



BGPE Discussion Paper

No. 103

**Interdependence of Liquidity Problems
in the Financial Sector**

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July 2011

ISSN 1863-5733

Editor: Prof. Regina T. Riphahn, Ph.D.
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30 June 2011

Abstract

The paper analyses problems arising from the interdependence of liquidity provision in the financial system. Findings document, that liquidity shortage of minor financial players can translate into liquidity shortage for systemic relevant players, thereby putting the proper functioning of the overall financial system on the line. As contractual mechanisms to safeguard against this threat are absent in the wake of market frictions, prudent regulation and governmental provision of external liquidity is asked for in order to solve these problems.

JEL classification: G01, G21

Keywords: Illiquidity, Insolvency, Contagion

I. Introduction

Unlucky investment decisions can trigger liquidity problems for financial players. While contractual agreements can prevent these problems to spread to the overall financial system in frictionless markets, they don't work if market mechanisms are disturbed. This model shows how frictions can render risk insuring state contingent contracts infeasible. Instead, it will be demonstrated that a debt contract is dominating in such a market place. However, it turns out, that a debt contract, while insuring the lender in normal times, can lead to spreading illiquidity in the wake of liquidity shocks.

In the second section, the general setup of the model is explained and it is shown that there are efficient state contingent contracts, which can prevent perturbations in a frictionless world and consequently safeguard systemic relevant institutions from default.

However, the implementation of frictions in form of missing management incentives renders the before efficient state contingent contracts inefficient and reveals the incentive of rational players to deviate to other forms of contractual agreements. The dominance of a debt contract is derived in section three.

Recent history proved that financial institutions are not always prepared to handle critical situations, whose occurrence is considered to be too unrealistic. Consequently, the incidence of such a worst case scenario might lead to severe financial disturbances. The debt contract is designed to make liquidity problems of systemic relevant players extremely improbable, however allowing for shortages in very rare cases. This problematic is addressed in the fourth section.

Section five outlines the differences between idiosyncratic and aggregate shocks. All issues already present without an aggregate shock can easily be shown to worsen dramatically.

In section six, policy recommendations are provided to avert the threats from the financial system. Even though the liquidity situation of minor players in this model only depends on good and bad luck (i.e. a good or bad shock), it can nevertheless be shown, that prudent regulatory action can allow for more unobstructed functioning of the financial system. Specifically, higher equity capital requirements for currently less regulated minor financial service providers and governmental provision of liquidity in shock situations can break up the interdependence of liquidity problems between minor players and systemic relevant players.

The seventh section concludes.

There is extensive literature on reasons for liquidity shortages in the financial system, on contracts fighting these problems, the drawbacks of those contracts and on measures mitigating those drawbacks. The selected overview of related literature serves the purpose of identifying important contributions to this field, which also influenced the direction and progress of this work.

This model explains liquidity shortages resulting from a combination of asymmetric information and unlucky investment behaviour. The investment values generated turn out to be insufficient to meet all liquidity requirements when they are due, thereby rendering certain market participants liquidity constraint. Holmström and Tirole (2008) define this scenario as a shortage of inside liquidity. They argue, that too low inside liquidity can be mitigated by the supply of outside liquidity. Outside liquidity refers to liquidity provided by international markets or the government. A scenario, in which the government uses its taxation power to redistribute liquidity from consumers to the financial system once it is hit by a bad enough shock is also discussed here as a reasonable policy measure.

In the analysis of markets with frictions, debt contracts are implemented in this model, because such a contractual agreement solves problems arising from the implemented frictions. Dang, Gorton and Holmström (2009) identify debt as welfare maximising contract. In their paper it is shown that debt minimizes asymmetric information problems. Similar to this paper, they also conclude that the implementation of debt contracts can cause a systemic risk to the liquidity provision for the economy if an aggregate shock occurs. Their reason for this result lies in the fact that adverse selection is caused because debt then can become information-sensitive.

As long as investments serve as collateral for debt capital financing, the investment value has a crucial impact on the ability of market participants to borrow funds. If the investment value is high, funding is easily available. Once a negative shock devalues the investment, funding dries out and the only source left for liquidity is proceeds from the sale of assets. However, the asset sale, which is potentially carried out at undervalued prices as liquidity requirements are immediate, further debases the investment value thereby rendering debt capital financing even more difficult. Brunnermeier and Pedersen (2008) refer to this as market and funding liquidity. They model the link between the ease with which assets can be traded and the ease with which external funding can be obtained. Once a shock reduces the capital of investors, a downturn spiral due to decreased collateral value forces investors to de-lever their positions and thereby weakens the market liquidity further. Finally, this process can lead to a shortage of liquidity.

Here, the liquidation value of collateral contains a haircut and is exogenously given. Benmelech and Bergman (2010) endogenize the liquidation value of collateral, thereby implementing further frictions which influence the availability of credit to market participants relying on collateralized borrowing. Assuming that insured lenders are not able to operate the received collateral by themselves, they are forced to sell it. With limited liquidity of players capable to operate collateral, the asset values decline thereby tightening the credit constraint of borrowers. Credit traps, i.e. situations in which liquidity provided by central banks is hoarded by intermediaries instead of lent out, are described as a result of endogenized collateral values. This problem doesn't arise here; however the distribution of collateral leads to redistribution of credit provisions and thereof resulting liquidity shortages.

Ivashina and Scharfstein (2008) show that during the crisis, concretely in the last quarter of 2008, loan provision to below investment grade borrowers decreased equally strong as loans to investment grade borrowers. Consequently, different credit quality doesn't necessarily play the crucial role in the decision for debt provision. Diamond and Rajan (2009) explain illiquidity of assets as a potential reason for depressed lending. Potential lenders assess future prospects of troubled borrowers even worse and therefore expect even further depressed prices of illiquid assets. Consequently, once the holder of the illiquid asset deteriorates more, higher liquidity premiums on the illiquid assets can be gained. In comparison to that, this model describes tightened lending conditions neither as a result of credit quality nor of illiquidity. It is shown that pure chance, i.e. good and bad shocks, can bring debt availability down, once the economic situation allows for an unlucky combination of investment projects. Allen and Gale (2005) show the occurrence of price volatility and financial crises caused by defaulted debt repayments in the wake of small liquidity shocks. One of their model's crucial components is the also here implemented implications of a backward bending supply curve of assets. While in complete markets with contingent contracts no asset sale in order to meet liquidity requirements is necessary, tapping the market liquidity of assets is indispensable in incomplete markets. In order to cover the opportunity costs of holding liquidity, the suppliers charge a premium in form of low asset prices once the illiquid demander is in a fire sale situation. In this model it will be shown that such a liquidity premium in some cases is not even necessary to trigger the occurrence of a crisis.

Finally, Kiyotaki and Moore (1995) construct a model which shows persistent fluctuations in output and asset prices as a result of temporary shocks. Market participants are only allowed to borrow against assets, so as to guarantee a repayment of loans. Consequently, the value of the collateralized assets represents the credit constraints of the borrowers. However, in their

dynamic model these constraints influence the asset prices and the asset prices influence the credit constraints vice versa. Also in this model, similar endogenized credit constraints are crucial to analyze the liquidity situation of single market participants.

II. Results in a Frictionless Market

1. Setup

The market participants of this model operate in the financial system. There are risk neutral providers of debt capital, which represent the major financial players and which are assumed to be systemic relevant. One can e.g. think of commercial banks, credit unions, depository banks or other providers of credit. Among the customers demanding credit are the minor players in the financial system.

Those minor players are risk neutral financial service providers. Within this group, there is any kind of broker and dealer whose business is crucially dependent on the access to external debt capital, e.g. hedge funds, private equity companies, investment vehicles or banks with no or not noteworthy deposit business. Those companies operate in particular specialized fields, i.e. they have superior knowledge and experience in their particular fields compared to any other company not operating in this specialized area. In order to make long-term investments in projects representing their particular specialized field, the service providers use equity capital but additionally require external debt capital. The service providers can buy and sell the assets underlying their investment projects if the counterpart of the trade operates within the same specialization. A priori, except for the own specialization, all members of this second group of market participants are equal, i.e. there are no differences when it comes to experience, ability or efficiency. However, pure chance (i.e. a shock) differentiates the financial service providers into groups with lucky or unlucky investment behaviour.

