}(t)}{m_{-ij}(t)}\right)^{\alpha_j}}{\left(1 + \left(\frac{m_{ij}(t)}{m_{-ij}(t)}\right)^{\alpha_j}\right)^2} \quad \text{and} \quad x_{-ij}(t) = \frac{\alpha_j}{m_{-ij}(t)} \frac{\left(\frac{m_{-ij}(t)}{m_{ij}(t)}\right)^{\alpha_j}}{\left(1 + \left(\frac{m_{-ij}(t)}{m_{ij}(t)}\right)^{\alpha_j}\right)^2} \,. \tag{7}$$

Finally, using these expressions and (4), we can express the leader's and laggard's profits in industry j at time t conditional on the technology gap $n_j(t)$ and the substitutability measure α_j:

$$\pi_{ij}(t) = \frac{1 + (1 - \alpha_j)\left(\gamma^{-n_j(t)}\right)^{\alpha_j}}{\left[1 + \left(\gamma^{-n_j(t)}\right)^{\alpha_j}\right]^2} \quad \text{and} \quad \pi_{-ij}(t) = \frac{1 + (1 - \alpha_j)\left(\gamma^{n_j(t)}\right)^{\alpha_j}}{\left[1 + \left(\gamma^{n_j(t)}\right)^{\alpha_j}\right]^2} \,. \tag{8}$$

For the sake of completeness, the market prices of the consumer goods' varieties are

$$p_{ij}(t) = \frac{m_{ij}(t)}{\alpha_j} \left[1 + \left(\frac{m_{-ij}(t)}{m_{ij}(t)}\right)^{\alpha_j}\right] \quad \text{and} \quad p_{-ij}(t) = \frac{m_{-ij}(t)}{\alpha_j} \left[1 + \left(\frac{m_{ij}(t)}{m_{-ij}(t)}\right)^{\alpha_j}\right] \,. \tag{9}$$

In spite of well-known limitations of this understanding, the industry-specific parameter α_j can be used as an indicator of the degree of product market competition.[10] This interpretation follows Aghion et al. (2001), who state that, *"although α is ostensibly a taste parameter, we think of it as proxying the absence of institutional, legal or regulatory impediments to entering directly into a rival firm's market [...]. Under this interpretation α reflects in particular the influence of anti-trust policy."*[11] Since α_j mathematically captures the extent to which one variety is able to generate a utility similar to the other, it can be interpreted as a measure to what degree both varieties belong to the same market and, therewith, engage in direct competition. The mentioned impediments to enter a rival's market (e.g., due to a more or less efficient anti-trust policy) influence the resemblence of both varieties with regard to the utility they generate. Thus an efficient anti-trust policy, which induces a high degree of competition, corresponds to the case where α_j is close to 1. By contrast, the absence of an efficient anti-trust policy corresponds to the case in which α_j is close to 0, since, then, the varieties do not generate a similar utility and cannot substitute one another. Aghion et al. (2001) substantiate this interpretation by additionally pointing out that α_j corresponds to standard measures of competition, particularly the price cost margin.[12] To ensure comparability and due to quite intricate alternatives, I will henceforth refer to α_j as the degree of competition.

The profit function (8) plays a crucial role in the model. As in every Schumpeterian growth model it determines the benefits of technologically outperforming the rivals. In step-by-step models

10 Regarding the limitations of this approach see Königer/Licandro (2004) and Boone et al. (2007).
11 See Aghion et al. (2001), p. 471.
12 See Aghion et al. (2001), p. 472. A related measure (share of profits in value added) is used by Nickell (1996).

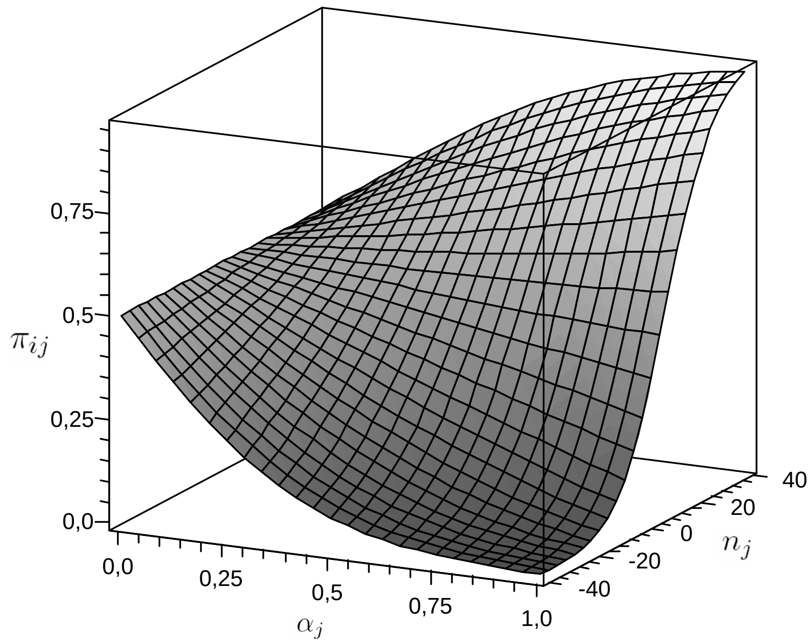


Figure 1: Profits as a function of competition α_j and technological gap n_j ($\gamma = 1.1$).

of endogenous growth the profit function is particularly important, because the potential innovator weighs ex-post research profits against its ex-ante revenue. So in contrast to leap-frogging models, where R&D is always done by an outsider firm, what determines research incentives in this model is not the overall research profit but the *difference* between postinnovation and preinnovation profits and its behavior with respect to the technological gap. These *incremental profits* are crucial for R&D incentives of both, technological leader and laggard. It is, therefore, worth taking a closer look at the characteristics of the profit function $\pi_{ij} = \pi_{ij}(\alpha_j, n_j(t), t)$ illustrated in figure 1.

The shape of the curve in figure 1 is in line with the well-known fact that oligopolistic competition à la Cournot exhibits less extreme behavior than Bertrand price competition. While at $\alpha_j = 1$ and in neck-to-neck state ($n_j(t) = 0$) price competition would yield a zero-profit situation (see Aghion et al. (2001)), Cournot ensures profits for both firms ($\pi_{ij}(1, 0, t) = 0.25$). The reason is the more long-term character of competition in quantities mentioned above, due to which producers are not able to react as flexible to the competitor's supply as in the course of price competition. In the other extreme case of $\alpha_j \to 0$, in which both varieties are complementary goods, the industry exhibits no product market competition. Consequently, the extent to which both producers differ technologically becomes irrelevant for the profits, so that both firms' profits will be $\lim_{\alpha_j \to 0} \pi_{ij}(\alpha_j, n_j(t), t) = 0.5$ regardless of their R&D effort. Besides, in a Cournot oligopoly even the technologically less advanced firm is able to realize profits to a certain degree. Only a very large technological gap causes the laggard's profits to asymptotically become zero, while the leader's profits increase asymptotically to 1, so that $\lim_{n \to -\infty} \pi_{-ij}(\alpha_j, n_j(t), t) = 0$ and $\lim_{n \to \infty} \pi_{ij}(\alpha_j, n_j(t), t) = 1$.

Figure 1 shows that, as α_j increases, the leader-laggard difference in profits becomes increasingly sharp, and the relationship between technological lead and profits assumes the shape of a logistic function. This is an important feature regarding the discussion of optimal patent protection below, because incremental (not total) profits are the main driving force of innovation incentives in this model. A logistic shaped function exhibits an inflection point which separates the convex part of the function (small $n_j(t)$) from the concave part (high $n_j(t)$). This implies that a leader with a large technological advantage has relatively little incentives to innovate, because the difference between preinnovative and postinnovative profits is small for high $n_j(t)$. By contrast, given a small productivity difference between technological leader and laggard, the logistic function exhibits a steep slope, and the potential R&D benefits for an innovator are high. Figure 2 illustrates the logistic behavior of the profit function w.r.t. $n_j(t)$ for the most distinct case of $\alpha_j = 1$.

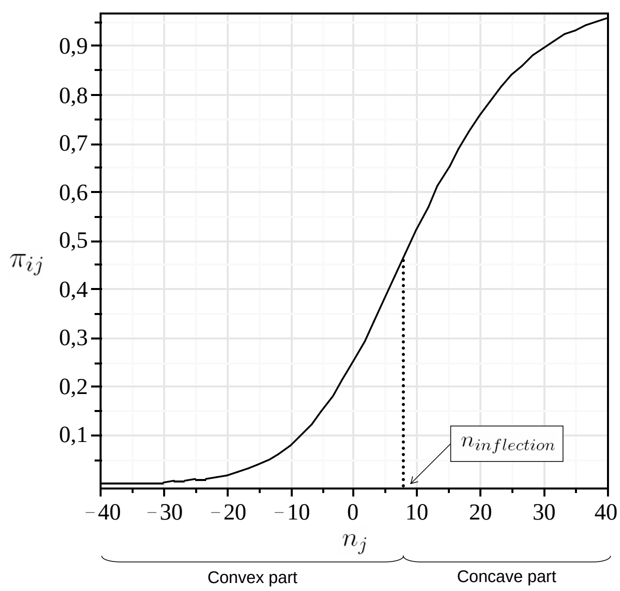


Figure 2: Profits as a function of the technological gap n_j ($\alpha_j = 1$, $\gamma = 1.1$).

Finally, it can be shown formally, that total industry profits are lower in neck-to-neck competition than if one firm has a cost advantage, i.e., $\pi_{ij}(\alpha_j, n_j(t), t) + \pi_{-ij}(\alpha_j, -n_j(t), t) > 2 \cdot \pi_{\pm ij}(\alpha_j, 0, t)$. To see this, remember that for negative $n_j(t)$ the profit function is convex. If the inflection point of the logistic is $n_{inflection} > 0$, the function still exhibits a convex behavior at neck-to-neck competition $n_j(t) = 0$ (see figure 2). This would connote that an innovator trying to escape the neck-to-neck state would raise his profits by a higher margin than his technological lead would lower the rival's profits, i.e., $\frac{\partial \pi_{ij}(\cdot)}{\partial n_j(t)}|_{n_j(t)=n} + \frac{\partial \pi_{-ij}(\cdot)}{\partial n_j(t)}|_{n_j(t)=-n} > 0$ or expressed differently $\pi_{ij}(\alpha_j, n_j(t), t) - \pi_{ij}(\alpha_j, 0, t) > \pi_{-ij}(\alpha_j, 0, t) - \pi_{-ij}(\alpha_j, -n_j(t), t)$. So in order to show that (8) exhibits the lowest total industry profits if firms are neck-to-neck, we need to prove that $n_{inflection} > 0$.

Proposition 1. *Given two firms competing à la Cournot in an industry j, whose profit function follows* (8)*, incremental profits of a firm i leaving neck-to-neck state are strictly greater than incremental profits of a catching up firm −i, i.e.,* $\pi_{ij}(\alpha_j, n_j(t), t) - \pi_{ij}(\alpha_j, 0, t) > \pi_{-ij}(\alpha_j, 0, t) - \pi_{-ij}(\alpha_j, -n_j(t), t)$ *for all* $\alpha_j \in (0, 1]$*, all* $n_j(t) \in \mathbb{N}$ *and* $i, -i \in \{A, B\}$ *with* $i \neq -i$.

Proof. See Appendix A.1. $\square$

It follows, that even though the innovator experiences the highest increase in incremental profits at $n_{inflection}$, total industry profits are lowest and, therefore, exhibit the highest potential increase via innovation, if firms compete neck to neck. This property, which Aghion et al. (2001) show for a profit function resulting from Bertrand competition, carries over to the Cournot set-up.[13] It enables us to reason in the following that neck-to-neck R&D investments are strictly greater than R&D investments by the laggard. This, in turn, is an important prerequisite for the analysis of the optimal patent strength's behavior to changes in sector-specific variables in section 3.

2.3 R&D and patent protection

The model does not allow for entry in R&D, so each industry's R&D sector comprises two duopolists. Both conduct research in order to advance technologically and lower their marginal costs for production. Each industry can either be in neck-to-neck state, where the technological gap is zero, or in leader-laggard state. Following Aghion et al. (1997), Mukoyama (2003), and Aghion et al. (2005), I restrict the maximum permissable lead to one step. This assumption implies that, if the leader innovates, the laggard automatically catches up by one step, as technology older than one step becomes common knowledge. Consequently, a leader does not further invest in research.[14]

For now, both firms exhibit identical R&D costs regardless of being leader or laggard.[15] Innovations occur stochastically and follow a memoryless Poisson process, whose arrival rate is determined by investments in R&D (in units of labor). More specifically, we focus on the case in which the innovation possibilities frontier for a firm i in industry j is $\varphi_{ij}(t) = \sqrt{2\,\beta_j\,R_{ij}(t)}$, where $\varphi_{ij}(t)$ is the flow rate of innovation, $\beta_j > 0$ is the R&D productivity in industry j, and $R_{ij}(t)$ is the number of workers employed in research by firm i in industry j.[16] Consequently, R&D costs in units of labor is the inverse of the R&D production function times wage, i.e., $w(t)\,\mathcal{G}(\varphi_{ij}(t))$, where

$$\mathcal{G}(\varphi_{ij}(t)) = \frac{1}{2\,\beta_j}\,(\varphi_{ij}(t))^2\,. \tag{10}$$

(10) enables us to simplify the first derivative of $\mathcal{G}(\varphi_{ij}(t))$ w.r.t. $\varphi_{ij}(t)$ to $\frac{\partial\,\mathcal{G}(\varphi_{ij}(t))}{\partial\,\varphi_{ij}(t)} = \frac{1}{\beta_j}\,\varphi_{ij}(t)$. This will turn out to be useful regarding the derivation of R&D intensities, because it makes sure that the optimal neck-to-neck R&D intensity is independent of the optimal laggard's intensity.

13 See Proposition 1c) in Aghion et al. (2001), p. 472.

14 This one-step lead assumption has no distortionary impact on the results, if the parameter specification of profits and R&D costs is such that the leader is not able to break even when introducing self-replacing innovations (see Mukoyama (2003), p. 369). Aghion et al. (2001) give an example for a specification that fulfills this no self-replacement condition: large innovations. Given $\alpha > 0$, for large γ even a one-step lead is sufficient to raise return on innovation to almost maximum level. This leaves relatively little potential to further augment profits by making additional effort to expand the technological frontier. However, even if the no self-replacement requirement is not met, the one-step lead assumption turns out to be a useful, albeit not particularly precise approximation of the intuition behind the following results, as it enables us to derive optimal R&D intensities analytically. Aghion et al. (2001) show that the main results regarding the behavior of g^* to changes of patent length carry over to the self-replacement case of small innovations. So considering the fact, that the qualitative implications behind (8) remain unchanged, the lack of precision seems a little price to pay for simplicity.

15 This assumption will be lifted in section 3.4, where we take a closer look at variations in the ease of imitation.

16 See also Budd et al. (1993), Aghion et al. (1997), Aghion et al. (2005), who use a similar R&D specification.

The basic model includes two alternative ways for the laggard to catch up with the leader. Firstly, we assume a patent regime that, if enforced perfectly, leaves *inventing around* as the only possibility to draw level with the leader. I.e., if the patent never expires, the imitator has to achieve the same technological level by means of a variation of the leader's idea. The variation has to be large enough not to violate the existing patent but, still, technologically serve the purpose of reducing production costs of consumer goods at the same rate as the patented idea. This specification, explicitly used in Acemoglu/Akcigit (2008) and Acemoglu (2009), implies a standardized scope of patent protection, i.e., patent breadth, which defines the technical bandwidth of protection, is neither zero nor infinitely high. If it were the latter, it would be impossible to invent around and it would make no sense for the laggard to invest in research. If it were the former, patent length would become virtually ineffective, since an infinitesimal technical variation of the imitation compared to the patented product would be enough not to violate the patent. Secondly, we allow for the possibility that patents expire at a Poisson rate h_j, where $h_j > 0$ and the index j indicates the possibility of an industry-specific differentiation of patent length, discussed below. If expiration occurs, the imitator can copy the idea without having to invest additional time and effort in inventing around.