The setup can best be explained with a constructed example. One can think of the financial service providers as being two private equity companies. Those two companies are located in countries B and C. Each company is specialized in investments concerning its own country and both are specialized in investments in the field of natural resources. Besides the unique experience in home country investments both companies are identical.

The long-term investment of the private equity companies consists in buying companies, collecting returns from them and afterwards selling them again. In order to realize that investments, the private equity companies rely on leverage, i.e. they need to receive some credit. The provider of debt capital can be thought of as a commercial bank, denoted as bank A.¹

Within the economy, there is a finite but large number N_0 of profitable investment opportunities which both B and C finance in equal parts. After investing, each project is represented by an asset, so the terms project and asset are interchangeable. The projects have to be financed in $t=0$ and realize their inherent value in $t=2$. Besides that payoff, each project generates a return of Ω per period. The aggregated inherent value of the entire N_0 projects within the portfolio is known in $t=0$. In detail, there are two different types of projects in the portfolio, some known fraction η of the projects has a value of Z and the remaining fraction $(1-\eta)$ has a value of $V < Z$.² In total, this yields a value of the entire portfolio provided in (1).

$$\eta N_0 Z + (1-\eta) N_0 V \tag{1}$$

In $t=0$, nobody knows which specific project will have a high value and which one will have a low value, consequently the two different types of projects are not distinguishable yet.

In $t=1$, the progress of the different projects can be observed. From that point on, the type of asset can be recognized by every market participant in the economy. Still, a fraction η of all N_0 projects has a value of Z and a fraction $(1-\eta)$ a value of V , leading to the identical portfolio value as in $t=0$, but now the two types of assets can be distinguished. Additionally, the good development of the high value projects allows them to be tradable in $t=1$. For the low value projects it is assumed, that they are not tradable.³

Continuing the example, one can think of countries B and C sharing the same border.

In $t=0$, both private equity companies B and C purchase an equally sized company for land development on its respective side of the border. Put it differently, the private

¹ The results of the model can best be derived for an example with one major financial player A and two minor players B and C. However, generalizations are possible. For an informal discussion refer to Appendix 6.

² The investment value is assumed to be ex ante determined. Even in the absence of volatility in the investment value, all further results hold. Uncertainty in the quality of the investment isn't necessary for the analysis.

³ Following Caballero and Krishnamurthy (2001), one can also think of tradable goods as being internationally demanded and of non-tradable goods as being domestically demanded.

equity companies buy units of land which formerly belonged to the land developers. There are N_0 units of land in total, half of it in country B, the other half in country C. Initially it is known that the land might have exploitable natural resources. In detail it is known, perhaps from earlier geological research, that a fraction η of the N_0 units has resources with a net worth of Z . The other fraction $(1-\eta)$ lacks resources, can only be used for construction and therefore only has a smaller net value V . The different types of units can not be distinguished before the land development has started. However, between periods $t=0$ and $t=1$ the land is developed and consequently afterwards the type of land unit becomes common knowledge. While working on both types of land, some side product can be realized on either kind of land, therefore each unit of land, independently of the type, additionally generates per period return of Ω .

From $t=1$ on, only the assets underlying the units with natural resources can be traded. Both private equity companies B and C are specialized in the field of natural resources, consequently they can trade that international assets in between themselves. The assets underlying the construction units can not be traded. Each company is only specialized in its own home country and the construction sites are immobile and country specific. As a consequence, these national assets are non-tradable.

Realizing some fraction of the N_0 projects in $t=0$ costs exactly that fraction of the value of the entire project portfolio, i.e. the price of one single of the N_0 projects is $\eta Z + (1-\eta)V$. B and C split the whole amount of projects equally in between themselves (both B and C finance $\frac{N_0}{2} = N_1$) and require debt capital in order to finance those investments, so they need to borrow from A. In order to safeguard against default, A strives to implement some mechanism which guarantees full repayment of the funds provided to B and C.

2. *State Contingent Contract*

In a frictionless world it is easy for A to offer an efficient state contingent contract to B and C. Efficient means, that A has a contractual secured guarantee, that at the end of the world, i.e. in $t=2$, it will receive full repayment of all the funds provided in $t=0$. Such a contract would work as follows: In $t=0$, A provides long-term financing (i.e. financing from $t=0$ until $t=2$) for B and C. As B and C are equal, they receive the identical amount of funds and have to put up themselves the identical amount of equity capital. Both B and C receive a fraction $\rho < 1$ of their investment capital in $t=0$ and A contractually obliges them to repay the received funds in $t=2$

conditionally on the state of the world. The timing of this kind of contracting is provided in figure one. Above the timeline, the development of the projects is explained, below the timeline, the actions and capital flows of A, B and C are described.

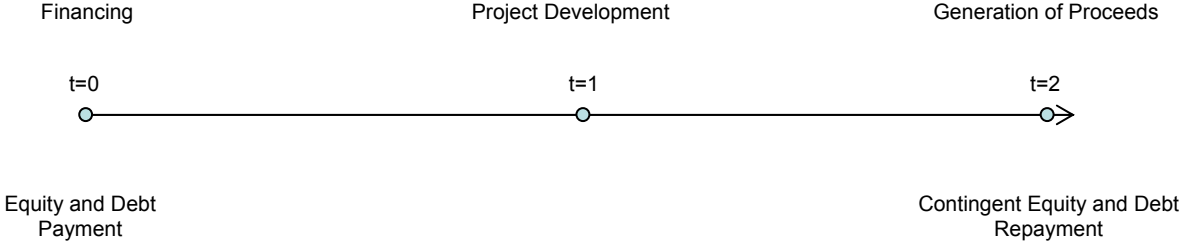


Figure 1: Timing in a state contingent contract

The whole investment volume of B and C serves as collateral for A, so that B and C can't simply run away with the money obtained. A does only provide a fraction $\rho < 1$ and not the whole financing for B and C because of limited pledgeability.⁴ The investments of B and C are worth less for A, because A isn't specialized in their particular fields of investment. Once A receives the investments as collateral, A is not able to operate those adequately and can only liquidate them with a payoff discounted by the factor ρ . Consequently, by providing exactly only a fraction of ρ as financing capital, A could still generate exactly as much proceeds from the assets as it lent to B and C, if A received all the collateral immediately after B and C invested.

In $t=2$, B and C have to repay the same fraction ρ of their proceeds. The contract is state contingent, as the repayment is higher in a good state of the world with high proceeds and lower in a bad state of the world with low proceeds. Using this kind of contract, A can totally safeguard its own interests, as full repayment of all funds is guaranteed. This is proven in the following.

In $t=0$, both B and C buy N_1 assets, and each market participant (A, B and C) expects for both B and C, that a fraction η of their investment is of high value Z and a fraction $(1-\eta)$ is of low value V . Consequently, the two portfolios of B and C combined have exactly the known aggregated value of the N_0 projects in the economy's investment portfolio given in (1). The investment of each B and C therefore costs (2).