2.4 Steady State Equilibrium

2.4.1 Markov Perfect Equilibrium

Following Acemoglu (2009), the next step is to derive optimal R&D intensities, where we denote $\varphi_{0j}(t)$ as the R&D decision of a neck-to-neck firm, $\varphi_{1j}(t)$ of the leader and $\varphi_{-1j}(t)$ of the laggard. We further denote $\mu_{0j}(t)$ as the probability that an industry is in neck-to-neck state and $\mu_{1j}(t)$ as the probability that an industry's technology gap equals one (leader-laggard state). In equilibrium, the optimal R&D intensities in the dynamic problem setting under consideration require being optimal responses to each other, given the history of the state variable to be subgame perfect. Yet, in accordance with Aghion et al. (2001), Acemoglu/Akcigit (2008) and Acemoglu (2009), we can further simplify by focussing on symmetric stationary Markov Perfect Equilibria, as the including Markov strategies only depend on the payoff-relevant state of the game (each firm's current technological state) and not on its historic values.

The list of decisions a firm $\pm i$ has to make can be expressed by $\mathcal{D}_{\pm n}(t) \equiv \lceil \varphi_{\pm nj}(t), x_{\pm ij}(t), p_{\pm ij}(t) \rfloor$, where $i \in \{A, B\}$ and $n_j(t) \in \{0, 1\}$. Futhermore, D indicates the whole sequence of decisions of firms at every state, so $D(t) \equiv \{\mathcal{D}_1(t), \mathcal{D}_0(t), \mathcal{D}_{-1}(t)\}$. An allocation is given by time paths of decisions of firms, $[D(t)]_{t=0}^{\infty}$, by the time path of the interest rate $[r(t)]_{t=0}^{\infty}$, and by the time paths of each industry's probability distribution regarding technology gaps $[\mu_{nj}(t)]_{t=0}^{\infty}$.[17] The Markov Perfect Equlibrium is represented by time paths $[D^*(t), r^*(t), Y^*(t)]_{t=0}^{\infty}$ such that

[17] This set up follows the similar step-by-step growth model with Bertrand competition by Acemoglu (2009), section 14.4.2. As in the Bertrand model, here a further notational simplification is used: Although the sequences $[p^*_{\pm ij}(t)]_{t=0}^{\infty}$ and $[x^*_{\pm ij}(t)]_{t=0}^{\infty}$ are stochastic (since they include $k_{\pm ij}(t)$ via $m_{\pm ij}(t)$), we ignore their stochastic nature, because it has no effect on the analysis and the rest of the objects are not stochastic.

a) $[p^*_{\pm ij}(t)]_{t=0}^{\infty}$ and $[x^*_{\pm ij}(t)]_{t=0}^{\infty}$ implied by $[D^*(t)]_{t=0}^{\infty}$ satisfy (7) and (9),

b) R&D policies $[\varphi^*(t)]_{t=0}^{\infty}$ maximize the expected profits of the respective firms taking aggregate output $[Y^*(t)]_{t=0}^{\infty}$, interest rate $[r^*(t)]_{t=0}^{\infty}$ and the optimal R&D investment choices of the rival firm $[\varphi^*(t)]_{t=0}^{\infty}$ as given,

c) aggregate output $[Y^*(t)]_{t=0}^{\infty}$ is given by (2), and

d) the stock market clears at all times given $[r^*(t)]_{t=0}^{\infty}$.

The stock market channels consumer savings to R&D projects by valuating each innovation according to the expected discounted profits it generates. The associated risk can be neglected since households hold a balanced portfolio of shares of all firms. Because one of the two firms will surely be the next innovator, this diversification leads to the fact that there is no risk premium. Shareholders receive two kinds of returns on assets: firstly, in form of dividends, which in the current context corresponds to profits given by (8), and secondly, in form of an appreciation in firm value, that the innovator experiences due to a change in the state variable (technological level). The stock market will be in equilibrium, if the asset pays out the required rate of return $r(t) = \rho$. The corresponding no-arbitrage condition for the stock market implies an annuity given by the following Hamilton-Jacobi-Bellman Equation (in stationary form, as we analyze the steady-state):

$$\begin{aligned} r(t)\,V_n(t) - \dot{V}_n(t) \;&= \max_{\varphi_{nj}(t)} \quad \langle [\pi_{nj}(t) - \mathcal{G}(\varphi_{nj}(t))] + \varphi_{nj}(t)\,[V_{n+1}(t) - V_n(t)] \\ &\qquad -(\varphi_{-nj}(t) + h_j)\,[V_n(t) - V_0(t)] \rangle \qquad , \end{aligned} \tag{11}$$

where $n \in \{0, 1\}$. $V_n(t)$ is the value of a firm with lead n, and $\dot{V}_n(t)$ is the change of this value with respect to time.[18] (11) is also refered to as the *no-arbitrage asset value equation.* The return on investment on the right hand side comprises the profits minus R&D cost, the increase in the firm's value due to innovation weighted by the probability that it succeeds, and the decrease in the firm's value due to the rival's innovation weighted by the probability that the rival succeeds. The expression plays a crucial role in the following derivation of steady-state R&D decisions.

2.4.2 Steady-state R&D decisions

Since in steady-state the aggregate growth rate g^* of the economy, as well as profits, industry structure, and R&D intensities are constant over time, we can drop their time subscript in the following calculations.[19] In order to derive the steady-state R&D intensities, we use the above

[18] Note that in a model specification, in which consumers' saving behavior follows a standard Euler Equation ($g = \frac{\dot{c(t)}}{c(t)} = r(t) - \rho$), $\dot{V}_n$ grows at the same rate as consumption. Since here $r(t) = \rho$, there is no saving additional to the rate of time depreciation, so $\dot{V}_n = 0$. Both yield the same annuity $\rho\, V_n$ paid to the investors.

[19] The reason for this stationarity is that those variables, as opposed to output, prices and marginal costs, only depend on the technology gap n_j, but not on the constantly expanding technology level of the respective firm, $k_{ij}(t)$. Consequently, profits π_{1j}, π_{0j}, π_{-1j}, industry structure μ_{1j}, μ_{0j}, value functions V_{1j}, V_{0j}, V_{-1j}, and R&D intensities φ_{0j}, φ_{-1j} are stationary in steady state.

given no-arbitrage asset value equation, (11), and the fact that $r = \rho$ to write the leader's value function

$$\rho\, V_{1j} = \pi_{1j} - (\widetilde{\varphi}_{-1j} + h_j)\, [V_{1j} - V_{0j}] \ , \tag{12}$$

the neck-to-neck firms' value function

$$\rho\, V_{0j} = \left[\pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j}\right] + \varphi_{0j}\, [V_{1j} - V_{0j}] - \widetilde{\varphi}_{0j}\, [V_{0j} - V_{-1j}] \ , \tag{13}$$

and the following firm's value function

$$\rho\, V_{-1j} = \left[\pi_{-1j} - \frac{(\varphi_{-1j})^2}{2\,\beta_j}\right] + (\varphi_{-1j} + h_j)\, [V_{0j} - V_{-1j}] \ , \tag{14}$$

where $\sim$ denotes all variables chosen by the rival firm. Based on these functions, we can derive the profit maximizing amount of labor employed in research by taking the first derivative with respect to the steady-state R&D intensities for each firm respectively. Note that, due to the one-step lead assumption, the optimal research level of the leading firm is zero. The first order conditions are

$$\frac{\varphi_{0j}}{\beta_j} = (V_{1j} - V_{0j}) \tag{15}$$

for a neck-to-neck firm, and

$$\frac{\varphi_{-1j}}{\beta_j} = (V_{0j} - V_{-1j}) \tag{16}$$

for the laggard. Expressions (12) to (16) suffice for being able to derive the profit maximizing R&D intensities of both firms. Starting with the neck-to-neck firms' first order condition, we can use (12) and (13) to rewrite (15) to $\rho\, \frac{\varphi_{0j}}{\beta_j} = \pi_{1j} - (\widetilde{\varphi}_{-1j} + h_j)\, \frac{\varphi_{0j}}{\beta_j} - \left[\pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j} + \frac{(\varphi_{0j})^2}{\beta_j} - \widetilde{\varphi}_{0j}\, \frac{\varphi_{-1j}}{\beta_j}\right]$. Rearranging this expression yields the profit maximizing R&D intensity of a neck-to-neck firm:

$$\varphi_{0j} = -(\rho + h_j) + \sqrt{(\rho + h_j)^2 + 2\,\beta_j\, (\pi_{1j} - \pi_{0j})} \ . \tag{17}$$

Similarly, we can derive the profit maximizing R&D intensity of the laggard, which yields

$$\varphi_{-1j} = -(\rho + h_j + \varphi_{0j}) + \sqrt{(\rho + h_j + \varphi_{0j})^2 + 2\,\beta_j\, (\pi_{0j} - \pi_{-1j}) + (\varphi_{0j})^2} \ . \tag{18}$$

Equations (17) and (18) are (along with the growth rate) the main equations in this model. On the one hand, they include R&D incentives determining parameters, which in section 3 determine flexible patent scope. R&D productivity β_j enters the expressions directly, while the degree of competition α_j enters both equations via incremental profits. On the other hand, the expressions capture the effects imposed by patent protection on the firm's R&D-behavior. The neck-to-neck R&D intensity decreases with a lower patent protection, i.e., an increase in h_j results in a strict decrease of φ_{0j}. This fact reflects the *disincentive effect* of a mitigation of patent strength. It manifests itself in the rather intuitive result that the sooner the technological lead will be taken away from a firm, the less this firm is apt to invest in achieving the lead in the first place.

Regarding the impact of a change in h_j on imitation, the picture is less clear-cut. Solely taking a look at (18) for the impact of an ease of patent strength on imitation yields the counter-intuitive result, that lower patent protection strictly reduces imitation. However, considering that catching up occurs via inventing around (φ_{-1j}) *and* via the expiration of the leader's patents (h_j), a reduction of imitative research turns out to be a logical reaction to the fact, that catching up with the leader has been alleviated without having to invest more ressources in inventing around.

2.4.3 Steady-state industry structure and growth rate

In order to obtain the growth rate, it turns out to be useful to derive the structure of each industry, i.e., each industry's probability of being in neck-to-neck and leader-laggard state respectively, denoted by μ_{nj}, where $0 < \mu_{nj} < 1$. Since $n_j \in \{0, 1\}$, we can write $\mu_{0j} + \mu_{1j} = 1$. The probability distribution of technology gaps depends on optimal R&D intensities (17) and (18), because a higher intensity in one technological state makes an industry less likely to remain in that state. For example, if neck-to-neck competition in an industry is very intense, firms will try to achieve an advantage over their rival as soon as possible and invest in R&D to escape competition. Consequently, the probability for that industry to be in neck-to-neck is relatively small.

Since μ_{nj} itself is stationary, the probability of going into the state of gap n_j must equal the probability of leaving that state. For the neck-to-neck state this yields $\mu_{1j}\,(\varphi_{-1j} + h_j) = 2\,\mu_{0j}\,\varphi_{0j}$. Aside from the laggard's R&D intensity, the left hand side contains the flow rate of patent expiration, as both are alternative ways of catching up with the leader. The right hand side represents the flow out of the neck-to-neck state and includes two times the profit maximizing R&D levels, because both firms simultaneously try to become the next leader. Since $\mu_{1j} = 1 - \mu_{0j}$, it follows that

$$\mu_{0j} = \frac{\varphi_{-1j} + h_j}{2\,\varphi_{0j} + \varphi_{-1j} + h_j}\;. \tag{19}$$

The sector's probability of finding itself in neck-to-neck state equals the share of the arrival rate of (inventing around) imitations plus the Poisson hazard rate of patent expiration in the total arrival rate of any firm advancing by one technological step. Similarly, we can write

$$\mu_{1j} = \frac{2\,\varphi_{0j}}{2\,\varphi_{0j} + \varphi_{-1j} + h_j}\;. \tag{20}$$

Using these expressions, we can derive the steady-state growth rate. In a similar model, Aghion et al. (2001) use the fact that aggregate output equals $ln\,Y = \int_0^1 ln\,X_j\,dj$ (see (2)). Each industry's output X_j grows according to its innovations, whose occurrence follows an i.i.d. stochastic process. Aghion et al. (2001), thus, assume that the aggregate steady-state growth rate $g^* = \dot{Y} = \frac{d\,lnY}{dt}$ and the growth rate of each industry asymptotically are the same in the long run, i.e., $g^* = \lim\limits_{\Delta t \to \infty} \frac{\Delta\, ln\, X_j}{\Delta t}$. In contrast to that, in this model one sector's growth rate cannot be considered representative for aggregate growth, since here we analyze inter-industry differences by assuming substantially different parameters constituting the market situation in each sector. Consequently, while the above specification implies that $g^* = \lim\limits_{\Delta t \to \infty} \frac{\Delta \ln X_1 + \Delta \ln X_2 + ... + \Delta \ln X_j}{j\, \Delta t} = \lim\limits_{\Delta t \to \infty} \frac{j\, \Delta \ln X_j}{j\, \Delta t}$ as industry outputs expand symmetrically in the long run, the aggregate growth rate under the assumption of sector-specific differences is $g^* = \lim\limits_{\Delta t \to \infty} \frac{\int_0^1 \Delta \ln X_j \, dj}{j\, \Delta t}$. In order to maximize aggregate growth we, therefore, need to take variations in sector characteristics into account and maximize each sector's growth rate given by the following Proposition.

Proposition 2. *Let an industry be characterized by its degree of competition α_j, its R&D productivity β_j, and the sector-specific strength of patent protection h_j. Given the firm's profits in this industry follow* (8)*, their R&D decisions follow* (17) *and* (18)*, and the industry's probability of exhibiting technology gap n_j is given by* (19) *and* (20)*, then the sector-specific growth rate is*

$$g_j^* = \left[\frac{2\, \varphi_{0j}\, (\varphi_{-1j} + h_j)}{2\, \varphi_{0j} + \varphi_{-1j} + h_j} \right] \ln \gamma \; . \tag{21}$$

Proof. See Appendix A.2. □

A sector's output grows with each step a neck-to-neck firm advances technologically. This occurs at a probability determined by the optimal R&D intensity of neck-to-neck firms, φ_{0j}. Hence, at first glance, it seems that the laggard's R&D-intensity, φ_{-1j}, is not relevant for growth. Yet imitation exhibits an indirect growth effect via the probability that the sector is in neck-to-neck state, μ_{0j}. The higher the laggard's propensity to catch up, the more likely the respective sector is in neck-to-neck state. Consequently, imitation has positive influence on growth by bringing the industry back into the state, in which both firms engage in innovative research.

A similar observation can be made with regard to the impact of changes in patent protection on growth. While the described *disincentive effect* of lower patent protection on φ_{0j} naturally results in less growth, a relaxation of patent strength can have the opposite growth effect by increasing the sector's probability of being in neck-to-neck state. This so-called *composition effect* of patent protection on growth can, depending on the strength of h_j, mitigate or overcompensate the *disincentive effect*. Growth hence exhibits an ambivalent reaction to changes in patent strength. In order to show this ambivalence mathematically, it is useful to set up the following Proposition.

Proposition 3. *Consider an industry j, in which R&D costs are given by* (10)*, and optimal research intensities are given by* (17) *and* (18)*, where $\rho > 0$, $\beta_j > 0$, and $h_j \geq 0$. Under the premise that Proposition 1 holds true, it follows that a) $\varphi_{0j} > 0$ and $\varphi_{-1j} > 0$, and b) $\varphi_{0j} > \varphi_{-1j}$.*

Proof. See Appendix A.3. □

It follows from Proposition 3 that the growth rate given by Proposition 2 is strictly positive. Besides, we can infer the ambivalent reaction of growth to changes of patent protection by showing that $\frac{\partial g_j^*}{\partial h_j}$ is strictly greater than zero for small h_j and strictly smaller than zero for large h_j. This implies that increasing h_j in a relatively strict patent regime yields a positive growth effect (*disincentive effect* < *composition effect*), while doing the same in a relatively weak regime yields a negative growth effect (*composition effect* < *disincentive effect*). Aghion et al. (2001) show this for a Bertrand oligopoly. Proposition 3 ensures that $\varphi_{0j} > \varphi_{-1j}$ carries over to the Cournot set-up.

In Appendix A.4 I show that $\frac{\partial \varphi_{0j}}{\partial h_j} = -\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}$ and $\frac{\partial \varphi_{-1j}}{\partial h_j} = -\frac{\varphi_{-1j}+\left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)(\varphi_{0j}-\varphi_{-1j})}{\varphi_{-1j}+\rho+h_j+\varphi_{0j}}$. Since $\varphi_{0j} > 0$, $\varphi_{-1j} > 0$ and $\varphi_{0j} > \varphi_{-1j}$, both expressions are strictly less than zero, so $\frac{\partial \varphi_{0j}}{\partial h_j} < 0$ and $\frac{\partial \varphi_{-1j}}{\partial h_j} < 0$. Using this, in Appendix A.5 it is shown that

$$\frac{\partial g_j^*}{\partial h_j} = \frac{2\,\varphi_{0j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma}{(2\,\varphi_{0j}+\varphi_{-1j}+h_j)^2}\left[\frac{\varphi_{-1j}+h_j}{\varphi_{0j}}\left(\frac{\partial \varphi_{0j}}{\partial h_j}\right)+\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j}\left(\frac{\partial \varphi_{-1j}}{\partial h_j}+1\right)\right]. \tag{22}$$

From this expression it can be infered that the growth rate's reaction to changes in patent strength is ambivalent (see Appendix A.6). Moreover, (22) is an important prerequisite for the analysis of the behavior of optimal patent strength, which will be subject to the following section.