⁴ Holmström and Tirole (2009) provide plenty of reasons defending the assumption of limited pledgeability.

$$\frac{\eta N_0 Z + (1-\eta) N_0 V}{2} = \eta N_1 Z + (1-\eta) N_1 V \quad (2)$$

For both B and C, A provides a fraction ρ of the investment capital in form of liabilities; the fraction $(1-\rho)$ needs to be equity financed. A pays out the same amount to B and C, i.e. in $t=0$ A has a total payout of (3).

$$-2[\eta N_1 Z + (1-\eta) N_1 V] \rho = -[\eta N_0 Z + (1-\eta) N_0 V] \rho \quad (3)$$

In $t=1$, the development of the assets, i.e. the type of the assets can be recognized. It turns out that one service provider (without loss of generality assumed to be B in the following) invested (without knowledge) in a lower than average fraction $\varepsilon < \eta$ of the high value asset and consequently in a higher than average fraction $(1-\varepsilon) > (1-\eta)$ of the low value asset. Contrary to that, C must have invested (also without knowledge) in a higher than average fraction $\gamma > \eta$ of the high value asset and a lower than average fraction $(1-\gamma) < (1-\eta)$ of the low value asset. The value of B's portfolio in $t=1$ is therefore lower than it was expected in $t=0$ (4a), the value of C's portfolio in $t=1$ is higher than expected in $t=0$ (4b).

$$\varepsilon N_1 Z + (1-\varepsilon) N_1 V < \eta N_1 Z + (1-\eta) N_1 V \quad (4a)$$

$$\gamma N_1 Z + (1-\gamma) N_1 V > \eta N_1 Z + (1-\eta) N_1 V \quad (4b)$$

For rational expectations, the expected value of the two investment portfolios of B and C combined must be equal in $t=0$ and $t=1$. Or put it differently, the amount of high value assets and the amount of low value assets must be equal in $t=0$ and $t=1$. This is assured by (5).

$$[\eta N_1 Z + (1-\eta) N_1 V] + [\eta N_1 Z + (1-\eta) N_1 V] = [\varepsilon N_1 Z + (1-\varepsilon) N_1 V] + [\gamma N_1 Z + (1-\gamma) N_1 V] \text{ or}$$

$$\varepsilon N_1 + \gamma N_1 = \eta(N_1 + N_1)$$

$$\Rightarrow \varepsilon = 2\eta - \gamma \quad (5)$$

However, even though the value of the investments changed for B and C in $t=1$, the credit conditions are not renegotiated, as A provided long-term financing which lasts until $t=2$.

In $t=2$, the investments yield exactly a payoff worth the assets' value in $t=1$ and a return of Ω per period. As the repayment in $t=2$ is state contingent, B might have to repay less than it received in $t=0$, and C has to repay more than it received in $t=0$. Specifically, the repayment of B and C amounts to:

$$[\varepsilon N_1 Z + (1 - \varepsilon) N_1 V] \rho + 2 N_1 \Omega \rho \quad (6a)$$

$$[\gamma N_1 Z + (1 - \gamma) N_1 V] \rho + 2 N_1 \Omega \rho \quad (6b)$$

This kind of payment schedule allows for a secured repayment of all funds for A, as the total of the repayments in $t=2$ (6a+6b) is higher than the payout in $t=0$ (3) under rational expectations (5). As a consequence, this kind of contract is efficient in the aforementioned way for A. A even receives interest amounting to $4\Omega N_1 \rho$.

3. Feasibility

The state contingent contract works as insurance. As both B and C don't know which one of them will make the unlucky investment choice, they can sign such a contract in order to insure against a loss situation. The unlucky service provider might still generate a loss in total, however it might repay less funds in $t=2$ than it received in $t=0$ (7a). The first term represents the equity payout in $t=0$, the second and third term the payment to equity in $t=2$. The lucky service provider still gains from the situation, however it has to spend some of its profits for buying an insurance which is then not used by itself, but by the unlucky one (7b).

$$\begin{aligned} & -[\eta N_1 Z + (1 - \eta) N_1 V](1 - \rho) + [\varepsilon N_1 Z + (1 - \varepsilon) N_1 V](1 - \rho) + 2 N_1 \Omega (1 - \rho) = \\ & = -(Z - V)(\eta - \varepsilon) N_1 (1 - \rho) + 2 N_1 \Omega (1 - \rho) \end{aligned} \quad (7a)$$

$$\begin{aligned} & -[\eta N_1 Z + (1 - \eta) N_1 V](1 - \rho) + [\gamma N_1 Z + (1 - \gamma) N_1 V](1 - \rho) + 2 N_1 \Omega (1 - \rho) = \\ & = (Z - V)(\gamma - \eta) N_1 (1 - \rho) + 2 N_1 \Omega (1 - \rho) \end{aligned} \quad (7b)$$

Risk neutral service providers will make that kind of deal, as for B and C there are positive expected profits of $2\Omega N_1 \rho$ in such a state contingent contract, as will be shown in the following.

As all other parameters ($N_1, Z, V, \rho, \eta, \Omega$) are exogenously ex ante given, the expected value of the profit depends on the realizations of ε and γ . Within some boundaries there are several potential ε - γ -combinations.⁵ Examining one arbitrary combination, ε_1 - γ_1 , it is straightforward that independently of the distribution of ε and γ , the probability for B to draw ε_1 is equal to the probability of C to draw γ_1 , as $\varepsilon_1=2\eta-\gamma_1$ by rational expectations (5). Reason to the same argument it must be true that also the probability for B to draw γ_1 must be equal to the probability for C to draw ε_1 . Finally, recognizing that B and C are exactly identical when drawing the projects, it is clear that also the probability for B to draw $\varepsilon_1<\eta$ must be equal to the probability for C to draw $\varepsilon_1<\eta$. This yields in total that B and C have an equal chance of drawing ε_1 and that probability is also equal for drawing γ_1 and that is also true for each other possible ε_i - γ_i -combination.

$$\text{Prob}(B = \varepsilon_i) = \text{Prob}(C = \gamma_i) = \text{Prob}(C = \varepsilon_i) = \text{Prob}(B = \gamma_i) \quad \forall i \quad (8)$$

$$\sum_{i=1}^{\infty} P(B = \varepsilon_i) = \sum_{i=1}^{\infty} P(B = \gamma_i) = \frac{1}{2}$$

Using these equal drawing probabilities (8), rational expectations (5) and the payout structures of the unlucky (7a) and the lucky service provider (7b), equation (9) shows exemplarily for B that the service providers have an expected investment value of $2N_1\Omega\rho$ in $t=0$.

$$\begin{aligned} E_0(B) &= \sum_{i=1}^{\infty} \text{Prob}(B = \varepsilon_i) [-(Z - V)(\eta - \varepsilon_i)N_1(1 - \rho) + 2N_1\Omega\rho] + \\ &+ \sum_{i=1}^{\infty} \text{Prob}(B = \gamma_i) [(Z - V)(\gamma_i - \eta)N_1(1 - \rho) + 2N_1\Omega\rho] = 2N_1\Omega\rho \end{aligned} \quad (9)$$

Concluding, one can say that in a frictionless world, the commercial banks can offer a state contingent contract which guarantees full repayment of all funds provided. In addition to that, the brokers and dealers are willing to sign such contracts, as there are positive expected profits. The state contingent contract is feasible in frictionless markets; however things change if there are frictions.

⁵ The boundaries are derived in Appendix 1.

III. Results in a Market with Missing Management Incentives

The results of section two don't hold once frictions are implemented in the model. Using a potentially bad management as a vivid example of a friction, it will be shown that a state contingent contract can't be offered anymore. The seemingly efficient debt contract which is implemented instead reveals a deceptive security.

The friction disturbing the proper functioning of the markets can be described as follows. Both B and C install managers responsible for the surveillance of diligent working within their companies. The salary of those managers is derived from the increase in equity capital in $t=2$, consequently the managers receive some form of performance payment. As long as the managers themselves work diligently, all projects develop as they should do. However, once the managers don't display the due diligence, each project sustains a fixed value reduction amounting to σ per period. Additionally, the projects won't generate a return of Ω .

Between $t=0$ and $t=1$, the managers don't know if they work for the lucky or the unlucky service provider and consequently work diligently during that period, as performance payment still seems possible for both. In $t=1$ it turns out that C is the lucky and B is the unlucky service provider. From that moment on it can be recognized that B will generate a decrease and C an increase in equity capital. While for the manager of C the performance payment is still an incentive to work diligently, the manager of B has no incentive anymore and will display a lack of diligence between $t=1$ and $t=2$. Therefore, each of the N_1 projects of B will sustain a value reduction of σ and won't generate a return during $t=1$ and $t=2$.

1. State Contingent Contract

Under a certain condition a state contingent contract delivers guaranteed losses for A due to the missing management incentives. As the management of C works diligently all the time, the repayment of C remains identical as in (6b). However, due to B's mismanagement, no return is generated in the second period and the investment value decrease in the amount of σ . Therefore, the friction decreases the repayment to (10).

$$(\varepsilon N_1(Z - \sigma) + (1 - \varepsilon)N_1(V - \sigma))\rho + N_1\Omega\rho \quad (10)$$

Using rational expectations (5), a payout situation for A as given in (11) can be shown. The first term captures the debt payout of A in $t=0$, the second and third term the repayment of B and C in $t=2$.

$$\begin{aligned}
& -2(\eta N_1 Z + (1-\eta)N_1 V)\rho + (\varepsilon N_1(Z - \sigma) + (1-\varepsilon)N_1(V - \sigma))\rho + \\
& + (\gamma N_1 Z + (1-\gamma)N_1 V)\rho + 3N_1 \Omega \rho = N_1 \rho (3\Omega - \sigma)
\end{aligned} \tag{11}$$

This payout situation represents a guaranteed loss once $3\Omega < \sigma$ is assumed. The state contingent contract delivers inefficient results for A in this case. Consequently, the potentially bad management makes it impossible for A to offer the state contingent contract. A different kind of security mechanism must instead be installed.

2. *Debt Contract*

A debt contract is another possibility for A to safeguard the funds provided to B and C. In such a contract, A lends a particular amount to both B and C and requires the investments as collateral. Consequently, B and C borrow against the expected value of their specific investment and as soon as one of them is not able to repay as the timing schedule requires, A receives the collateral and the defaulted service provider disappears from the marketplace. The margin is not only due to the limited pledgeability, but also in order to diminish the default probability. As the contract is not state contingent, potential value losses of one service provider are not directly captured by the other one. Therefore, the margin not only threatens B and C to run away with the capital received, but also assures that potential value losses are initially captured by subordinated equity capital. Consequently, even if there are losses, A still has the chance to receive the complete repayment of the loan.⁶ A charges per period interest amounting to Φ per asset which was financed. The exact value of Φ is derived later from the participation condition of A, for now it is only important to assume that it is smaller than the exogenously given return Ω .

A now provides short-term financing, i.e. the credit conditions are renegotiated according to the collateral value in $t=1$. If the collateral value decreased, a part of the funds has to be paid back, if the collateral value increases, more funds can be obtained. Additionally, the high value assets can be traded within that period. In $t=2$, after the projects realized their proceeds,

⁶ Such margin constraints in collateralized borrowing are common practice in reality. Brunnermeier and Pedersen (2008) dedicate a whole appendix to real-world margin constraints.

the remaining debt capital has to be repaid. The timeline for this contract is provided in figure two. Such a contract is not efficient in a frictionless world, as will be shown in the next section.

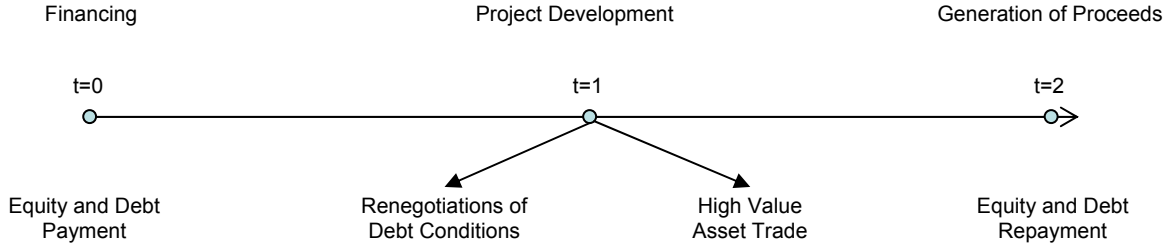


Figure 2: Timing in a debt contract

2.1 Debt Contract: Failure in Frictionless Markets

In $t=0$, A provides some fraction δ of B and C's investment value as debt capital. With the beginning of $t=1$, the realizations of ε and γ become common knowledge. As there is no potential for a bad management in a frictionless market, all proceeds and returns which both B and C will have gathered until $t=2$ are also common knowledge then. Consequently, if there are no frictions, two scenarios are possible in the following.

In a first scenario, the future funds of the unlucky service provider B are higher than the funds which A paid to B and B is solvent. As A knows that B is currently solvent and no bad management can diminish the future solvency, A has no reason to renegotiate the credit conditions in $t=1$. All repayments and all interest will be paid by B in $t=2$. As the lucky service provider C pays back in any case, A won't incur losses in that scenario.

In a second scenario, however, the future funds of B are lower than the received credit and B is insolvent. The condition for that scenario is given in (12).

$$(\eta N_1 Z + (1 - \eta) N_1 V) \delta + 2 N_1 \Phi > \varepsilon N_1 Z + (1 - \varepsilon) N_1 V + 2 N_1 \Omega \quad (12)$$

$$\varepsilon < \varepsilon^* = \eta \delta - \frac{V}{Z - V} (1 - \delta) - \frac{2(\Omega - \Phi)}{Z - V} < \eta$$

By no means will B be able to make all payments as contractually agreed on. If ε turns out to be too small, B will not only default on the interest, but also on the repayments. If the

collateral is not enough to counterbalance that missed payments, A will incur a loss.⁷ The debt contract consequently is not efficient if there are no frictions.

2.2 *Debt Contract: Market with Missing Management Incentives*

The debt contract is nevertheless a reasonable option for A, once the model allows for management errors. Still the contract won't be efficient, however all problems arising from the friction can be ruled out and the contract dominates the contingent state contract.

If there are frictions, three scenarios are possible. In a first scenario, B is again insolvent, as is already explained in the above section. In the other two scenarios, B is solvent, is however only able to repay all funds under certain conditions. If these conditions are not met, B is solvent but illiquid.

2.2.1 *Debt Contract: Illiquidity*

If A offers a debt contract to B and B turns out not to be insolvent, there might emerge the problem of illiquidity for B. In a debt contract, in $t=0$ A pays debt capital amounting to (13) to both B and C.

$$-(\eta N_1 Z + (1 - \eta) N_1 V) \delta \tag{13}$$

In $t=1$, the credit conditions are renegotiated according to the development of the collateral value which B and C can offer. With the development of the projects, the collateral value of B decreases and it becomes harder for B to roll over the loans. B has to pay back the difference between the two short-term credits granted in $t=0$ and $t=1$ and therefore a liquidity requirement for B arises. Due to this shortage of funding liquidity, B needs to rely on the market liquidity of its assets, i.e. B needs to obtain the required liquidity by selling high value assets. If the price at which B can sell is high, B must sell less assets than if the price is low. Consequently, the resulting asset supply function (14) is downward sloping.⁸ In contrary to that, the underlying liquidity demand is upward sloping.⁹

⁷ The particular conditions for losses will be derived in the next section. This section only intends to demonstrate that a debt contract is not efficient if there are no frictions.

⁸ The asset supply function is derived in Appendix 2.

⁹ If $P < Z\delta$, the asset supply function turns into a demand function. The price is too low for B to remain liquid; consequently B's only chance to remain liquid is to buy undervalued assets which would grant better financing

$$Q_B(P) = \frac{(Z-V)(\eta-\varepsilon)N_1\delta - N_1(\Omega-\Phi)}{P-Z\delta} \quad (14)$$

The collateral value of C increases with the development of the projects in $t=1$, therefore C receives more debt capital from A as it already received in $t=0$. This additional liquidity can be used to buy assets from B. For the same reasons, this asset demand function (15), which represents C's liquidity supply, is also downward sloping (Derivation similar as for B).¹⁰

$$Q_C(P) = \frac{(Z-V)(\gamma-\eta)N_1\delta + N_1(\Omega-\Phi)}{P-Z\delta} \quad (15)$$

As B and C trade the assets in between themselves,¹¹ their aggregated portfolio value remains identical in $t=0$ and $t=1$. Consequently, as A provides funds against that portfolio value, also the total funds provision remains the same as long as every market participant pays like contractually agreed on; there is only redistribution: The funds which B loses directly are readdressed to C. In addition to that, the returns Ω which B and C receive are greater than the interest Φ which they have to pay each period. This improves the liquidity situation of both B and C. As a result thereof, the asset demand of C (15) at any price is higher than the asset supply of B (14), what can be derived using the rational expectations condition (5).

In order to remain liquid, B needs to collect enough proceeds from its sale of assets. If the asset price falls, B must sell more assets in order to meet its financial obligations. If the asset price falls to a certain boundary value, B must sell all of its high value assets at that boundary price \underline{P}_B . If the price falls below that value, B has no chance to remain liquid. This boundary price (16) can be calculated by using the fact, that at this price, B needs to sell the entire amount of high value assets it possesses. Consequently, the asset supply at this price must be equal to B's high value assets in $t=0$.

$$Q_B(\underline{P}_B) = \varepsilon N_1$$

conditions due to their expected value. This demand function can be neglected, as it is straightforward that no market participant would sell at such an undervalued price.