3 Differentiation of patent protection

3.1 Objectives, motivation, and benchmark values of the numerical calibration

Based on the framework in section 2, we can now address the question how sector-specific parameters can serve as independent variables determining the flexible component of a differentiated patent regime, and, more specifically, how variations in their scale change the optimal (i.e., growth-maximizing) patent strength. Recall from the introduction that the parameters determining the variation of patent strength in this model are: R&D productivity β_j, the degree of product market competition α_j, and the imitator's capability to reverse engineer ι_j (see 3.4). All three have an impact on the innovator's R&D-decision by influencing i) how resource-consuming is the innovation, ii) how much return can be expected from it, and iii) how long will the innovation yield profit for. They can, therefore, be used to approximate sector-specific differences in innovation incentives.

In order to find the growth-maximizing patent strength for each sector, the next step would be to find the root of (22). The solution would be a function $h_j^* = h_j^*(\alpha_j, \beta_j, \iota_j)$, whose first derivative w.r.t. any one of those three variables needs to be zero for uniform patent policy to be optimal, while deviance from zero would indicate optimality of a sector-specific differentiation of patent strength. However, finding the root of (22) requires solving a sextic equation, which according to the *Abel-Ruffini Theorem* cannot be solved in radicals.[20] In the following we, therefore, concentrate on numerical calibrations in order to scrutinize the optimal sector-specific patent protection.

The numerical analysis aims not at providing a detailed calibration of the modeled economy. Instead, its purpose is to highlight the growth rate's reaction to changes in patent strength on the one side, and the three sector differences determining variables on the other. I follow Mehra/Prescott (1985), who find an average real return on S&P 500 firms between 1889 and 1978 of 6.98 %, and set the annual discount rate to $\rho = 0.07$ throughout the calibration. The parameter specification

[20] Regarding the *Abel-Ruffini Theorem* see King (1996). For the sextic equation itself please contact the author.

will be geared to a benchmark case, in which patent length will be in accordance with the standard TRIPs term of patent protection, $h_{j,benchmark} = 0.05$[21], and the extant parameters will be specified so that the sector-specific growth rate in the benchmark case equals 2 %, which is consistent with the average US GDP per capita growth from 1950 to 1994, reported by Jones (2005).[22] This implies $\alpha_{j,benchmark} = 0.8$, $\beta_{j,benchmark} = 5$ and $\gamma_{benchmark} = 1.1$.[23] Depending on which of the three patent differentiation determining variables is under consideration, I will check for robustness of the results by gradually deviating from these benchmark values.

3.2 Sector-specific product market competition

Empirical findings on the impact of product market competition on innovation and growth are mixed. While Blundell et al. (1995) find a negative correlation and, thereby, corroborate the implications of basic Schumpeterian growth models (e.g., Aghion/Howitt (1992)), Nickell (1996) and Blundell et al. (1999) find the opposite effect. In their seminal empirical paper Aghion et al. (2005) integrate the opposing findings and empirically scrutinize an inverted-U relation between competition and innovation. Subsequent studies, however, cannot fully substantiate the inverted-U relation.[24] Prasad (2008) explains this with the composition of industries in the underlying data sets, as sector-specific characteristics have crucial impact on the inverted-U's existence. By testing the prediction of an inverted-U for several sectors separately, he finds a wide variation across industries. The model presented here accounts for these sector differences by modelling the familiar *Schumpeterian effect* and *escape competition effect* of competition on innovation depending on α_j.[25]

How should a flexible optimal patent life react to changes in a sector's product market competition? α_j affects (17) and (18) via incremental profits of an innovator advancing one technological step $(\pi_{1j} - \pi_{0j})$ and of an imitator catching up $(\pi_{0j} - \pi_{-1j})$. In figure 3 it becomes apparent that incremental profits of neck-to-neck firms that innovate grow with α_j, regardless of the size of innovations. Even for small γ, where profits decrease with α_j (l.h.s. of figure 3), incremental profits increase, because neck-to-neck profits decline with a higher rate than profits of the leader. Hence, R&D-investments in neck-to-neck state are more worthwhile the higher the degree of competition, because a firm can escape competition by innovating (*escape competition effect*). Consequently, the fact that an industry's naturally inherent research incentives are higher, when it exhibits more intense competition, mitigates the necessity to compensate innovators via patent protection.

By contrast, the *Schumpeterian effect* implies that more competition destroys incentives to

[21] Part II, sect. 5, art. 33 of the TRIPs Agreement stipulates a patent length not less than 20 years. Since we set $\rho = 0.07$, a standardized time intervall corresponds to one year. Thus, 20 years of expiration implies $h_j = 0.05$.

[22] See Jones (2005), p. 1091, who finds an average growth rate of 1.95 %.

[23] The latter value lies within the parameter range used in numerical calibrations of similar models: e.g. Aghion et al. (2001) set innovation size to $\gamma = 1.135$; Mukoyama (2003) gives an example in which $\gamma = 1.09$; Acemoglu/Akcigit (2008) choose $\gamma = 1.05$ and, then, check the robustness of the results with $\gamma = 1.01$ and $\gamma = 1.2$.

[24] See, e.g., Poldahl/Tingvall (2006) and Prasad (2008).

[25] See regarding the effects Aghion et al. (2001), Mukoyama (2003), and Aghion et al. (2005). Note that, compared to the latter, this model tends to marginalize the Schumpeterian effect. This is because, firstly, in a Cournot dyopoly the laggard's incremental profits are less likely to drop with an increase in α_j, and, secondly, unlike Aghion et al. (2005), changes in α_j are assumed to affect not only π_{0j}, but π_{1j} and π_{-1j} as well.

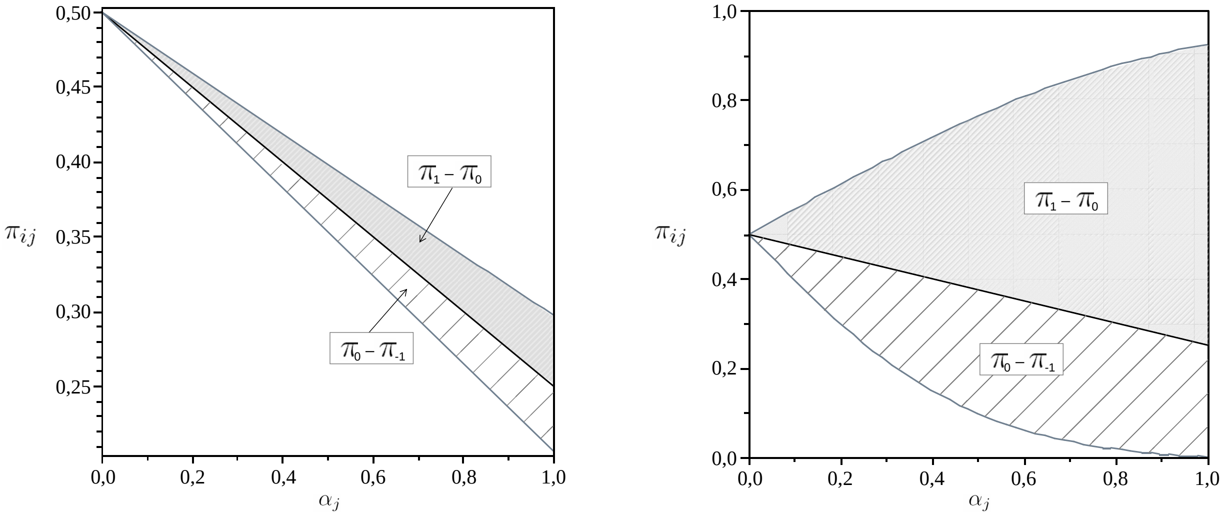


Figure 3: Neck-to-neck, leader and laggard profits' reaction to changes in α_j (*left:* $\gamma = 1.2$, *right:* $\gamma = 25$).

innovate. This becomes apparent by taking a closer look at the laggard's incremental profits for large γ and their ambivalent reaction to an increase of α_j (r.h.s. of figure 3). While for small α_j $\pi_{0j} - \pi_{-1j}$ rise with an increase of α_j, for high α_j they begin to fall again. The latter aspect induces imitative research to decline (*Schumpeterian effect*). The ambivalence only becomes apparent for high γ, because, unlike Bertrand, in the Cournot set-up the laggard still generates profits, if γ is small. Only if γ is large, he finds himself in a zero-profit situation, even when α_j is not close to 1.

What does this imply with regard to flexible patent length? In Appendix B.1 it is shown that

$$\frac{\partial(\pi_{1j} - \pi_{0j})}{\partial \alpha_j} > \frac{\partial(\pi_{0j} - \pi_{-1j})}{\partial \alpha_j} \tag{23}$$

for all $\alpha_j \in (0, 1]$. It follows that the discrepancy between innovating and catching up incremental profits increases with more intense competition. Again, the implication for optimal patent strength is that more competition corresponds to less protection, since the threat of losing the technological edge over the rival and, to that extent, the necessity to compensate the innovator diminishes.

Larger R&D incentives for leader and laggard when competition is more intense and, more importantly, the increasing gap between those incentives also influence optimal R&D intensities. Variations of (17) and (18) with competition are induced solely by changes of incremental profits. Consequently, since according to Proposition 3b) $\varphi_{0j} > \varphi_{-1j}$, (23) implies that

$$\frac{\partial \varphi_{0j}}{\partial \alpha_j} > \frac{\partial \varphi_{-1j}}{\partial \alpha_j} \tag{24}$$

for any $\alpha_j \in (0, 1]$ and $\gamma > 1$. Expression (24) has crucial impact on the composition of growth rate (21) inasmuch as it induces that components of the latter react differently to changes in α_j. The fact that the relation of imitative to innovate research falls with more intense competition, mitigates the *composition effect*. This, in turn, suggests a higher need to weaken patent protection, since a high neck-to-neck R&D-intensity is useless without imitation.

What can we infer from the analysis so far with regard to the growth rate's reaction on variations of product market competition? All three theoretical implications of changes in α_j derived up to this point suggest that a more intense competition corresponds to a lower optimal patent strength. The next step is to carry out a numerical calibration to test these deliberations. As it turns out, the benchmark case, which is shown in figure 4, corroborates the intuitive results derived before (see Result 2). Besides, it renders possible to substantiate the ambivalence of the growth rate's reaction to changes in patent strength (see Result 1).

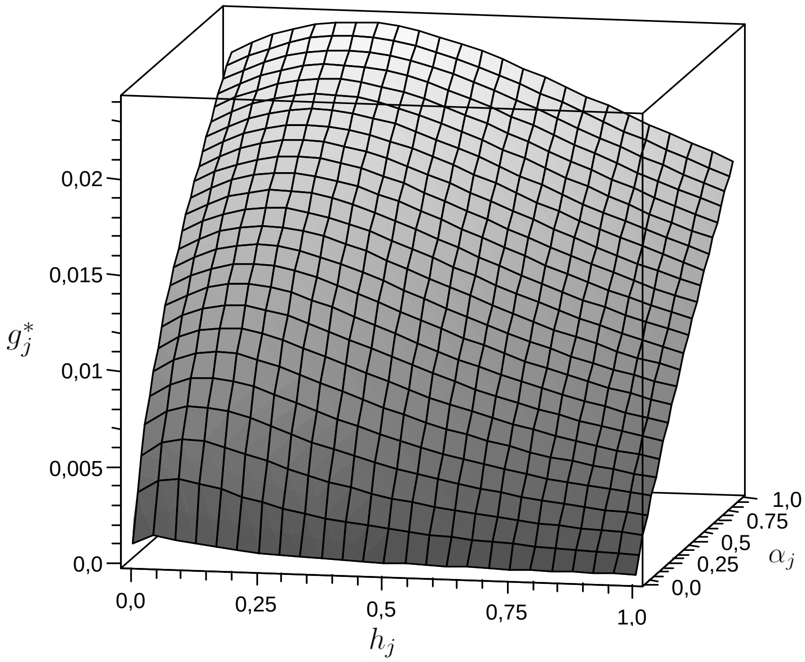


Figure 4: The growth rate's reaction to changes in patent length and product market competition α_j in the benchmark case ($\gamma = 1.1$, $\rho = 0.07$, $\beta_j = 5$).

Result 1. *Given identical R&D costs for leader and laggard, the ambivalent reaction of growth to changes in patent length, shown in Appendix A.6, takes the form of an inverted-U relationship.*

In Appendices B.2 and B.3 it is shown that this result is robust to numerous variations in parameter specifications over the given range of values for each parameter. Hence, we can infer that the familiar inverted-U relationship between growth and patent protection, inter alia established by similar step-by-step models with Bertrand competition[26], carries over to a Cournot set-up. Result 1 is consistent with empirical findings of Comanor (1967) and Qian (2007).[27] Moreover, since the inverted-U's maximum represents the growth-maximizing patent strength, this characteristic relation between growth and patent protection implies that the calibration yields an unambiguous solution for the optimal (differentiated) patent strength within a realistic parameter range.

[26] See, e.g., Aghion et al. (2001) and Mukoyama (2003), where the latter uses a slightly different interpretation of the relevant parameter. In a related theoretical model Horowitz/Lai (1996) find an inverted-U relation between patent length and innovation, resulting from two opposing effects similar to the ones present in this model.

[27] Comanor (1967) observes that too high and too low technical entry barriers impair research incentives. Qian (2007) finds the existence of an optimal patent level above which stronger protection discourages innovation.

Result 2. *Given a duopolistic sector j, whose growth rate follows* (21)*, an increase in the sector-specific degree of product market competition implies an according adaptation of optimal patent protection in the form of shorter differentiated patent lives and vice versa.*

This result, also, is robust to numerous variations of parameter values (see Appendix B.2).

3.3 Sector-specfic R&D productivity

The idea of flexible patent length according to a sector's R&D productivity can be traced back to the seminal work of Nordhaus (1969). Based on a simple model trading off dynamic efficiency and static inefficiency of patents, the model implies that, under the assumption of a concave welfare function, a higher productivity corresponds to a shorter optimal patent life. Using a similar but more elaborated model, Cornelli/Schankerman (1999) reach the opposite conclusion. They assume large technology steps to be of socially higher value than many small steps, because significant inventions are more likely to generate positive externalities and the related products obtain a lower elasticity of demand. Due to the resulting convex welfare function, they find that more productive inventors should be granted longer protection in order to tilt their R&D effort towards large inventions.

In contrast to these welfare models, the approach presented here accounts for sequential innovations and, thereby, focuses on dynamic efficiency and its trade off between innovation and imitation. The model also differs from the previous inasmuch as it explicitly models the firms' competitive behavior, instead of assuming a given relation between R&D output and profits. Under this specification it contradicts the result of Cornelli/Schankerman (1999) and reestablishes the rather intuitive implication of the Nordhaus model. A sector-specific patent regime depending on an industry's level of R&D costs ought to grant greater protection to those sectors, which require more intense research to take a technological step forward. Since the level of R&D costs can be approximated by a measure for how productive a researcher in one sector is compared to another, we utilize a sector's average R&D productivity to determine its optimal patent strength.

As opposed to product market competition, R&D productivity β_j has direct impact on (17) and (18) by rescaling incremental profits of neck-to-neck and catching up firms. According to (10), a higher productivity leads to a lower need for researchers to achieve a given flow rate of innovation and, to that extent, lower R&D costs. Put differently, a higher productivity implies that the same amount of workers employed in research potentially generates an innovation sooner, so that higher profits accrue to the innovating firm. It becomes apparent that a higher β_j yields stronger incentives to innovate and, therewith, higher optimal research intensities of neck-to-neck and following firms.