¹⁰ If $P < Z\delta$, the asset demand function turns into a supply function which can also be neglected. C isn't in liquidity trouble; therefore it won't sell assets at an undervalued price.

¹¹ For A it is not reasonable to engage in that trade because the discount A would sustain because of the limited pledgeability is assumed to be too high. Refer to Appendix 3 for a detailed discussion of that assumption.

$$\Rightarrow \underline{P}_B = \frac{\eta\delta(Z-V) - (\Omega - \Phi)}{\varepsilon} + V\delta \quad (16)$$

C can use its additional liquidity to buy assets from B. If the asset price falls, C can buy more assets with the additional liquidity at hand. If the asset price falls to a certain boundary value \bar{P}_C , C has exactly enough liquidity to buy all of B's high value assets. The asset demand at this price (17) is consequently equal to B's amount of high value assets in $t=0$.

$$Q_C(\bar{P}_C) = \varepsilon N_1$$

$$\Rightarrow \bar{P}_C = \frac{\eta\delta(Z-V) + (\Omega - \Phi)}{\varepsilon} + V\delta \quad (17)$$

Using the fact that the asset demand is higher than the supply, or checking the rational expectations condition (5) on (16) and (17) leads to the conclusion that the minimum price which B needs to receive for all of its Z-assets is smaller than the maximum price which C could pay for the same amount of Z-assets. Consequently, a scenario in which C uses all of its surplus liquidity to buy high value assets of B, thereby filling B's liquidity gap is possible in some, but not in all situations.

In a scenario denoted as case one B won't become illiquid. In this scenario, $\underline{P}_B < Z$, i.e. the minimum price which B needs to receive is smaller than the fundamental value of the high value asset. C is in a position of power, as only B is in liquidity trouble and C has surplus liquidity to provide. Consequently, C could charge a liquidity premium from B. That premium would have the form of undervalued prices at which C buys the high value assets from B. Consequently, C doesn't pay out the fair value for B's assets, but only the minimum price \underline{P}_B which B needs to receive. As $\bar{P}_C > \underline{P}_B$, C is able to pay that price and doesn't even need to use all of its surplus liquidity in the process. B sells its high value assets and receives exactly the amount of liquidity required to pay all funds to A as contractually obliged.

In the second case B becomes illiquid and defaults on its debt. If $\underline{P}_B > Z$, the boundary price which B needs to receive is an overvalued price. Put it differently, the price at which B could remain liquid contains a negative liquidity premium, i.e. a subsidy from C to B. While C would potentially be able to pay that overvalued price with the available liquidity, the

fundamental price Z is the maximum price C is willing to pay. Thereby a liquidity premium of zero is charged.¹² C buys all of B 's assets for Z instead of \underline{P}_B thereby not using all of its additional liquidity. B is forced to sell all of its high value assets, but nevertheless it isn't able to collect enough liquidity, as the price is too low. B isn't able to repay its debt in $t=1$ and therefore becomes illiquid in case two. The remaining low value assets, which serve as collateral in such a case of default go to A afterwards. B disappears from the marketplace. The condition for illiquidity is given in (18).

$$\begin{aligned} \underline{P}_B &> Z \\ \Rightarrow \varepsilon < \varepsilon^{**} &= \frac{\eta\delta(Z-V) - (\Omega - \Phi)}{Z - V\delta} < \eta \end{aligned} \tag{18}$$

At this point it needs to be clarified, why B can be called illiquid in that scenario. B is illiquid, if it can't receive funds for the continuation of a project, which is worth continuing.¹³ If B was able to continue its business, the low value assets not sold to C could be operated. Factoring in the mismanagement, the proceeds from these low value assets in $t=2$ are $(1-\varepsilon)N_1(V-\sigma)$. If the business of B is terminated, A receives the low value assets as collateral and liquidates them. The thereof resulting proceeds in $t=2$ are $(1-\varepsilon)N_1V\rho$. If the continuation value is higher than the liquidation value, a worthwhile project of B is stopped because B lacks the required liquidity in $t=1$. Consequently, assuming (19), the situation of B can be described as liquidity problem.

$$\begin{aligned} (1-\varepsilon)N_1(V-\sigma) &> (1-\varepsilon)N_1V\rho \\ \Rightarrow V(1-\rho) &> \sigma \end{aligned} \tag{19}$$

If case two occurs, B is liquidity constraint. The shortage of liquidity is triggered by B 's unlucky investment in a too bad portfolio. While B 's investment is also below average in case one, it is still good enough to meet all debt repayments. However, this good enough investment grade isn't given anymore in case two. The sudden downward shift in the secured debt level of B forces B to de-lever its position too strongly. While in case one, C is induced to use its additional cash to support B 's de-leveraging process by spending enough surplus

¹² A positive liquidity premium would make the illiquidity problems worse.

¹³ Refer to Tirole (2009) for this formulation of illiquidity.

liquidity, in case two it is more profitable for C to hoard more of its cash. One can think of case two as a severe shock. Once this shock is big enough, the cash redistribution between the credit constraint service provider and the cash providing service provider is not sufficient and therefore disables B to meet its contractual liquidity requirements. In section four it will be shown, that such a shock turns out to be a rare event.

The asset supply and demand functions are shown in figure three. In case one, the fundamental price Z is higher than the minimum price which B needs to receive. B can sell all assets for $\underline{P}_B < Z$ and remains liquid, because the cash generated from the tradable assets is sufficient to meet the repayment obligations caused by the tighter funding conditions. In case two, the fundamental price is lower than the boundary price. As C won't pay above the fundamental price B becomes illiquid. All tradable assets need to be sold at a too low price. The market liquidity turns out to be inefficient to fill the liquidity gap.

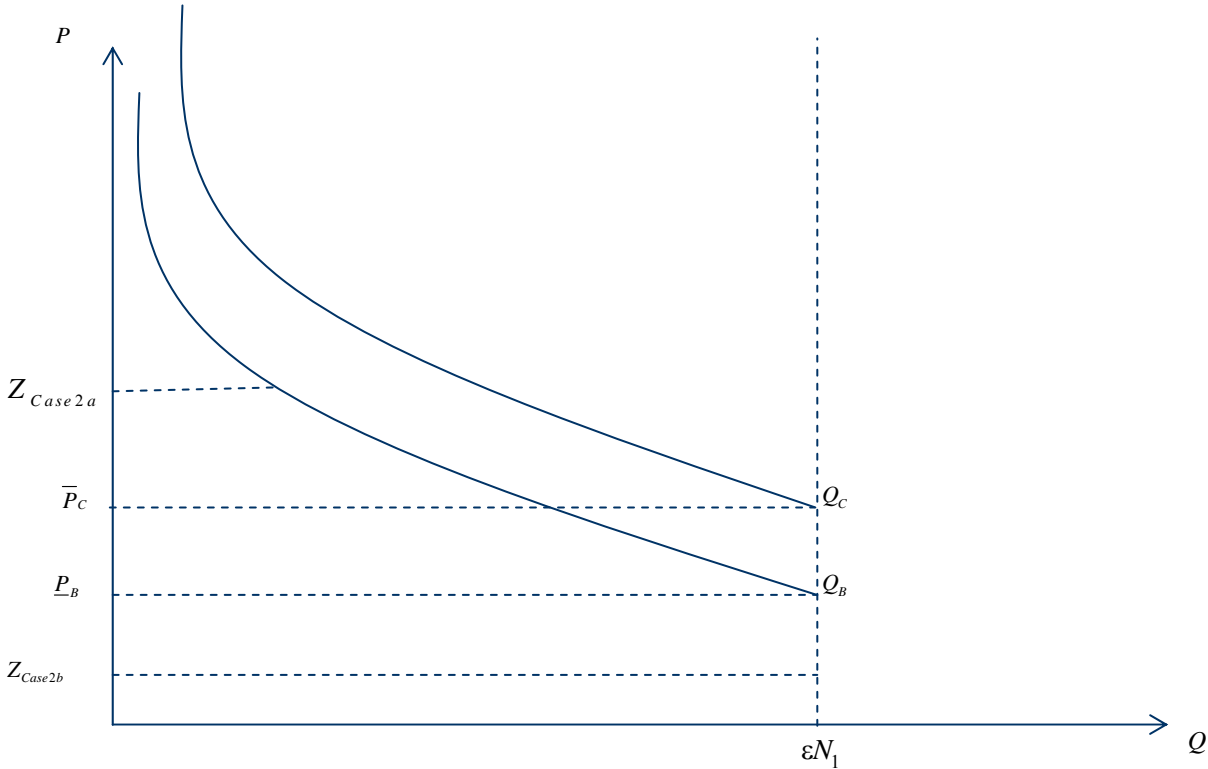


Figure 3: Asset supply and demand

2.2.2 Debt Contract: Inefficiency for A

The last section outlined that a debt contract can lead to situations, in which the unlucky service provider B defaults on its contractually obliged payments: A scenario in which B is insolvent or a scenario in which B is solvent but illiquid. For A, there might nevertheless be