The extent to which φ_{0j} and φ_{-1j} react to changes in R&D productivity differs, however. This follows from Proposition 1. Since incremental profits of neck-to-neck firms are always greater than incremental profits of laggard firms, scaling up both also scales up this difference, so that

$$\frac{\partial \varphi_{0j}}{\partial \beta_j} > \frac{\partial \varphi_{-1j}}{\partial \beta_j} \,. \tag{25}$$

As a consequence, by altering the relation between imitative and innovative research, changes in R&D productivity have crucial impact on growth and optimal patent strength. For instance, a higher sector-specific productivity induces a stronger increase in innovative R&D incentives than in the propensity to imitate. Hence, the sector under consideration is less likely to be in neck-to-neck and the *composition effect* on growth declines. It follows, that optimal (differentiated) patent strength decreases with higher β_j, since more imitation is needed in order to increase the probability to be in neck-to-neck state and to benefit from the increased innovation incentives in that state.

An alternative way to think of it is to consider the interpretation of β_j as a measure for R&D costs. An average research project in an industry, that exhibits a smaller β_j, is more expensive than average projects in sectors with a higher productivity. That is why in the latter kind of sectors the necessity for patent authorities to intervene and compensate innovators is lower than in the former kind of sectors. A flexible patent regime adjusting the strength of protection according to R&D productivity should grant more protection to sectors with costlier R&D.

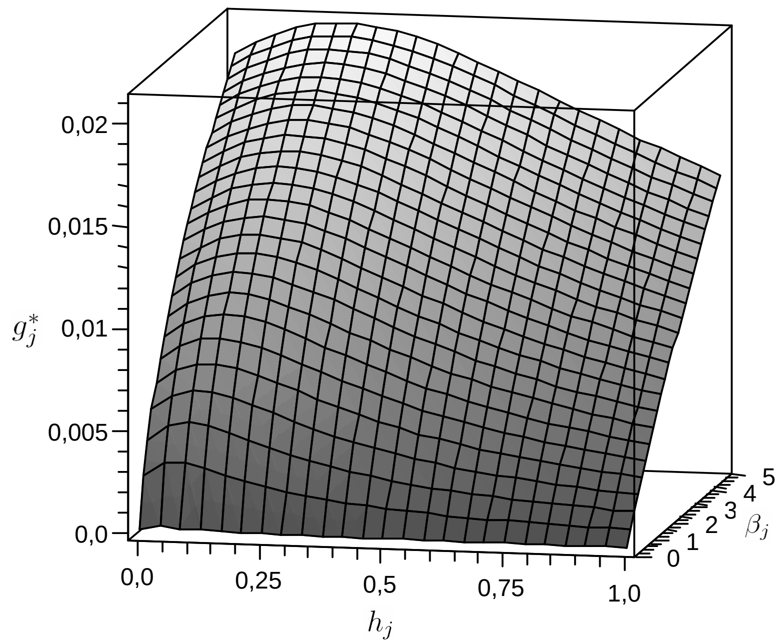


Figure 5: The growth rate's reaction to changes in patent length and R&D productivity β_j in the benchmark case ($\gamma = 1.1$, $\rho = 0.07$, $\alpha_j = 0.8$).

The numerical analysis of the reaction of optimal patent strength to changes in sector-specific R&D productivity corroborates this reasoning. Figure 5 illustrates that, similar to the case of product market competition, the maxima of the inverted-U relationship correspond to higher h_j^*, the higher sector-specific R&D productivity β_j is. Hence, we can state

Result 3. *Given a duopolistic sector j, whose growth rate follows* (21)*, an increase in the sector-specific R&D productivity implies an according adaptation of optimal patent protection in the form of shorter differentiated patent lives and vice versa.*

This result is robust to numerous variations of parameter values (see Appendix B.3).

3.4 Sector-specific imitation costs

The necessity for governmental intervention to compensate innovators via patent policy originates from the well-known problem that imitation impairs the innovating firm's ability to cover its R&D expenses. However, inter-industry differences regarding the technical complexity of products imply that sectors may also differ with respect to the capability to reverse engineer the state-of-the-art technology. The pace of imitation, in turn, determines how long the leader is able to accrue monopoly profits. Inter-industry differences, therefore, have an impact on a market's natural ability to compensate innovators and, to that extent, on the necessity of patent protection.

Note that costly innovations are not necessarily costly to imitate. If the correlation between innovative and imitative costs were close to one, a higher necessity to compensate innovators would always coincide with a higher naturally inherent capability of the market to do that. In this case a uniform patent protection might be appropriate, since the need for governmental intervention would not fluctuate with the sector-specific differences in the technical complexity of products. Yet empirical evidence suggests that the correlation between innovation and imitation costs differs from sector to sector. While, for instance, literature and music can be copied without considerable costs, imitating most products is more laborious. Mansfield et al. (1981) find that in chemical and drug industries a vast majority of innovations (project size $\geq$ \$ 1 million) imply imitation costs that are larger than 60 % of innovation costs. In case of chemicals, for 38 % of new products imitation is even more expensive than innovation itself, because the technological leader possesses highly specialized R&D know-how, which is inaccessible to imitators. As opposed to that, in electronics and machinery industries the majority of new products can be imitated at less than 60 % of innovation costs. Hence, the threat of losing the technological lead in an industry varies from sector to sector.

Since the goal here is to differentiate patent strength according to a sector's necessity to compensate inventors beyond its natural capability to do so, I take the inter-industry differences in the discrepancy between innovation and imitation costs found by Mansfield et al. (1981) into account. In accordance with them, the decisive parameter used here is the amount of imitation costs in percent of innovations costs. In the following, this *imitation-innovation costs relation* will be denoted ι_j. Considering the specific R&D costs function (10), it follows that $\mathcal{G}_{j,innovator} \neq \mathcal{G}_{j,imitator}$, where $\mathcal{G}_{j,imitator} = \iota_j \, \mathcal{G}_{j,innovator}$. Hence, ι_j rescales the R&D cost function (10) according to whether leader or laggard conducts research. While the leader's R&D costs remain unchanged to the previous specification of the model, the imitator's R&D costs are now given by

$$\mathcal{G}(\varphi_{-1j}) = \frac{\iota_j}{2\,\beta_j} \, (\varphi_{-1j})^2 \, . \tag{26}$$

This alternative specification has crucial repercussions on the optimal research intensities of neck-to-neck and following firms ((17) and (18)). Based on (26), it becomes apparent that the imitator's first order condition becomes $\frac{\iota_j}{\beta_j} \, \varphi_{-1j} = (V_0 - V_{-1})$. Moreover, in Appendix B.5 I show that the optimal research intensity of a neck-to-neck firm becomes

$$\varphi_{0j} = -[\rho + h_j + \varphi_{-1j}\,(1-\iota_j)] + \sqrt{[\rho + h_j + \varphi_{-1j}\,(1-\iota_j)]^2 + 2\,\beta_j\,(\pi_{1j} - \pi_{0j})}\,, \tag{27}$$

and, similarly, the optimal research intensity of the firm trying to catch up becomes

$$\varphi_{-1j} = -(\rho + h_j + \varphi_{0j}) + \sqrt{(\rho + h_j + \varphi_{0j})^2 + \frac{2\,\beta_j}{\iota_j}\,(\pi_{0j} - \pi_{-1j}) + \frac{1}{\iota_j}\,(\varphi_{0j})^2}\;. \tag{28}$$

As opposed to the previous specification of the model, in which $\iota_j = 1$, the optimal neck-to-neck intensity here depends on the laggard's intensity. This is due to the fact that the special case, in which the risk of losing the technological lead times the lost value equals the risk of becoming laggard times the lost value, only applies when imitation and innovation costs are the same ($\iota_j = 1$). The fact that ι_j is now assumed to fluctuate sector-specifically complicates the solution of the model.

What is the intuition behind it? ι_j directly enters the optimal laggard's intensity by lowering incremental profits $(\pi_{0j} - \pi_{-1j})$ and the term $(\varphi_{0j})^2$. The former constitute the immediate profits from imitation, while the latter term represents the additional firm value from having gained the opportunity to potentially become the next leader.[28] A higher ι_j leads to a decline of both terms and, to that extent, a decline of the optimal φ_{-1j}, because both types of benefits come at a higher price. In other words, since according to the imitator's first order condition above, the marginal cost of research must be equal to the value added, an increase in the imitator's marginal costs causes the same level of value added to correspond to a lower optimal research intensity. This yields the following intuitive result for the direct effect of ι_j on the laggard's optimal research intensity: a more complicated reverse engineering induces lower investments in imitative research.

In view of (27), it becomes apparent that ι_j has direct impact on φ_{0j} by reducing the threat of imitation. Since in this specification of the model the imitation-innovation costs ratio may deviate from 1, the term $\varphi_{-1j}\,(1-\iota_j)$ enters the equation before and under the square root. It represents the difference in the threat of falling one step behind in a leader-laggard state ($\varphi_{-1j}\,(V_1 - V_0) = \varphi_{-1j}\,\frac{\varphi_{0j}}{\beta_j}$) compared to a neck-to-neck state ($\varphi_{0j}\,(V_0 - V_{-1}) = \varphi_{0j}\,\frac{\iota_j}{\beta_j}\,\varphi_{-1j}$). In case of $\iota_j = 1$ the threat of falling one step behind is the same regardless in which state the sector is situated, because there is no difference between innovation and imitation costs. If, however, ι_j deviates from 1, it changes the leading firm's threat of being imitated relative to a neck-to-neck firm's threat of falling behind. Consequently, a higher ι_j corresponds to a higher optimal research intensity of a neck-to-neck firm. This is because the threat to imitate the leader decreases relative to the threat for a neck-to-neck firm to fall behind, so that incentives to leave the neck-to-neck state increase.

Again, this reasoning only captures the direct effect of ι_j on optimal R&D investment. Yet, in both cases, direct and indirect effects of ι_j on the optimal intensities go in the same direction. According to (27), a higher φ_{-1j} lowers optimal neck-to-neck intensity φ_{0j}. Since the direct effect

[28] The benefit from potentially becoming next leader is the probability of a successful innovation in neck-to-neck state times the value added by innovating minus R&D costs: $\varphi_{0j}\,(V_1 - V_0) - \frac{1}{2\,\beta_j}\,\varphi_{0j}^2 = \frac{1}{\beta_j}\varphi_{0j}^2 - \frac{1}{2\,\beta_j}\,\varphi_{0j}^2 = \frac{1}{2\,\beta_j}\,\varphi_{0j}^2$. From solving the FOC for φ_{-1j}, $2\,\beta_j$ cancels out and ι_j enters the term, so that both terms go on the other side.

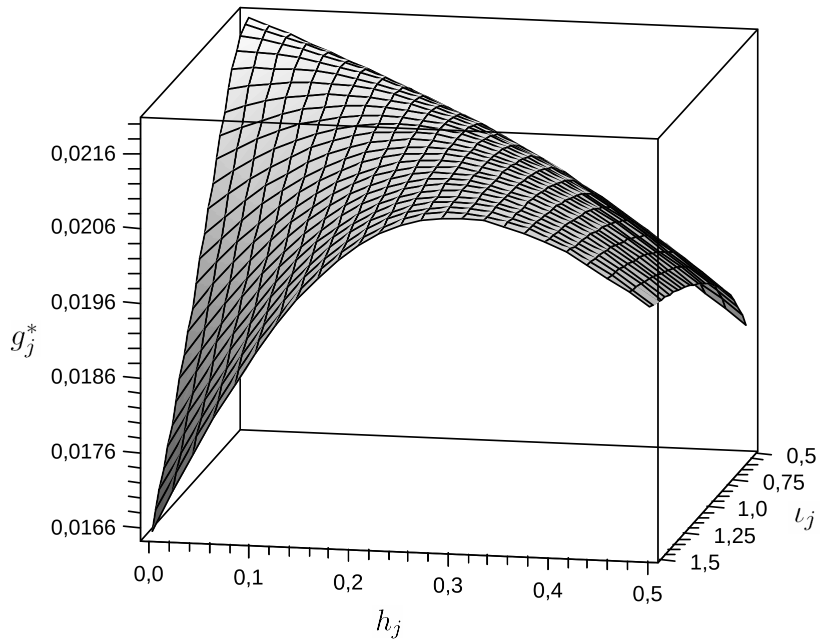


Figure 6: The growth rate's reaction to changes in patent length and sector-specific reverse engineering capability ι_j in the benchmark case ($\gamma = 1.2$, $\rho = 0.04$, $\alpha_j = 1$, $\beta_j = 1$).

implies a high φ_{-1j} when ι_j is small, the indirect effect on φ_{0j} corroborates the direct effect by additionally leading to the fact that a small ι_j corresponds to a small optimal neck-to-neck research intensity. Similarly, the direction of the indirect effect of ι_j on the laggard's optimal intensity equals the direct effect's direction, since a higher ι_j induces a higher φ_{0j} which, in turn, lowers φ_{-1j}.

What are the implications of a variation in ι_j on optimal differentiated patent protection? In a sector with relatively laborious reverse engineering, φ_{0j} exceeds φ_{-1j}, since a higher ι_j lets φ_{0j} increase while φ_{-1j} decreases. Recall from section 2.4.3 that such a sector does not necessarily grow faster, because with a simultaneously lower φ_{-1j} the sector is less probable to be situated in neck-to-neck. It follows that in order to benefit from the neck-to-neck firms' high propensity to invest in R&D, it is essential to ease patent protection and, by this means, increase the pace of catching up (*composition effect*). Hence, high ι_j correspond to low optimal patent protection. Similarly, small ι_j imply a high optimal patent strength. This is due to the fact that easier imitation induces a shorter technological lead, which impairs the innovator's ability to break even.

Figure 6 illustrates how optimal patent strength varies with the sector-specific propensity to imitate.[29] It becomes apparent that, as opposed to previous specifications of the model, growth does not exhibit an inverted-U relation with respect to h_j in any case. This is due to the fact that for small ι_j the precondition for an inverted-U relation, $\varphi_{0j} > \varphi_{-1j}$ (Proposition 3b), does not hold. Since a decline in imitation costs increases the propensity to imitate and, beyond that, lowers the propensity to innovate, for small ι_j we have that $\varphi_{-1j} > \varphi_{0j}$. Moreover, as for $\iota_j \to 0$ the imitation costs become negligible, we have that $\lim_{\iota_j \to 0} \varphi_{-1j} = \infty$ and $\lim_{\iota_j \to 0} \varphi_{0j} = 0$. As a consequence, the growth rate tends to zero. Note that this is the fact even with maximal patent protection ($h_j = 0$), because

[29] Since the empirical results in Mansfield et al. (1981) imply $0.5 < \iota_j < 1.5$ to be a realistic range for the imitation-innovation costs relation, figure 6 and the figures in the Appendix focus on that range.

patents only shield innovators from copying their idea, not from inventing around it. Since the specification of patent protection in this model follows Aghion et al. (2001) and Acemoglu/Akcigit (2008) by modelling patent length and implicitly assuming a standardized patent breadth, maximal patent protection implies infinite patent length, while patent breadth remains unchanged. Hence, patents in this model can only mitigate but not prevent the decomposition of innovation incentives for extremely small ι_j. Nevertheless, we can summarize the following result, which also is robust to numerous variations of parameter values (see Appendix B.4).

Result 4. *Given a duopolistic sector j, whose growth rate follows* (21)*, a sector-specific increase in the imitator's ability to reverse engineer technologically superior products, i.e., a higher imitation-innovation costs relation* ι_j*, where* $\iota_j \in [0.5, 1.5]$*, implies shorter optimal patent lives and vice versa.*

4 Conclusion

This paper deals with a step-by-step innovation model of endogenous growth in a Cournot duopoly set-up. In order to account for previous empirical findings on industry differences in the impact and importance of patents, it is used to scrutinize how sector-specific parameters can be used to determine the flexible component of a differentiated patent regime, and, more specifically, how variations in their scale change the optimal (i.e., growth-maximizing) patent length. It is found that patent protection of a sector ought to be weaker, the more intense its product market competition, the higher its R&D productivity, and the more laborious imitation by a firm trying to catch up are. In the latter case, the model resorts to the imitation-innovation costs relation, empirically elicited by Mansfield et al. (1981), in order to account for the ease of imitation. It is found that for most empirically plausible relations the inverted-U relation between patent length and growth, carries over from Bertrand competition to the Cournot set-up. Only in sectors with inherently easy imitation the composition effect disappears, so that it is optimal to grant maximum patent length.

The model takes a first step towards a theoretical basis for an assessment of industry-specific modifications of patent length already implemented in practice (e.g., supplementary protection certificates and pediatric exclusivity). Recall from the introduction that supplementary protection certificates aim at compensating inventors in pharmaceutical and biotech industries for the loss of effective patent term due to regulatory delays in the launch of new products. However, the fact, that this concerns subsequent innovations as well, prolongs the opportunity to accrue profits and casts the necessity for additional protection into doubt. Since the model proposes a differentiation of patent length according to how much and how long the innovation yields profit, it accounts for this dynamic aspect of the evaluation of supplementary protection certificates, and, to that extent, can provide a theoretical basis for empirical studies on the necessity to grant them.

A common objection to previous propositions of a differentiated patent regime is that implementation costs will overcompensate the undoubtly existent social benefit of such a system. While for low aggregation levels (e.g., firm or project level) it seems difficult to remedy these concerns, three reasons come to mind for why regarding a sector-specific differentiation they lack persuasive power. Firstly, since, as mentioned before, the current system already comprises a specific protection for some sectors, part of the corresponding implementation costs accrue anyway. Secondly, in contrast to the firm or project level, an industry-specific differentiation does not require a reevaluation of patent strength for every single application, which implies significantly less costs. Yet it, still, can efficiently approximate differences in the necessity to compensate inventors, since parameters constituting these differences either exhibit a sector-specific nature or, as numerous empirical studies indicate, are comparatively easy to obtain on a sectoral level. Thirdly, the current country-specific patent system includes implementation costs, that would not accrue under the proposed sector-specific system. Even though WTO member countries under TRIPs agreed on a uniform patent length, application, grant and litigation of patents usually fall under the jurisdiction of every member country, each having to bear the corresponding costs. Within Europe, the European Patent Convention (EPC) aims at lowering these costs by harmonizing patent applications and grants. However, since the EPC does not include a joint judicial authority, litigations remain under each country's jurisdiction. Hence, a European patent is no single, centrally enforceable patent but rather a bundle of patents with ambiguous legal certainty. The related costs could be avoided by a transnational uniform system of a sector-specific differentiation, in which countries potentially cooperate to scrutinize a sector's optimal patent length. It follows that, only if implementation costs of the proposed system exceed costs of the current one by a higher amount than its social benefit, the above objection applies. The given reasons lead us to the conclusion that this is unlikely the case. Nevertheless, the verification of this conclusion deserves further study.

Possible extensions of the model include a specification which accounts for the whole range of patent design mechanisms besides patent length. The paper follows Aghion et al. (2001) and Acemoglu/Akcigit (2008) by modelling patent length explicitly, while implicitly assuming a standardized patent breadth. Since the latter cannot be influenced by patent policy decisions in the model, maximal patent protection cannot entirely shield innovators from imitation. Firms trying to catch up can still imitate by inventing around the technological leader's design. Because patent breadth (leading or lagging) determines how much effort the inventing around implies, its consideration enables us to model maximal patent protection to entirely shield innovators from imitation. This might have interesting implications on optimal patent strength in a flexible regime.

Moreover, one might extend the model by additionally taking secrecy, lead time advantages, and the use of complementary marketing and manufacturing capabilities as alternative means to protect ideas from imitation into account. Empirical studies, such as Levin et al. (1987) and Cohen et al. (2000), suggest that, most notably, secrecy is more important in this regard. Since patents include a disclosure of technical details of an invention after a certain period of protection, simply keeping a lid on the idea is an effective alternative to protect intellectual property. Accounting for it might yield interesting results regarding optimal differentiated patent protection.

Appendices

Appendix A (section 2)

A.1

Objective. Given (8), show that $n_{inflection} > 0$.

Proof. At first, I take the second derivative of (8) with respect to n_j and simplify to

$$\frac{\partial^2 \pi_{ij}(\alpha_j, n_j)}{\partial n_j^2} = \frac{\alpha_j^2 \, ln\gamma \left((\gamma^{-n_j})^{3\alpha_j} + 4\alpha_j \, (\gamma^{-n_j})^{2\alpha_j} - \alpha_j \, (\gamma^{-n_j})^{3\alpha_j} - \alpha_j \, (\gamma^{-n_j})^{\alpha_j} \right)}{\left(1 + (\gamma^{-n_j})^{\alpha_j}\right)^4} .$$

The next step is to set this expression equal to zero and solve for $n_{inflection}$. The resulting expression only holds for $0 < \alpha_j < 1$, since $ln \left(-\frac{\sqrt{3\alpha_j^2+1} - 2\alpha_j}{\alpha_j - 1} \right)$ is not defined for $\alpha_j = 1$. Hence, again, I take the second derivate of (8), but instead of solving for $n_{inflection}$ directly, I evaluate the expression at $\alpha_j = 1$ and, then, solve for $n_{inflection}|_{\alpha_j=1}$. Consequently, the inflection point is given by

$$n_{inflection} = \begin{cases} -\dfrac{ln\left(-\frac{\sqrt{3\alpha_j^2+1}-2\alpha_j}{\alpha_j-1}\right)}{\alpha_j \, ln\gamma} & \text{if } 0 < \alpha_j < 1 \\ \frac{ln2}{ln\gamma} & \text{if } \alpha_j = 1 \end{cases}$$

Since $0.5 < \left(-\frac{\sqrt{3\alpha_j^2+1} - 2\alpha_j}{\alpha_j - 1} \right) < 1$, both expressions can be shown to be positive for any $\gamma > 1$ and for each defined range of α_j respectively. Note that for all $\gamma > 1$ and $\alpha_j \in (0, 1]$ we have that $\frac{\partial^3 \pi(\cdot)}{\partial n_j^3}\Big|_{n_j = n_{inflection}} \neq 0$, so the inflection points exist. $\square$

A.2

Objective. Show that the sector-specific growth rate of sector j equals $g_j^* = \left[\frac{2\,\varphi_{0j}\,(\varphi_{-1j} + h_j)}{2\,\varphi_{0j} + \varphi_{-1j} + h_j} \right] ln\,\gamma$.

Proof. Without the one-step lead assumption, each industry j experiences a cycle, expressed by the sequence $\{0, 1, ..., \upsilon - 1, \upsilon, 0\}$ where $\upsilon \in \mathbb{N}$, so that $ln\,X_j$ grows at rate $\upsilon\,ln\,\gamma$ between the beginning and the end of that cycle. υ can be interpreted as the number of technology steps the industry moved forward per cycle. With the one-step lead assumption we have that $\upsilon = 1$, so that for a long time interval we can write $\Delta\,ln\,X_j \approx \mathcal{V}_j\,(ln\,\gamma)$, where $\mathcal{V}_j$ is the number of cycles that industry j experiences within the time interval under consideration. Plugging this into the growth rate of a sector j, $g_j^* = \lim_{\Delta t \to \infty} \frac{\Delta\,ln\,X_j}{\Delta t}$, which follows from the aggregate growth rate $g^* = \lim_{\Delta t \to \infty} \frac{j\,\Delta\,ln\,X_j}{j\,\Delta t}$ given in section 2.4.3, yields

$$\begin{aligned} g_j^* &= \lim_{\Delta t \to \infty} \left[\mathcal{V}_j \left(\frac{ln\,\gamma}{\Delta t} \right) \right] \\ &= \left(\lim_{\Delta t \to \infty} \frac{\mathcal{V}_j}{\Delta t} \right) (ln\,\gamma) \\ &= \mu_{1j}\,(\varphi_{-1j} + h_j)\,(ln\,\gamma) . \end{aligned} \tag{29}$$

The last step uses the fact that the asymptotic frequency of cycles with υ steps, $\lim_{\Delta t \to \infty} \frac{\mathcal{V}_\upsilon}{\Delta t}$, in steady-state underlies the stationarity condition that the flow out of a state must equal its inflow. Since the laggard is always able to close the whole technological gap at once, the frequency of a cycle with υ steps equals the probability μ_{1j} of being in leader-laggard state times the Poisson arrival rate of falling back into neck-to-neck, i.e., $(\varphi_{-1j} + h_j)$. Finally, using (20) we can rewrite the growth rate as

$$g_j^* = \left[\frac{2\,\varphi_{0j}\,(\varphi_{-1j} + h_j)}{2\,\varphi_{0j} + \varphi_{-1j} + h_j}\right]\, ln\,\gamma\;. \tag{21}$$

□

A.3

Objective. Under the premise that Proposition 1 holds and given that $\rho > 0$, $\beta_j > 0$, and $h_j > 0$, show that for the optimal research intensities (17) and (18) holds true that a) $\varphi_{0j} > 0$ and $\varphi_{-1j} > 0$, and b) $\varphi_{0j} > \varphi_{-1j}$.

Proof. I begin by proving a). From Proposition 1 and the fact that all parameters are positive, it follows that the square roots in the optimal research intensities (17) and (18) are greater than zero. Even if $(\pi_{1j} - \pi_{0j})$ and $(\pi_{0j} - \pi_{-1j})$ were not greater than but equal to zero, the positive squared part under the roots would compensate the negative first part of the expressions. Consequently, we can infer $\varphi_{0j} > 0$. Since we can apply the same procedure to the optimal laggard intensity, we also have that $\varphi_{-1j} > 0$.

In order to prove b), I use (15) and (16) to show that the optimal neck-to-neck R&D intensity is strictly greater than the optimal laggard R&D intensity, if and only if the difference of leader and neck-to-neck firm values is strictly greater than the difference of neck-to-neck and laggard firm values, so

$$\varphi_{0j} > \varphi_{-1j} \quad \Leftrightarrow \quad (V_{1j} - V_{0j}) > (V_{0j} - V_{-1j})\;, \tag{30}$$

Hence, we can prove b) by proving the term on the r.h.s. Using (12), (13) and (14), we can write

$$\rho\,(V_{1j} - V_{0j}) = \pi_{1j} - \pi_{0j} - (\varphi_{0j} + \varphi_{-1j} + h_j)\,(V_{1j} - V_{0j}) + \varphi_{0j}\,(V_{0j} - V_{-1j}) + \frac{(\varphi_{0j})^2}{2\,\beta_j} \tag{31}$$

and

$$\rho\,(V_{0j} - V_{-1j}) = \pi_{0j} - \pi_{-1j} - (\varphi_{0j} + \varphi_{-1j} + h_j)\,(V_{0j} - V_{-1j}) + \varphi_{0j}\,(V_{1j} - V_{0j}) - \frac{(\varphi_{0j})^2}{2\,\beta_j} + \frac{(\varphi_{-1j})^2}{2\,\beta_j}\;. \tag{32}$$

Substracting (32) from (31) and combining similar terms yields

$$[(V_{1j} - V_{0j}) - (V_{0j} - V_{-1j})]\;(2\,\varphi_{0j} + \varphi_{-1j} + \rho + h_j) - \frac{(\varphi_{0j})^2}{\beta_j} + \frac{(\varphi_{-1j})^2}{2\,\beta_j} = \pi_{1j} - 2\,\pi_{0j} + \pi_{-1j}\;. \tag{33}$$

According to Proposition 1, the r.h.s. of (33) is strictly greater than zero. Using (15) and (16), we replace φ_{0j} by $\beta_j\,(V_{1j} - V_{0j})$ and φ_{-1j} by $\beta_j\,(V_{0j} - V_{-1j})$. Moreover, for notational simplicity we set $\mathcal{X} = (V_{1j} - V_{0j})$ and $\mathcal{Y} = (V_{0j} - V_{-1j})$. This yields

$$[\mathcal{X} - \mathcal{Y}]\;(2\,\beta_j\,\mathcal{X} + \beta_j\,\mathcal{Y} + \rho + h_j) - \beta_j\,\mathcal{X}^2 + \frac{\beta_j\,\mathcal{Y}^2}{2} > 0$$

and respectively

$$\beta_j\,\mathcal{X}^2 + (\rho + h_j)\,\mathcal{X} \;>\; \frac{\beta_j}{2}\,\mathcal{Y}^2 + \beta_j\,\mathcal{X}\,\mathcal{Y} + (\rho + h_j)\,\mathcal{Y}\;. \tag{34}$$

Based on (34), we can prove by contradiction that $\mathcal{X} > \mathcal{Y}$ must hold. Supposing $\mathcal{X} \leq \mathcal{Y}$, (34) simplifies to

$$i)\ \mathcal{X} = \mathcal{Y} \ \Rightarrow\ \beta_j\,\mathcal{X}^2 + (\rho + h_j)\,\mathcal{X} \ > \ \frac{\beta_j}{2}\,\mathcal{X}^2 + \beta_j\,\mathcal{X}^2 \ + (\rho + h_j)\,\mathcal{Y} \Leftrightarrow \ 0 > \frac{\beta_j}{2}\,\mathcal{X}^2 \quad \rightarrow \ \textit{contradiction}$$

$$ii)\ \mathcal{X} < \mathcal{Y} \ \Rightarrow\ \beta_j\,\mathcal{X}^2 + (\rho + h_j)\,\mathcal{X} \ > \ \frac{\beta_j}{2}\,(\mathcal{X} + \mathcal{E})^2 + \beta_j\,\mathcal{X}\,(\mathcal{X} + \mathcal{E}) \ + (\rho + h_j)\,(\mathcal{X} + \mathcal{E})$$

$$\Leftrightarrow \ 0 > \frac{\beta_j}{2}\,(\mathcal{X} + \mathcal{E})^2 + \beta_j\,\mathcal{X}\,\mathcal{E} + (\rho + h_j)\,\mathcal{E} \quad \rightarrow \ \textit{contradiction}$$

Because $\beta_j > 0$, $\mathcal{E} > 0$, $\mathcal{X} > 0$, and since from a) we have that $\varphi_{0j} > 0$, both expressions imply a contradiction. Consequently, we can infer that $\mathcal{X} > \mathcal{Y}$ and, therewith, using (15) and (16), $\varphi_{0j} > \varphi_{-1j}$. □

A.4

Objective. Show that $\frac{\partial \varphi_{0j}}{\partial h_j} = -\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}$ and $\frac{\partial \varphi_{-1j}}{\partial h_j} = -\frac{\varphi_{-1j}+\left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)(\varphi_{0j}-\varphi_{-1j})}{\varphi_{-1j}+\rho+h_j+\varphi_{0j}}$.

Proof. From (17) the first derivative of φ_{0j} with respect to h_j is

$$\frac{\partial \varphi_{0j}}{\partial h_j} = -1 + \frac{\rho + h_j}{\sqrt{(\rho + h_j)^2 + 2\,\beta_j\,(\pi_{1j} - \pi_{0j})}} = \frac{\rho + h_j - \sqrt{(\rho + h_j)^2 + 2\,\beta_j\,(\pi_{1j} - \pi_{0j})}}{\sqrt{(\rho + h_j)^2 + 2\,\beta_j\,(\pi_{1j} - \pi_{0j})}}\,,$$

which again using (17) becomes

$$\frac{\partial \varphi_{0j}}{\partial h_j} = -\frac{\varphi_{0j}}{\varphi_{0j} + \rho + h_j}\,. \tag{35}$$

Since $\varphi_{0j} > 0$, it becomes apparent that this expression is strictly less than zero, so $\frac{\partial \varphi_{0j}}{\partial h_j} < 0$. Moreover, for large h_j it tends to zero, so that $\lim_{h_j \to \infty} \varphi_{0j} = 0$. To see this, note that in (17) the expression under the square root is always positive and strictly greater than the negative expression before the square root, since incremental profits are always positive. Yet the latter terms fade into the background as h_j becomes large, so that the expression goes to zero. Consequently, we obtain $\lim_{h_j \to \infty} \frac{\partial \varphi_{0j}}{\partial h_j} = 0$.

Next, I take the first derivative of φ_{-1j} with respect to h_j. From (18) we get

$$\frac{\partial \varphi_{-1j}}{\partial h_j} = -1 - \frac{\partial \varphi_{0j}}{\partial h_j} + \frac{(\rho + h_j + \varphi_{0j})\left(1 + \frac{\partial \varphi_{0j}}{\partial h_j}\right) + \varphi_{0j}\,\frac{\partial \varphi_{0j}}{\partial h_j}}{\sqrt{(\rho + h_j(\tau_j) + \varphi_{0j})^2 + 2\,\beta_j\,(\pi_{0j} - \pi_{-1j}) + (\varphi_{0j})^2}}\,. \tag{36}$$

For the sake of clarity, I set $\sqrt{(\rho + h_j(\tau_j) + \varphi_{0j})^2 + 2\,\beta_j\,(\pi_{0j} - \pi_{-1j}) + (\varphi_{0j})^2} = \sqrt{\psi}$, so that again from (18) we can write $\sqrt{\psi} = \varphi_{-1j} + \rho + h_j + \varphi_{0j}$. By also using (35), we can further simplify the expression to

$$\begin{aligned}
\frac{\partial \varphi_{-1j}}{\partial h_j} &= -\frac{\sqrt{\psi} - \sqrt{\psi}\left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right) - (\rho + h_j) - \varphi_{0j}\left(-\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)}{\sqrt{\psi}} \\
&= -\frac{\varphi_{-1j} + \varphi_{0j} + \varphi_{0j}\left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right) - \sqrt{\psi}\left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)}{\varphi_{-1j} + \rho + h_j + \varphi_{0j}} \\
&= -\frac{\varphi_{-1j} + \left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)(\varphi_{0j} - \varphi_{-1j})}{\varphi_{-1j} + \rho + h_j + \varphi_{0j}}\,.
\end{aligned} \tag{37}$$

Furthermore, analogue to above, it can be shown that $\lim_{h_j \to \infty} \frac{\partial \varphi_{-1j}}{\partial h_j} = 0$. This immediately follows from the fact that $\lim_{h_j \to \infty} \varphi_{0j} = 0$ and $\lim_{h_j \to \infty} \varphi_{-1j} = 0$. □

A.5

Objective. Show that $\frac{\partial g_j^*}{\partial h_j} = \frac{2\,\varphi_{0j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma}{(2\,\varphi_{0j}+\varphi_{-1j}+h_j)^2}\left[\frac{\varphi_{-1j}+h_j}{\varphi_{0j}}\,\left(\frac{\partial\varphi_{0j}}{\partial h_j}\right)+\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j}\,\left(\frac{\partial\varphi_{-1j}}{\partial h_j}+1\right)\right]$.