hope to receive full repayment of all credit provided in $t=0$. B might only default on the interest but manage to make all down payments. In that case A would only sustain decreased interest revenues. But even if B also defaults on the regular repayments, the payoff from the collateral which A collected might still be high enough to prevent A from losses. This is examined in the following.

Once the condition for insolvency (12) or illiquidity (18) is met, A realizes a in- and outflow of funds, whose net position is given in (20).¹⁴

$$-\eta N_1(Z - V)\delta + \varepsilon N_1(Z - V\rho) - N_1V(\delta - \rho) + (2 + \varepsilon)N_1\Phi + N_1\Omega \quad (20)$$

Depending on ε , (20) can be either greater or smaller than zero, where a negative value represents a loss for A. The debt contract results in a loss for A, if condition (21) is met.

$$\varepsilon < \varepsilon^{***} = \frac{\eta\delta(Z - V) + V(\delta - \rho) - 2\Phi - \Omega}{Z - V\rho + \Phi} < \eta \quad (21)$$

The reasons for a loss of A require close attention. As the aggregate investment value doesn't deteriorate between the time of financing in $t=0$ and profit generation in $t=2$, additional financing needs in the intermediate period $t=1$ should seem to be unnecessary. Nevertheless, the aggregate supply of funds by A turns out to be insufficient to support the payment schedule of both B and C. The unlucky service provider faces tighter credit conditions from the funding side in $t=1$ (in the extreme case of insolvency B doesn't receive any more credit at all), but this should normally be absorbed by the lighter constraint of the lucky one. The problem however is that even though A doesn't receive all funds from B, it is nevertheless contractually obliged to pay out more funds to C, actually those funds which B should have repaid. Consequently, in $t=1$ A is forced to augment the debt capital outstanding compared to $t=0$. In $t=2$, A can use the low value assets which it received as collateral to counterbalance the additional debt payout, however the proceeds aren't necessarily enough to capture the missed repayment of B. The repayment of C is capped to the amount of credit C received from A and the interest payment. Consequently, A can't profit from the fact that C has more high value assets after the trade. The redistribution between B and C consequently can lead to losses for A in case two.

¹⁴ For the derivation of the flow of funds refer to Appendix 4.

A loss situation is guaranteed, if the random variable ε fulfils a pair of two conditions:

- i) The condition for insolvency (12) or the condition for illiquidity (18)
- ii) The condition for losses of A (21)

As long as B is solvent and liquid, there is always full repayment for A. However if B is insolvent or solvent and illiquid, the full repayment of all funds is not guaranteed. Offering a debt contract is nevertheless reasonable for A, as it adjusts the problems of bad management and makes a loss situation extremely unlikely, as is shown in the next two sections.

2.2.3 Debt Contract: Adjustment of Bad Management

Why should A offer a collateralized short term debt contract? As long as there are no frictions, A can provide credit in $t=0$ and afterwards B is either solvent or insolvent according to (12). No short-term readjustments in $t=1$ are necessary. In a short-term contract however, the credit conditions are directly related to the development of the collateral value. Because of the renegotiations of the credit conditions in $t=1$, a liquidity requirement for B emerges, posing the new threat of illiquidity, even if insolvency did not occur. Comparing (12) and (18), it can also be recognized that the condition for illiquidity is weaker than the condition for insolvency. However, it is nevertheless reasonable for A to rely on such a contractual agreement.

Once the investment value turns out in $t=1$, it is recognized if B is solvent. However, even if that is the case, the managers of B also recognize that they work for the company which won't have any increase but losses in equity. As a consequence, performance payment is not possible anymore. A knows that due to that missing incentive, all assets won't realize the expected payoff in $t=2$ but will have a discount of σ instead. In addition, B's assets won't generate return. Due to that knowledge, A is only willing to provide as much debt in $t=1$, that B's proceeds in $t=2$ are sufficient to make all repayments and interest payments even if the management of B behaves badly. Consequently, in $t=0$ A needs to install a fraction of debt capital provision δ such that full repayment in the case of bad management is ensured.¹⁵ Then, it is irrelevant for A if the management of B misbehaves. A's problem of this friction is healed by the debt contract.

¹⁵ For the derivation of δ refer to Appendix 5.

Besides delivering protection against the problems caused by missing management incentives, a debt contract additionally renders losses for A extremely unlikely. However, as also very unlikely events still occur with positive probability, the security of such a debt contract turns out to be deceptive.

2.2.4 Debt Contract: Probability of Spreading Problems

The insolvency or illiquidity problems of the unlucky service provider B spread out to the credit provider A, if the insolvency or illiquidity leads to losses for A. Put it differently, problems of minor financial players are spreading out, if the major financial players are affected by these problems. Losses for A in a debt contract only arise, if the set of conditions (12) and (21) or (18) and (21) are met. As the condition for insolvency (12) implies the condition for illiquidity (18), it is sufficient to examine when (18) and (21) are simultaneously fulfilled. For these two conditions, the random variable ε must be small enough, namely smaller than the boundaries ε^{**} and ε^{***} given in (18) and (21). Therefore the distribution of ε needs to be assessed.

In $t=0$, there is a pool of a limited number of investment projects N_0 , from which B and C draw their N_1 investment projects without replacement. Within the N_0 projects of the entire portfolio, there are M high value projects and (N_0-M) low value projects. M is also exogenously known, as $\eta = \frac{M}{N_0} = \frac{M}{2N_1}$. From the N_1 projects, the unlucky provider draws Ψ high value projects and $(N_1-\Psi)$ low value projects. This Ψ follows a hyper geometric distribution, as there is no replacement of the projects. ε represents the fraction of high value assets in the unlucky service provider's portfolio, consequently $\varepsilon = \frac{\Psi}{N_1}$. It follows that ε is a hyper geometrically distributed random variable divided by a constant; consequently ε itself is also hyper geometrically distributed.

The probability of the loss conditions (18) and (21) can be evaluated using the hyper geometric distribution. With increasing N_1 , i.e. with increasing draws of both B and C, the hyper geometric converges to a binomial distribution. Consequently Ψ , the number of high value projects B draws, converges exactly to half of the M high value projects in N_0 , i.e. to

ηN_1 and ε converges to η . Given the fact, that N_0 is a large number, the fulfilment of both conditions becomes extremely improbable.

However, given the fact that N_0 is finite, leaves a small probability for losses of A to occur. In a world with frictions, it is reasonable for A to offer a debt contract instead of a state contingent contract, because it better safeguards the funds of A. However, this security is deceptive, as in some situations, such a debt contract can lead to losses for A. If losses are substantial and A didn't protect against this very unlikely risk, serious problems for A arise. As this event is very rare, it suitably describes a situation, in which the functioning of the financial markets is normally not distracted, however a certain threat exists, that in extreme cases big problems arise.

2.2.5 Debt Contract: Participation Constraints

In a last step, it needs to be proven that all market participants would sign in such a contractual agreement. The contingent state contract turned out to be not implementable in the presence of frictions, as it generates negative expected profits for all market participants. The debt contract must deliver expected profits of zero at minimum for the risk neutral market participants to sign in.