Proof. The first derivative of (21) is

$$\frac{\partial g_j^*}{\partial h_j} = \frac{2\,\frac{\partial\varphi_{0j}}{\partial h_j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma}{2\,\varphi_{0j}+\varphi_{-1j}+h_j} + \frac{2\,\varphi_{0j}\,ln\,\gamma\left[-\frac{\partial\varphi_{0j}}{\partial h_j}+\frac{(\rho+h_j+\varphi_{0j})\left(1+\frac{\partial\varphi_{0j}}{\partial h_j}\right)+\varphi_{0j}\,\frac{\partial\varphi_{0j}}{\partial h_j}}{\sqrt{(\rho+h_j+\varphi_{0j})^2+2\,\beta_j\,(\pi_{0j}-\pi_{-1j})+(\varphi_{0j})^2}}\right]}{2\,\varphi_{0j}+\varphi_{-1j}+h_j}$$
$$-\frac{2\,\varphi_{0j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma\left[\frac{\partial\varphi_{0j}}{\partial h_j}+\frac{(\rho+h_j+\varphi_{0j})\left(1+\frac{\partial\varphi_{0j}}{\partial h_j}\right)+\varphi_{0j}\,\frac{\partial\varphi_{0j}}{\partial h_j}}{\sqrt{(\rho+h_j+\varphi_{0j})^2+2\,\beta_j\,(\pi_{0j}-\pi_{-1j})+(\varphi_{0j})^2}}\right]}{\left(2\,\varphi_{0j}+\varphi_{-1j}+h_j\right)^2}\,. \tag{38}$$

We can use equations (35) and (37) to further simplify (38). Note the similarity of (36) and the complicated expressions in the squared brackets in (38). Hence, we can utilize (37) (as a more manageable (36)) and substitute the expressions in squared brackets by $\frac{\partial\varphi_{-1j}}{\partial h_j}$ plus the respectively missing terms, so that

$$\begin{aligned}
\frac{\partial g_j^*}{\partial h_j} &= \frac{2\,\frac{\partial\varphi_{0j}}{\partial h_j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma}{2\,\varphi_{0j}+\varphi_{-1j}+h_j} + \frac{2\,\varphi_{0j}\,ln\,\gamma\left[\frac{\partial\varphi_{-1j}}{\partial h_j}+1\right]}{2\,\varphi_{0j}+\varphi_{-1j}+h_j} - \frac{2\,\varphi_{0j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma\left[\frac{\partial\varphi_{-1j}}{\partial h_j}+2\,\frac{\partial\varphi_{0j}}{\partial h_j}+1\right]}{\left(2\,\varphi_{0j}+\varphi_{-1j}+h_j\right)^2}\\
&= \frac{2\,\varphi_{0j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma}{2\,\varphi_{0j}+\varphi_{-1j}+h_j}\left[\frac{\partial\varphi_{0j}}{\partial h_j}\,\frac{1}{\varphi_{0j}}+\frac{\left(\frac{\partial\varphi_{-1j}}{\partial h_j}+1\right)}{\varphi_{-1j}+h_j}-\frac{\left(\frac{\partial\varphi_{-1j}}{\partial h_j}+2\,\frac{\partial\varphi_{0j}}{\partial h_j}+1\right)}{2\,\varphi_{0j}+\varphi_{-1j}+h_j}\right]\\
&= \frac{2\,\varphi_{0j}\,(\varphi_{-1j}+h_j)\,ln\,\gamma}{(2\,\varphi_{0j}+\varphi_{-1j}+h_j)^2}\left[\frac{\varphi_{-1j}+h_j}{\varphi_{0j}}\,\left(\frac{\partial\varphi_{0j}}{\partial h_j}\right)+\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j}\,\left(\frac{\partial\varphi_{-1j}}{\partial h_j}+1\right)\right]\,.
\end{aligned}$$

□

A.6

Objective. Show that *ceteris paribus* the growth rate exhibits an ambivalent reaction to changes of h_j.

Proof. Based on (22), it can be shown that $\frac{\partial g_j^*}{\partial h_j} > 0$ for small h_j. Since the first term on the r.h.s. of (22) is always positive, it is sufficient to scrutinize the sign of the second term on the r.h.s. (term in squared brackets). So, using Proposition 3 and (35) and (37), it should be true that for small h_j

$$\frac{\varphi_{-1j}+h_j}{\varphi_{0j}}\,\left(-\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)+\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j}\,\left(1-\frac{\varphi_{-1j}+\left(\frac{\varphi_{0j}}{\varphi_{0j}+\rho+h_j}\right)\,(\varphi_{0j}-\varphi_{-1j})}{\varphi_{-1j}+\rho+h_j+\varphi_{0j}}\right) > 0\,.$$

Since we investigate the behavior of $\frac{\partial g_j^*}{\partial h_j}$ for small h_j, it is sufficient to show that the inequality is greater than zero for $h_j = 0$. The reason for this is that $h_j = 0$ corresponds to a point compared to which the inflection point of the inverted-U, constituting the ambivalent behavior of g_j^*, is certainly on the right. Setting h_j to zero yields

$$\begin{aligned}
\frac{2\,\varphi_{0j}}{\varphi_{-1j}}\,\left(\frac{(\varphi_{0j}+\rho)^2-\varphi_{0j}\,(\varphi_{0j}-\varphi_{-1j})}{(\varphi_{-1j}+\varphi_{0j}+\rho)\,(\varphi_{0j}+\rho)}\right)-\frac{\varphi_{-1j}}{\varphi_{0j}+\rho} &> 0\\
\frac{2\,\varphi_{0j}}{\varphi_{-1j}}\,\frac{2\,\varphi_{0j}\,\rho+\rho^2+\varphi_{0j}\,\varphi_{-1j}}{\varphi_{-1j}\,\varphi_{0j}+\varphi_{0j}^2+2\,\varphi_{0j}\,\rho+\varphi_{-1j}\,\rho+\rho^2} &> \frac{\varphi_{-1j}}{\varphi_{0j}+\rho}\\
\frac{2\,\varphi_{0j}}{\varphi_{-1j}}\,\frac{3\,\varphi_{0j}\,\rho^2+2\,\varphi_{0j}^2\,\rho+\varphi_{0j}^2\,\varphi_{-1j}+\rho^3+\varphi_{0j}\,\varphi_{-1j}\,\rho}{\varphi_{-1j}^2\,\varphi_{0j}+\varphi_{0j}^2\,\varphi_{-1j}+2\,\varphi_{0j}\,\varphi_{-1j}\,\rho+\varphi_{-1j}^2\,\rho+\varphi_{-1j}\,\rho^2} &> 1\,.
\end{aligned} \tag{39}$$

In the last line it becomes apparent, that ρ simply scales up the expression on the l.h.s. of (39). This is due to the fact that $\frac{2\,\varphi_{0j}}{\varphi_{-1j}}\,\frac{3\,\varphi_{0j}\,\rho^2+2\,\varphi_{0j}^2\,\rho+\varphi_{0j}^2\,\varphi_{-1j}+\rho^3+\varphi_{0j}\,\varphi_{-1j}\,\rho}{\varphi_{-1j}^2\,\varphi_{0j}+\varphi_{0j}^2\,\varphi_{-1j}+2\,\varphi_{0j}\,\varphi_{-1j}\,\rho+\varphi_{-1j}^2\,\rho+\varphi_{-1j}\,\rho^2} > 0$ immediately follows from $\varphi_{0j} > 0$, $\varphi_{-1j} > 0$, and $\rho \geq 0$. Consequently, if the inequality (39) holds for $\rho = 0$, then it must hold for $\rho > 0$ a fortiori. Again, we are able to reduce the expression by setting an exogenous parameter to zero, so that $\rho = 0$. This yields

$$\frac{2\,\varphi_{0j}^3\,\varphi_{-1j}}{\varphi_{-1j}^3\,\varphi_{0j}+\varphi_{0j}^2\,\varphi_{-1j}^2} > 1\ . \tag{40}$$

If this inequality is true, $\frac{\partial g_j^*}{\partial h_j} > 0$ holds for $h_j = 0$ under the condition that $\varphi_{0j} > \varphi_{-1j}$ (Proposition 3). We can prove this by contradiction, supposing that $\varphi_{0j} \leq \varphi_{-1j}$. In this case (40) becomes

$$a)\ \ \varphi_{0j} = \varphi_{-1j} \quad \Rightarrow \quad \frac{2\,\varphi_{0j}^3\,\varphi_{0j}}{\varphi_{0j}^3\,\varphi_{0j}+\varphi_{0j}^2\,\varphi_{0j}^2} > 1 \quad \Leftrightarrow \quad \frac{2\,\varphi_{0j}^4}{2\,\varphi_{0j}^4} > 1 \quad \Leftrightarrow \quad 1 > 1 \quad \rightarrow \ \textit{contradiction}$$

$$b)\ \ \varphi_{0j} < \varphi_{-1j} \quad \Rightarrow \quad \frac{2\,\varphi_{0j}^3\,(\varphi_{0j}+\varepsilon)}{(\varphi_{0j}+\varepsilon)^3\,\varphi_{0j}+\varphi_{0j}^2\,(\varphi_{0j}+\varepsilon)^2} > 1 \quad \Leftrightarrow \quad \frac{2\,\varphi_{0j}^4+2\,\varphi_{0j}^3\,\varepsilon}{2\,\varphi_{0j}^4+5\,\varphi_{0j}^3\,\varepsilon+4\,\varphi_{0j}^2\,\varepsilon^2+\varphi_{0j}\,\varepsilon^3} > 1$$

$$\Leftrightarrow \quad 2\,\varphi_{0j}^4+2\,\varphi_{0j}^3\,\varepsilon > 2\,\varphi_{0j}^4+2\,\varphi_{0j}^3\,\varepsilon+\Gamma \quad \rightarrow \ \textit{contradiction}$$

where $\Gamma = 3\,\varphi_{0j}^3\,\varepsilon+4\,\varphi_{0j}^2\,\varepsilon^2+\varphi_{0j}\,\varepsilon^3 > 0$. b) uses the fact that, if $\varphi_{0j} < \varphi_{-1j}$, there is an infinitesimal $\varepsilon > 0$ that can be added to φ_{0j} in order to yield φ_{-1j}, which implies that $(\varphi_{0j}+\varepsilon) = \varphi_{-1j}$. Since $\varphi_{0j} > 0$ and $\varepsilon > 0$, Γ must be strictly greater than zero, so the right hand side cannot be smaller than the left hand side. Hence, we have established that $\frac{\partial g_j^*}{\partial h_j} > 0$, if h_j is sufficiently small.

Finally, we can complete proving the ambivalence by showing that $\frac{\partial g_j^*}{\partial h_j} < 0$ for large h_j. Using (22) and inserting (35) and (37), we can write

$$\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j}\left(\frac{(\varphi_{0j}+\rho+h_j)^2-\varphi_{0j}\,(\varphi_{0j}-\varphi_{-1j})}{(\varphi_{-1j}+\varphi_{0j}+\rho+h_j)\,(\varphi_{0j}+\rho+h_j)}\right)-\frac{\varphi_{-1j}+h_j}{\varphi_{0j}+\rho+h_j} < 0$$

As before, we set $\rho = 0$ and simplify the expression to

$$\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j}\left(\frac{2\,\varphi_{0j}\,h_j+h_j^2+\varphi_{0j}\,\varphi_{-1j}}{(\varphi_{-1j}+\varphi_{0j}+h_j)\,(\varphi_{0j}+h_j)}\right) < \frac{\varphi_{-1j}+h_j}{\varphi_{0j}+h_j}\ . \tag{41}$$

If the inequality holds for $h_j = \infty$, there is at least one h_j for which the direction of the above unequal sign is reversed and the ambivalence exists. Taking the limit of the right hand side of (41) yields

$$\lim_{h_j\to\infty}\frac{\varphi_{-1j}+h_j}{\varphi_{0j}+h_j} = 1\ ,$$

since, as we have seen before, $\lim\limits_{h_j\to\infty}\varphi_{0j} = 0$ and $\lim\limits_{h_j\to\infty}\varphi_{-1j} = 0$. Taking the limit of the first term on the left hand side gives us

$$\lim_{h_j\to\infty}\frac{2\,\varphi_{0j}}{\varphi_{-1j}+h_j} = 0\ ,$$

because the numerator tends to zero for large h_j. This leaves us with enough information to confirm that inequality (41) holds true, because we have $0\cdot\left(\frac{2\,\varphi_{0j}\,h_j+h_j^2+\varphi_{0j}\,\varphi_{-1j}}{(\varphi_{-1j}+\varphi_{0j}+h_j)\,(\varphi_{0j}+h_j)}\right) < 1$. Consequently, we can state that, while for small h_j, $\frac{\partial g_j^*}{\partial h_j} > 0$, for large h_j we have $\frac{\partial g_j^*}{\partial h_j} < 0$. Changes in patent protection, therefore, cause growth to exhibit an ambivalent reaction. □

Appendix B (section 3)

B.1

Objective. Show that $\frac{\partial(\pi_{1j}-\pi_{0j})}{\partial\alpha_j} > \frac{\partial(\pi_{0j}-\pi_{-1j})}{\partial\alpha_j}$.

Proof. According to (8), we can write the incremental profits as

$$\pi_{1j} - \pi_{0j} = \frac{1 + (1-\alpha_j)\left(\gamma^{-1}\right)^{\alpha_j}}{\left[1 + (\gamma^{-1})^{\alpha_j}\right]^2} - \left(\frac{1}{2} - \frac{1}{4}\,\alpha_j\right)$$

and

$$\pi_{0j} - \pi_{-1j} = \left(\frac{1}{2} - \frac{1}{4}\right)\alpha_j + \frac{1 + (1-\alpha_j)\,(\gamma)^{\alpha_j}}{\left[1 + (\gamma)^{\alpha_j}\right]^2}\ .$$

The first derivative of $(\pi_{1j} - \pi_{0j})$ w.r.t. α_j is

$$\frac{\partial(\pi_{1j} - \pi_{0j})}{\partial\alpha_j} = \frac{1}{4} + \frac{(1-\alpha_j)\,(\frac{1}{\gamma})^{\alpha_j}\, ln\,(\frac{1}{\gamma}) - (\frac{1}{\gamma})^{\alpha_j}}{\left[1 + (\frac{1}{\gamma})^{\alpha_j}\right]^2} - \frac{2\,\left[1 + (1-\alpha_j)\,(\frac{1}{\gamma})^2\right]\,(\frac{1}{\gamma})^{\alpha_j}\, ln\,(\frac{1}{\gamma})}{\left[1 + (\frac{1}{\gamma})^{\alpha_j}\right]^3}$$

which, for notational simplicity, we rewrite to $\frac{\partial(\pi_{1j}-\pi_{0j})}{\partial\alpha_j} = \frac{1}{4} + \mathcal{B}(\gamma^{-1}) - \mathcal{C}(\gamma^{-1})$. Similarly, we take the first derivative of $(\pi_{0j} - \pi_{-1j})$ w.r.t. α_j, which yields

$$\frac{\partial(\pi_{0j} - \pi_{-1j})}{\partial\alpha_j} = -\frac{1}{4} - \frac{(1-\alpha_j)\,(\gamma)^{\alpha_j}\, ln\,(\gamma) - (\gamma)^{\alpha_j}}{\left[1 + (\gamma)^{\alpha_j}\right]^2} + \frac{2\,\left[1 + (1-\alpha_j)\,(\gamma)^2\right]\,(\gamma)^{\alpha_j}\, ln\,(\gamma)}{\left[1 + (\gamma)^{\alpha_j}\right]^3}\ .$$

Again, we simplify the following notations by rewriting to $\frac{\partial(\pi_{0j}-\pi_{-1j})}{\partial\alpha_j} = -\frac{1}{4} - \mathcal{B}(\gamma) + \mathcal{C}(\gamma)$.

Since γ is strictly greater than 1 and $-\,ln(\frac{1}{\gamma}) > 0$, it must be true that $-\mathcal{C}(\gamma^{-1}) > \mathcal{C}(\gamma)$. In order to prove $\frac{\partial(\pi_{1j}-\pi_{0j})}{\partial\alpha_j} > \frac{\partial(\pi_{0j}-\pi_{-1j})}{\partial\alpha_j}$, we can, therefore, focus on showing that $\frac{1}{4} + \mathcal{B}(\gamma^{-1}) > -\frac{1}{4} - \mathcal{B}(\gamma)$. Due to the fact that

$$\lim_{\gamma\to 1} \mathcal{B}(\gamma^{-1}) = -0.25 \quad \text{and} \quad \lim_{\gamma\to\infty} \mathcal{B}(\gamma^{-1}) = 0$$

and similarly

$$\lim_{\gamma\to 1} \mathcal{B}(\gamma) = -0.25 \quad \text{and} \quad \lim_{\gamma\to\infty} \mathcal{B}(\gamma) = 0\ ,$$

and because $\mathcal{B}(\gamma^{-1})$ and $\mathcal{B}(\gamma)$ are always smaller than zero, it becomes apparent that for the given assumption of $\gamma > 1$

$$\frac{1}{4} + \mathcal{B}(\gamma^{-1}) > 0 \quad \text{while} \quad -\frac{1}{4} - \mathcal{B}(\gamma) < 0\ .$$

Hence, we have shown that

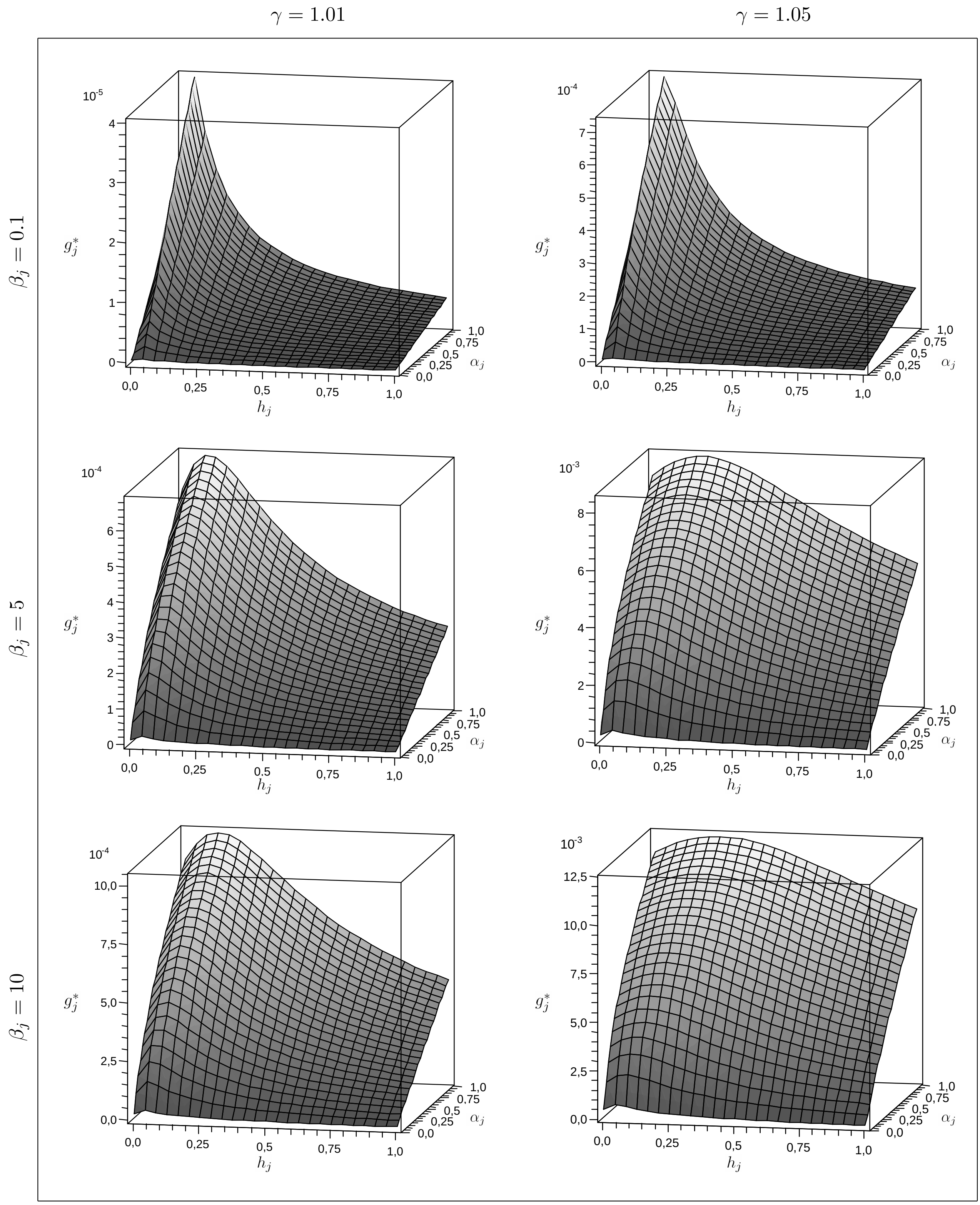
$$\frac{\partial(\pi_{1j} - \pi_{0j})}{\partial\alpha_j} = \frac{1}{4} + \mathcal{B}(\gamma^{-1}) - \mathcal{C}(\gamma^{-1}) > \frac{\partial(\pi_{0j} - \pi_{-1j})}{\partial\alpha_j} = -\frac{1}{4} - \mathcal{B}(\gamma) + \mathcal{C}(\gamma)\ ,$$

which concludes the proof. □

B.2

Objective. Show that Results 1 and 2 are robust to variations of the extant parameters (γ and β_j).

Numerical Calibration. Besides the benchmark case of $\gamma_{benchmark} = 1.1$, I will vary γ similarly to the calibration in Acemoglu/Akcigit (2008), where $\gamma = 1.01$, $\gamma = 1.05$ and $\gamma = 1.2$. Also, apart from $\beta_{j,benchmark} = 5$, I will check for robustness using $\beta_j = 0.1$ and $\beta_j = 10$. Note that since Mehra/Prescott (1985) found that $r(t) = 0.07$, I fix ρ to that rate throughout the calibration.



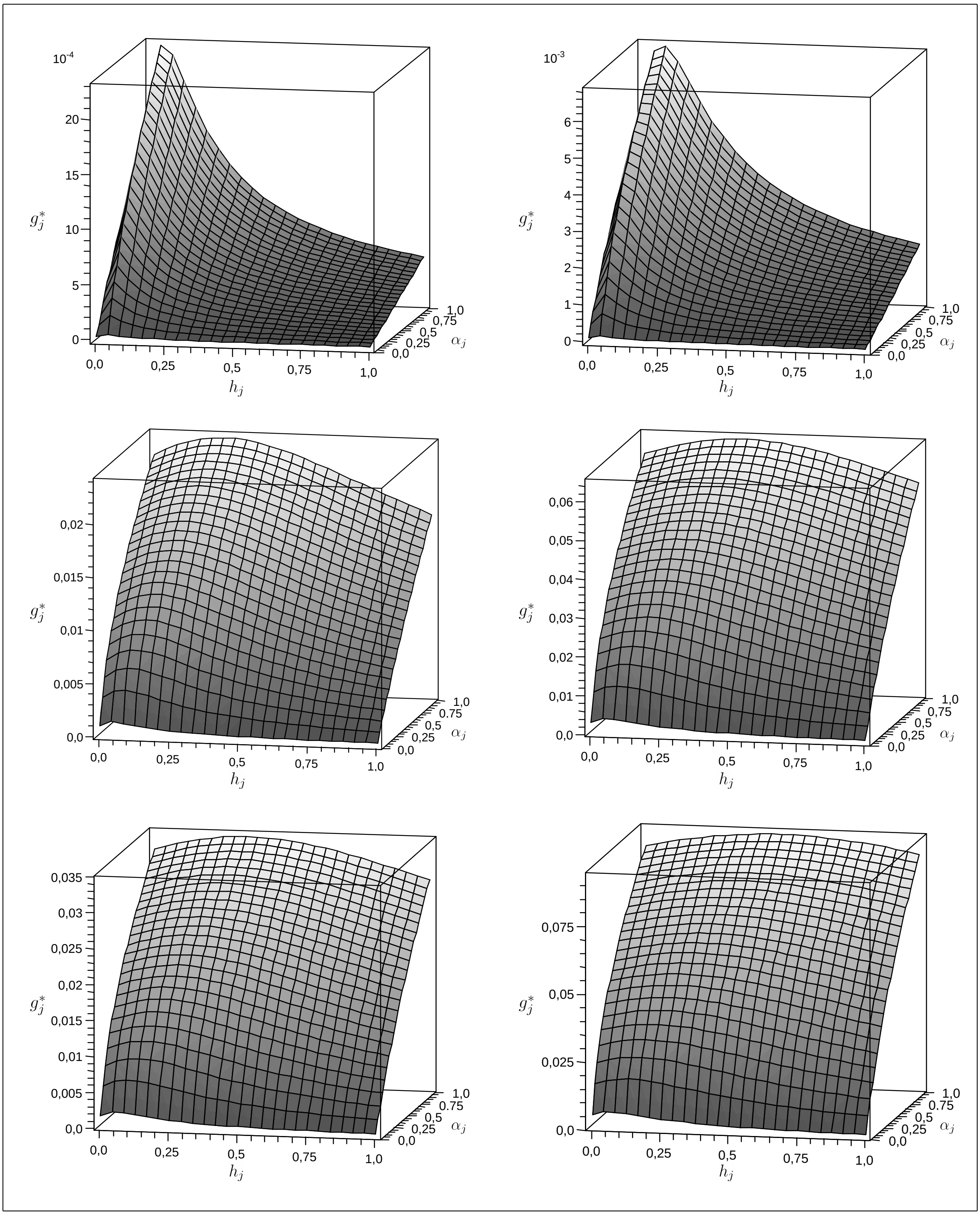
Note that I chose to keep the scale of h_j fixed in order to ensure comparability, although in the cases where $\beta_j = 0.1$ Results 1 and 2 could be seen more easily by rescaling h_j.

$\gamma = 1.1$ $\gamma = 1.2$

$\beta_j = 0.1$

$\beta_j = 5$

$\beta_j = 10$



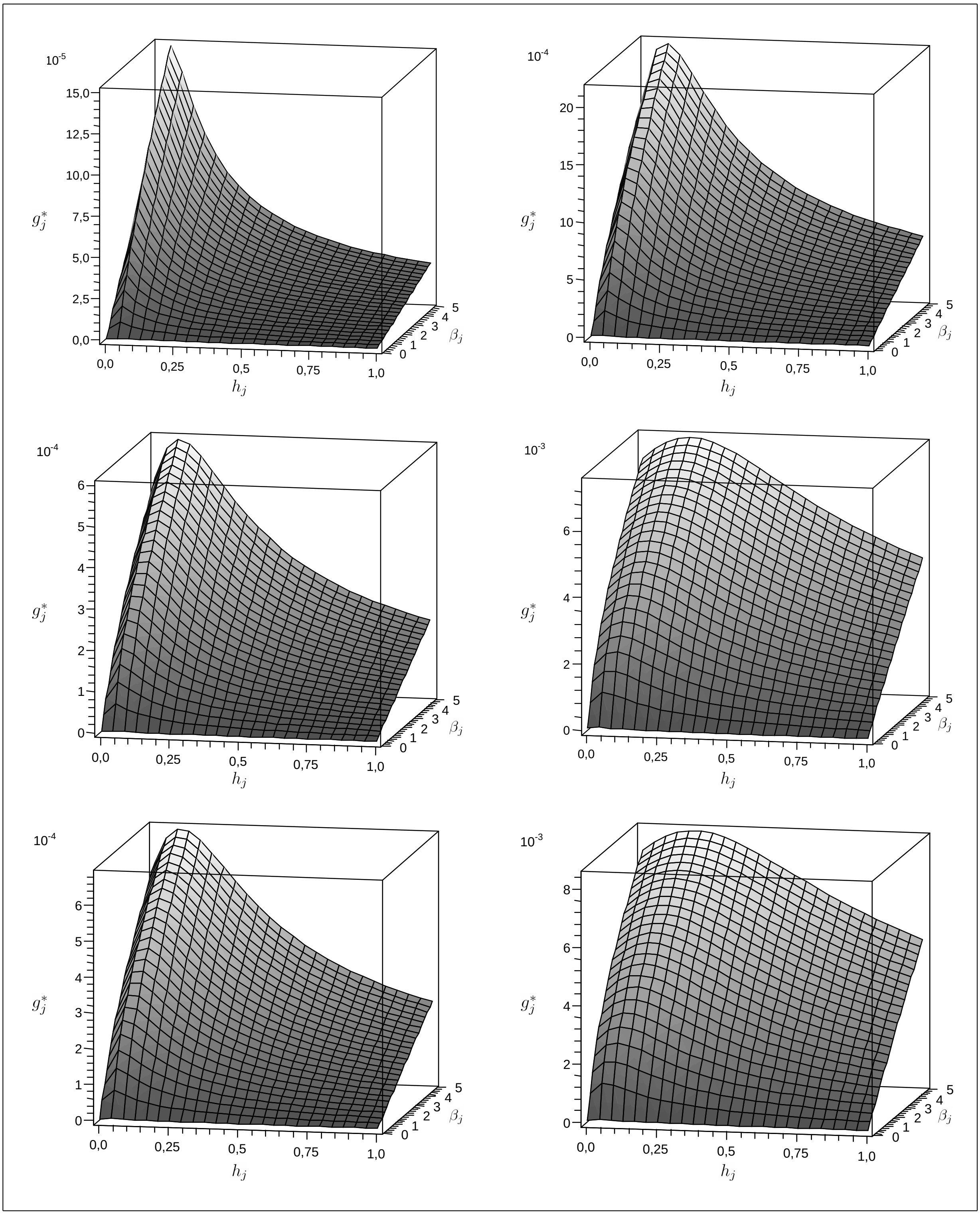
It becomes apparent that the inverted-U relation between growth (Result 1) and the direction of its reaction to changes in product market competition α_j (Result 2) are robust to several parameter variations.

B.3

Objective. Show that Results 1 and 3 are robust to variations of the extant parameters (γ and α_j).