For A, participation depends on the right choice of interest Φ . In the worst case, for A, the random variable ε turns out to be zero and A realizes the highest potential loss given in (20). In the best case, besides C also B makes all repayments and all interest payments and A generates a profit of $4N_1\Phi$. The situation for A always improves with increasing Φ , consequently A has to evaluate all probabilities of ε to set the interest $\Phi=\Phi^*$ so that the participation constraint for A given in (22) is fulfilled.

$$\Phi^* \rightarrow E(A/\Phi, \varepsilon) = \sum_{\varepsilon=\varepsilon_{\min}}^{\eta} [Payoff(\Phi, \varepsilon)]P(\varepsilon) \geq 0 \quad (22)$$

In comparison to A, B and C can't influence their participation by setting parameters endogenously but need to rely on exogenously given parameters. The unlucky service provider might be insolvent or illiquid. In that case, it will generate losses according to (A9) derived in appendix 3. If it is solvent, then fewer losses according to (A13) will be sustained. The lucky service provider generates profits amounting to (A10) if the unlucky one is insolvent/illiquid and profits amounting to (A14) otherwise. Both service provider are

identical in the beginning and have an equal chance of being the lucky or the unlucky service provider. Consequently, by evaluating the win and the loss situation for all probabilities of ε , an expected value can be calculated. Once this value is equal or bigger than zero, the participation constraint for the service providers is also fulfilled. While most of the parameters display an ambiguous effect on the participation likelihood of B and C, it can be seen from (A9), (A10), (A13) and (A14) that at each value of ε , the investment return Ω has a positive effect on the payoff for the service providers. Therefore, with a sufficiently high return, the participation constraint for B and C is also fulfilled.

V. Aggregate Shocks

In all results derived so far, an idiosyncratic shock scenario is assumed. The unlucky service provider suffers a decreased investment and therefore has a liquidity requirement which can often be served by the liquidity surplus of the lucky service provider. This model can easily be advanced to a situation, in which an aggregate shock hits both service providers, thereby rendering problems worse. A simple way to show this is to redefine the investment return Ω . Until now, it was assumed that $\Omega > \Phi$. If an unexpected aggregate shock hits the overall economy in $t=1$, the return decreases so that $\Omega < \Phi$.

With Ω falling below Φ , B will be insolvent or illiquid for sure. If B is solvent, the minimum boundary price it needs to receive for its assets (16) in order to meet its liquidity requirement is in any case higher than the maximum price C can afford (17). The aggregate supply of liquidity is smaller than the aggregate demand and consequently there is a lack of markets for the high value assets of B to be sold. Therefore, only one last condition (21) needs to be fulfilled in order to guarantee losses for A. Additionally, this condition becomes more likely, as ε^{***} increases with falling Ω . The problem of spreading illiquidity becomes more severe if an aggregate shock disturbs the financial markets.

VI. Policy recommendations

If the representatives of A are not able to contractually solve the default problem, politic measures need to be taken in order to safeguard A of own insolvency or illiquidity. This is crucial, as A is a systemic relevant bank. Once a systemic relevant player itself suffers severe problems, even if self-inflicted, politics are forced to bail it out as otherwise financial

problems could easily translate into problems for the overall financial system. To avoid such a bailout situation, ex ante measures need to be considered.

In order to avoid the existence of unlucky players, the government could force the service providers to merge to one big entity. This company would invest in all N_0 projects and thereby obviously in an exactly average portfolio. There wouldn't be losses from an unlucky service provider anymore; however a monopoly would have been created. As this uncompetitive market structure gives rise to other distortions, this possibility turns out to be useless; however other measures prove to be helpful.

1. Higher Equity Capital Requirements

Losses for A only occur, if the unlucky service provider draws some Ψ allowing for ε being smaller than both the boundaries given in (18) and (21). With large N_0 , ε converges to η . Consequently, the smaller ε^{**} and ε^{***} are, the less likely it is to draw a situation resulting in a loss for A.

Understanding the influence of the ex ante exogenously given parameters on the default probability can lead to policy recommendations which could be implemented. However, only the margin δ is a factor, which can be controlled by the government. The less debt capital B and C are allowed to use in order to finance the projects and consequently the more equity capital they have to raise (i.e. δ decreases), the less likely a default situation becomes. While the effect of higher equity capital requirements on the illiquidity condition (18) are ambiguous, decreasing δ always leads to a decrease in ε^{***} , thereby rendering losses of A more unlikely.

Raising equity capital requirements helps stabilizing the financial system. A lower amount of capital is put at risk if A provides less debt capital to B and C. Once the unlucky service provider defaults, the high value projects are redistributed from the unlucky to the lucky service provider. Such redistribution is only advantageous for the lucky service provider. A receives all the proceeds from the trade of the high value assets in $t=1$ and the remaining low value assets as collateral, if however the capital outstanding is too high, these two sources of liquidity are not enough. The problem lies in the fact that the unlucky service provider lost too less equity capital in comparison to the gains of the lucky one. Consequently, also A needs to incur some losses. If, however, the losses of the unlucky service provider are increased by augmenting the equity capital put at risk, A isn't forced to make own losses in order to capture the gains of the lucky one.

2. *Governmental Provision of Outside Liquidity*

Another potential measure to stabilize the financial system is an intermediate provision of governmental funds in $t=1$. If B is illiquid in $t=1$, A receives the low value assets as collateral and liquidates them with a discount. Due to that discount, it might be better if the assets were operated by B for the second period as well. However, as the management of B misses incentives for the second period, also then the assets will only generate diminished proceeds. Consequently, it is necessary to compare the liquidation proceeds with those proceeds the bad management of B would generate. If it was better to allow for a continuation of B's business, the government could help. By providing enough liquidity for B not to become illiquid in $t=1$, i.e. by fixing the price of high value assets via own purchases so that B remains liquid, the government ensures that B is allowed to keep its low value assets until $t=2$. The losses from this liquidity provision then can be rolled over to A in $t=2$ so the government is not responsible to carry any losses.

Summarizing, the government needs to assess if the aggregate results for A are better if B becomes illiquid or not. If the errors of a bad management are better than the discount due to limited pledgeability, it is reasonable to provide outside liquidity.¹⁶

VII. Conclusion

This model shows that there might be threats to the liquidity provision within the financial system, even if there are no *ex ante* differences between the good and bad market participants. While efficient contracts theoretically exist, they are ruled out once missing incentive structures disturb the markets. Debt contracts can prevent systemic relevant providers of debt capital from losses in many situations, however not in very rare and extreme shock cases, which are triggered by pure chance. The more unrealistic such situations are assessed to be, the less likely is a good preparation for such scenarios. Consequently, while normally functioning seemingly smoothly, the financial system can severely get into predicament in rare occasions.

As recent history showed that safeguarding for rare but catastrophic occasions is limited, governmental actions need to be put in place in order to force for such a preparation. *Ex post*

¹⁶ Refer to Appendix 6 for conditions favouring outside liquidity provision.

bailouts of systemic relevant institutions are expensive; therefore some useful ex ante regulation should be implemented instead. This model shows that even if the success of the borrower only depends on pure luck, mechanisms can be implemented to stabilize the financial system. A higher degree of equity capital financing decreases the likelihood and the magnitude of losses of capital providers. Also the provision of intermediate governmental liquidity might be useful in some occasions. While losses still can occur, they are more unlikely and smaller than in an ex post bailout. Relying on a prudent combination of equity capital requirements and provision of outside liquidity therefore leads to preferable outcomes with smaller losses for systemic relevant players and the government.

All measures guarantee that only investors directly hit by the shock suffer the worst losses. The investor's financier will remain less affected. The threat of insufficient liquidity endowment of the whole financial system is alleviated.