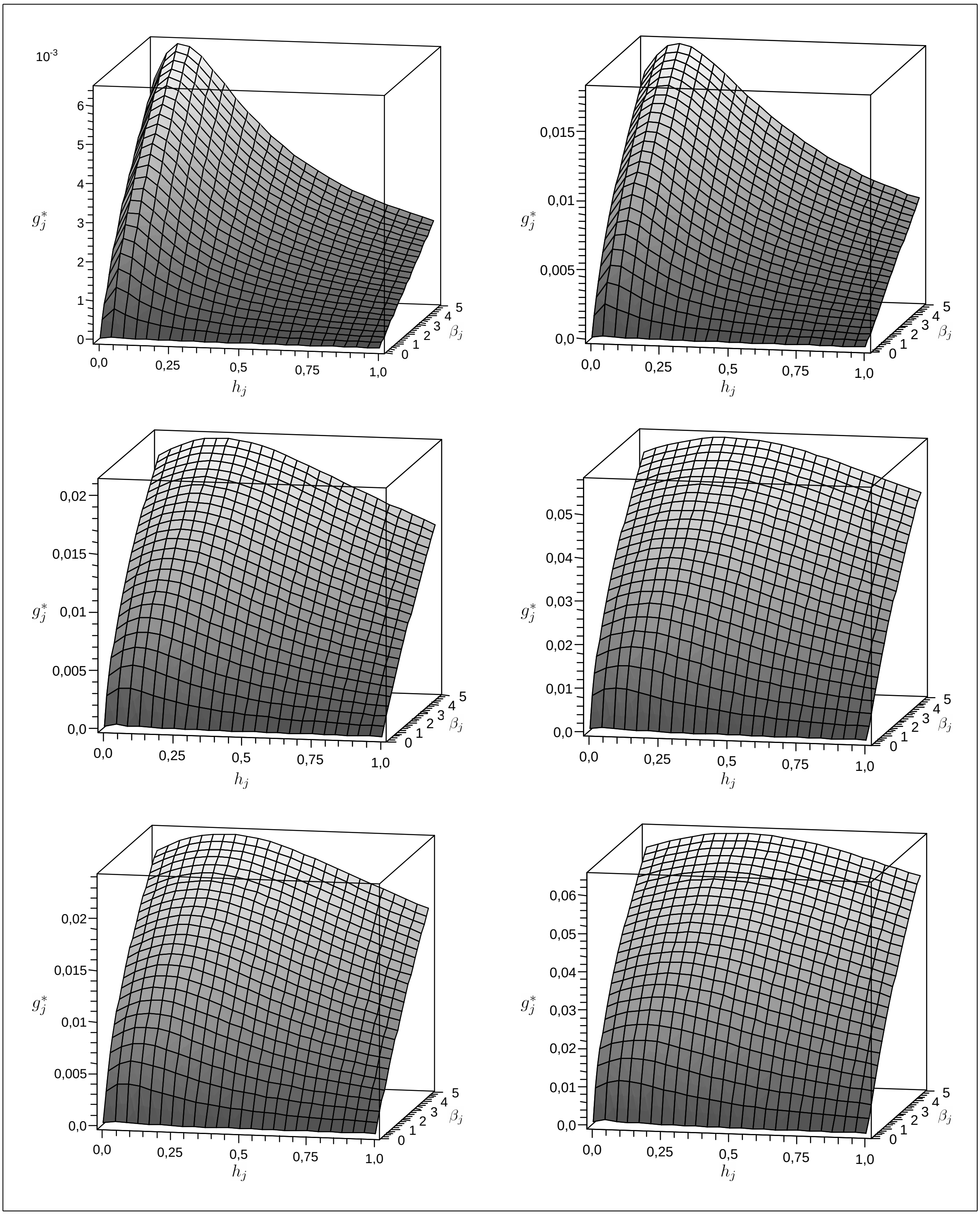
Numerical Calibration. Besides the benchmark case of $\gamma_{benchmark} = 1.1$, I will vary γ similarly to the calibration in Acemoglu/Akcigit (2008), where $\gamma = 1.01$, $\gamma = 1.05$ and $\gamma = 1.2$. Also, apart from $\alpha_{j,benchmark} = 0.8$, I will check for robustness using $\alpha_j = 0.1$ and $\alpha_j = 1$, since $\alpha_j \in (0, 1]$. As above, $\rho = 0.07$ throughout the calibration.

$\gamma = 1.01$ $\gamma = 1.05$



Note that, as above, I chose to keep the scale of h_j fixed in order to ensure comparability, although in the cases where $\alpha_j = 0.1$ Results 1 and 3 could be seen more easily by rescaling h_j.

$\gamma = 1.1$ $\gamma = 1.2$

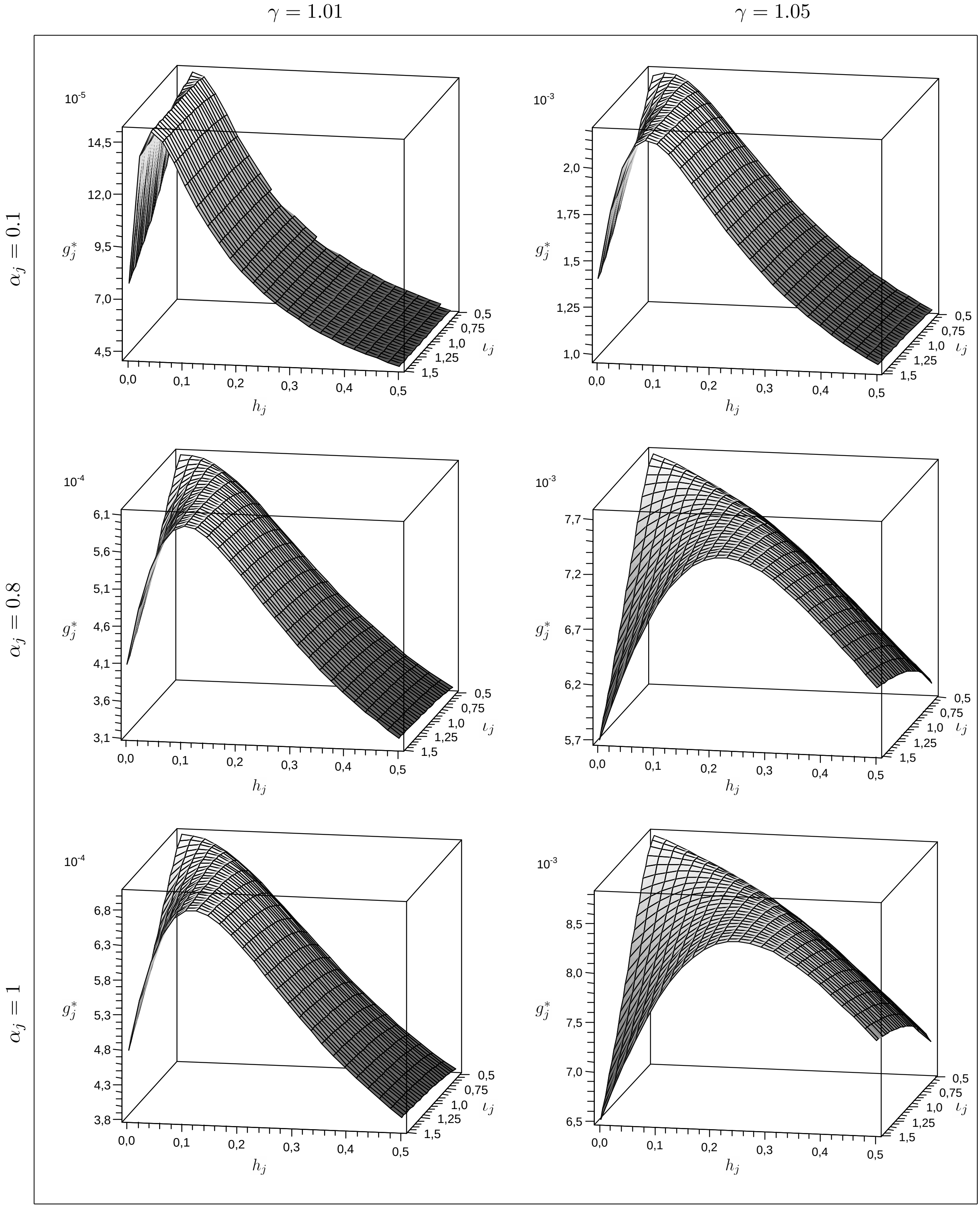


It becomes apparent that the inverted-U relation between growth (Result 1) and the direction of its reaction to changes in R&D productivity β_j (Result 3) are robust to parameter variations.

B.4

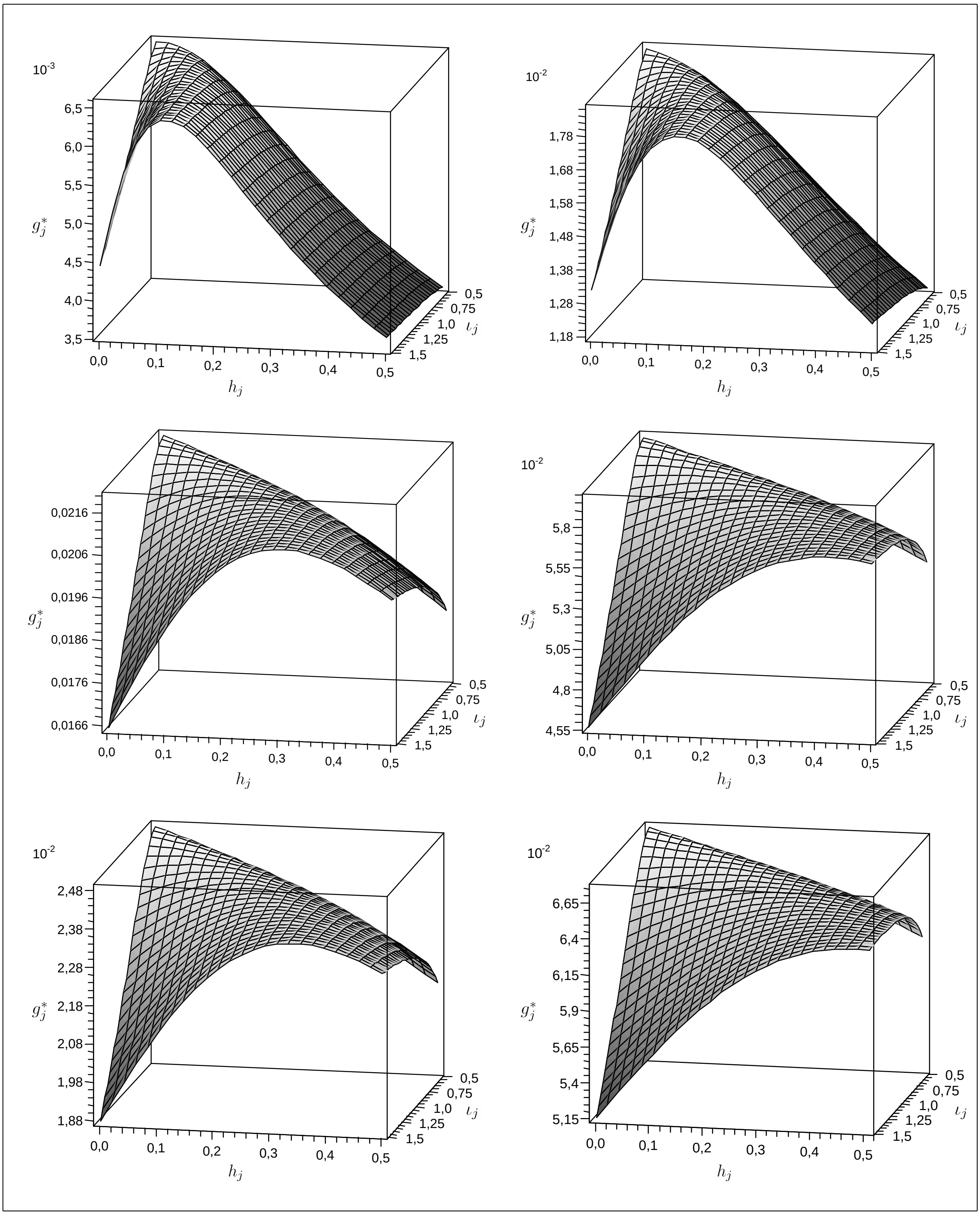
Objective. Show that Result 4 is robust to variations of the extant parameters (γ and α_j).

Numerical Calibration. Besides the benchmark case of $\gamma_{benchmark} = 1.1$, I will vary γ similarly to the calibration in Acemoglu/Akcigit (2008), where $\gamma = 1.01$, $\gamma = 1.05$ and $\gamma = 1.2$. Also, apart from $\alpha_{j,benchmark} = 0.8$, I will check for robustness using $\alpha_j = 0.1$ and $\alpha_j = 1$, since $\alpha_j \in (0, 1]$. As above, $\rho = 0.07$ throughout the calibration and β_j will be fixed to 1.



Note that, as above, I chose to keep the scale of h_j fixed in order to ensure comparability, although in the cases where $\alpha_j = 0.1$ Result 4 could be seen more easily by rescaling h_j.

$\gamma = 1.1$ $\gamma = 1.2$



It becomes apparent that the direction of the reaction of the growth rate's maximum to changes in the laggard's reverse engineering capability ι_j (Result 4) is robust to several parameter variations.

B.5

Objective. Under the premise that the imitator's R&D costs follow (26), show that the optimal research intensity of a neck-to-neck firm is given by (27) and the optimal research intensity of a following firm is given by (28).

Proof. Recall from section 2.4.2 that the leading firm's value function is

$$\rho\, V_1 = \pi_{1j} - (\widetilde{\varphi}_{-1j} + h_j)\,[V_1 - V_0]\ , \tag{12}$$

and the neck-to-neck firms' value function to

$$\rho\, V_0 = \left[\pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j}\right] + \varphi_{0j}\,[V_1 - V_0] - \widetilde{\varphi}_{0j}\,[V_0 - V_{-1}]\ . \tag{13}$$

Due to (26) the catching-up firm's value function changes to

$$\rho\, V_{-1} = \left[\pi_{-1j} - \frac{\iota_j}{2\,\beta_j}\,(\varphi_{-1j})^2\right] + (\varphi_{-1j} + h_j)\,[V_0 - V_{-1}]\ . \tag{42}$$

While the first order condition of a neck-to-neck firms remains unchanged (see (15)), as shown in section 3.4, the laggard's first order condition becomes

$$\frac{\iota_j}{\beta_j}\,\varphi_{-1j} = (V_0 - V_{-1})\ . \tag{43}$$

Based on this, we can derive the laggard's optimal research intensity. Using (43) and inserting (13) and (42) we can write

$$\begin{aligned}\frac{\iota_j}{\beta_j}\,\varphi_{-1j} \quad &= \quad \frac{1}{\rho}\left[\pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j} + \varphi_{0j}\,[V_1 - V_0] - \widetilde{\varphi}_{0j}\,[V_0 - V_{-1}]\right] \\ &\quad -\frac{1}{\rho}\left[\pi_{-1j} - \frac{\iota_j}{2\beta_j}\,(\varphi_{-1j})^2 + (\varphi_{-1j} + h_j)\,[V_0 - V_{-1}]\right]\ .\end{aligned}$$

As in section 2.4.2 I proceed by using (15) and (43) in order to simplify to

$$\rho\,\frac{\iota_j}{\beta_j}\,\varphi_{-1j} = \pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j} + \frac{(\varphi_{0j})^2}{\beta_j} - \widetilde{\varphi}_{0j}\,\frac{\iota_j}{\beta_j}\,\varphi_{-1j} - \left[\pi_{-1j} - \frac{\iota_j}{2\,\beta_j}\,(\varphi_{-1j})^2 + (\varphi_{-1j} + h_j)\,\frac{\iota_j}{\beta_j}\,\varphi_{-1j}\right]$$

Rearranging this to

$$0 = (\varphi_{-1j})^2 + 2\,\varphi_{-1j}\,(\rho + h_j + \varphi_{0j}) - \frac{2\,\beta_j}{\iota_j}\,(\pi_{0j} - \pi_{-1j}) - \frac{1}{\iota_j}\,(\varphi_{0j})^2$$

enables us to solve for the alternative profit maximizing R&D intensity of the laggard

$$\varphi_{-1j} = -(\rho + h_j + \varphi_{0j}) + \sqrt{(\rho + h_j + \varphi_{0j})^2 + \frac{2\,\beta_j}{\iota_j}\,(\pi_{0j} - \pi_{-1j}) + \frac{1}{\iota_j}\,(\varphi_{0j})^2}\ .$$

Similarly, based on the neck-to-neck first order condition (15), we can use (12) and (13) to write

$$\frac{\varphi_{0j}}{\beta_j} = \frac{1}{\rho}\,[\pi_{1j} - (\widetilde{\varphi}_{-1j} + h_j)\,[V_1 - V_0]] - \frac{1}{\rho}\left[\pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j} + \varphi_{0j}\,[V_1 - V_0] - \widetilde{\varphi}_{0j}\,[V_0 - V_{-1}]\right]\ .$$

Again, I utilize (15) and (43) to simplify to

$$\rho\,\frac{\varphi_{0j}}{\beta_j} = \pi_{1j} - (\widetilde{\varphi}_{-1j} + h_j)\,\frac{\varphi_{0j}}{\beta_j} - \left[\pi_{0j} - \frac{(\varphi_{0j})^2}{2\,\beta_j} + \frac{(\varphi_{0j})^2}{\beta_j} - \widetilde{\varphi}_{0j}\,\frac{\iota_j}{\beta_j}\,\varphi_{-1j}\right] \ .$$

After rearranging this expression to

$$0 = (\varphi_{0j})^2 + 2\,\varphi_{0j}\,[\rho + h_j + \varphi_{-1j}\,(1-\iota_j)] - 2\,\beta_j\,(\pi_{1j} - \pi_{0j}) \ ,$$

we can solve for the alternative profit maximizing R&D intensity of a neck-to-neck firm

$$\varphi_{0j} = -[\rho + h_j + \varphi_{-1j}\,(1-\iota_j)] + \sqrt{[\rho + h_j + \varphi_{-1j}\,(1-\iota_j)]^2 + 2\,\beta_j\,(\pi_{1j} - \pi_{0j})} \ .$$

This concludes the proof and the Appendices. □

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