Appendix

1. ε - γ -Boundaries

The ε - γ -combinations have natural boundaries. Obviously, the worst draw B can do is receiving zero high value projects. As a consequence, $\varepsilon \geq 0$. The best scenario C can reach is drawing only high value projects and consequently $\gamma \leq 1$. As $\gamma = 2\eta - \varepsilon$, this yields $\varepsilon \geq 2\eta - 1$.

$$\varepsilon_{\min} = \max\{0; 2\eta - 1\} \quad (\text{A1})$$

Calculating it the other way round, $\varepsilon \geq 0$ and $\varepsilon = 2\eta - \gamma$ yields $\gamma \leq 2\eta$.

$$\gamma_{\max} = \min\{1; 2\eta\} \quad (\text{A2})$$

2. Asset supply function

The liquidity requirement of B is the difference between the credit B receives in $t=0$ and in $t=1$. This difference is the amount of credit which B is obliged to pay back to A:

$$\text{Liabilities}_0 - \text{Liabilities}_1 = (\eta N_1 Z + (1 - \eta) N_1 V) \delta + N_1 \Phi - (N_{2,B}(P) Z + (1 - \varepsilon) N_1 V) \delta - N_1 \Omega \quad (\text{A3})$$

$(1 - \varepsilon) N_1$ is the amount of low value assets after the draw in $t=0$. Those assets can't be traded for liquidity. $N_{2,B}(P)$ is the amount of high value assets B possesses after trade took place in $t=1$. Obviously this amount of assets influences the collateral value. After the draw in $t=0$, B has εN_1 high value assets which can be traded. As B needs liquidity, it will trade those assets. But by selling assets the own portfolio value and thereby the collateral value decreases. This decreasing collateral value again reduces the funds A provides in $t=1$, consequently there is further amplification of the liquidity shortage and the credit limit is endogenized. As the number of assets B needs to sell in order to remain liquid depends crucially on the asset price, the amount of high value assets B possesses after trade also depends on the price P.

The liquidity requirement of B divided by the asset price determines how many assets need to be sold. This yields the following asset supply function:

$$Q_B(N_{2,B}(P)) = \frac{(\eta N_1 Z + (1-\eta)N_1 V)\delta - (1-\varepsilon)N_1 V\delta - N_1(\Omega - \Phi)}{P} - N_{2,B}(P) \frac{Z\delta}{P} \quad (\text{A4})$$

Now it is problematic, that the liquidity requirement of B also depends on the number of B's high value assets $N_{2,B}(P)$ after $t=1$. Consequently, the trading behaviour of B also influences the liquidity need. The asset supply function should determine the trading behaviour of B; therefore it is not convenient if this function itself is influenced by the trading behaviour. It is the goal to find an asset supply function of B, which is only dependent on P and independent of the number of B's high value assets after trade. Conveniently, there is another way to express the same asset supply function in dependence of $N_{2,B}(P)$. It is the difference of high value assets in $t=0$ and after trade in $t=1$. If this difference is positive, B sold assets, if it is negative, B bought assets.

$$Q_B(N_{2,B}(P)) = \varepsilon N_1 - N_{2,B}(P) \quad (\text{A5})$$

(A4) and (A5) can be combined to find an expression for $N_{2,B}(P)$.

$$\varepsilon N_1 - N_{2,B}(P) = \frac{(\eta N_1 Z + (1-\eta)N_1 V)\delta - (1-\varepsilon)N_1 V\delta - N_1(\Omega - \Phi)}{P} - N_{2,B}(P) \frac{Z\delta}{P} \quad (\text{A6})$$

$$N_{2,B}(P) = \frac{N_1 [\varepsilon P - \eta Z\delta - (1-\eta)V\delta + (1-\varepsilon)V\delta] + N_1(\Omega - \Phi)}{P - Z\delta}$$

(A6) needs to be used in (A5) to arrive at the asset supply function of (14).

3. Limited Pledgeability

It is assumed that A lacks the specialization in the investment fields of B and C and therefore it is not reasonable for A to invest in the assets of B and C. Put it differently, the discount ρ which A has to sustain if operating the assets is too high, so that it is not profitable to engage in the trade of assets between B and C. In particular that means that even if the trading price between B and C falls to its lowest value, this value is still too high for A to enter that trade.

Considering all scenarios, the lowest price for which C might buy the high value assets of B is the minimum boundary price of B given in (16). This price decreases with increasing ε . The highest value ε can take is its convergence value η . If A buys a high value asset, A is able to

liquidate that and realizes a value of $Z\rho$. If trade is assumed to be unreasonable for A the liquidation value needs to be smaller than the minimum trade price between B and C as shown in (A7).

$$\underline{P}_B(\varepsilon = \eta) = \frac{\eta\delta(Z - V) - (\Omega - \Phi)}{\eta} + V\delta > Z\rho \quad (\text{A7})$$

$$\Rightarrow \Phi > \eta Z(\rho - \delta) + \Omega$$

Consequently, the endogenously chosen interest rate Φ needs to be big enough, which is not possible as long as $\Phi < \Omega$. However, A encounters only problems in situations, in which the unlucky service provider B receives a price of Z for its high value assets, i.e. in which B is insolvent or illiquid. So even if condition (A7) is not met and A could engage in trade once B is liquid anyways, A could never engage in the trade once B is insolvent/illiquid, because $Z\rho < Z$. Consequently, all results also hold if one only assumes limited pledgeability for A and a thereof resulting $\rho < 1$.

4. Flow of Funds

The flow of funds can be derived for B being insolvent, illiquid and solvent. First of all, it needs to be recognized, that the flow of funds for B being insolvent or illiquid is identical. If B is illiquid, B has to sell all high value assets at a price of Z to C. B loses all equity and C gains from that situation. A receives all proceeds from the trade between B and C, the first period return of B and the low value assets as collateral. A might gain or lose from that situation, depending on condition (21).

If B is insolvent, A receives B's first period return and all high and low value assets as collateral. Due to the limited pledgeability, A is not able to operate these assets properly. A is stuck on the low value assets, because these are nontradable. However, the high value assets are tradable and consequently A is better off when selling those to C. As the insolvency condition (12) is met, condition (18) is also implied and $\bar{P}_C > \underline{P}_B > Z$. Consequently, C has enough surplus liquidity to buy all of B's high value assets from A for the fair value Z .¹⁷ B again loses all equity, C receives all of B's high value assets for the price Z and A receives an identical payoff as in the illiquidity case.

¹⁷ This assumes that C doesn't charge a liquidity premium from A. This assumption needs not to be made if there is more competition in the sector of financial service providers.

4.1 *Insolvency/Illiquidity*

In the case of insolvency/illiquidity, A receives the following flow of funds. In $t=0$, the payout to B and C is given in (13). In $t=1$, A receives all proceeds from the trade between B and C (A and C if insolvency) and B's first period returns, represented by the first two terms in (A8a). Additionally, A pays out more debt to C because of C's improved collateral value and receives C's first period interest payments, represented by the second term in (A8a).

$$\varepsilon N_1 Z + N_1 \Omega + [(\eta N_1 Z + (1 - \eta) N_1 V) \delta - ((\varepsilon + \gamma) N_1 Z + (1 - \gamma) N_1 V) \delta + N_1 \Phi] \quad (\text{A8a})$$

In $t=2$, A receives proceeds from the low value assets which it received as collateral from B. Those are given in the first term of (A8b). As discussed, the proceeds are reduced by the factor ρ due to the limited pledgeability. In addition, A receives full repayment of C's outstanding debt and C's interest payments for all invested assets of the second period.

$$(1 - \varepsilon) N_1 V \rho + ((\varepsilon + \gamma) N_1 Z + (1 - \gamma) N_1 V) \delta + N_1 \Phi (1 + \varepsilon) \quad (\text{A8b})$$

Aggregating (13), (A8a) and (A8b) lead to a total payoff for A as given in (20).

Due to its insolvency/illiquidity, the unlucky service provider B loses all equity capital paid in $t=0$, leading to losses of (A9).

$$-(\eta N_1 Z + (1 - \eta) N_1 V)(1 - \delta) \quad (\text{A9})$$

The lucky service provider C pays the same amount of equity in $t=0$. In $t=1$, C receives more debt capital, generates first period returns from the invested assets and has to pay first period interest. The thereof resulting net liquidity surplus is used to buy all of B's high value assets. Consequently, in $t=2$ C receives the proceeds from all high value assets and its own low value assets and the second period return of those. Not all surplus liquidity is needed to buy the high value assets. As the lucky service provider only pays a price of Z smaller than the maximum boundary price which it could potentially afford, there still remains surplus liquidity. This surplus also adds to the profit. Finally, the debt capital has to be repaid and second period

interest needs to be provided. Using the maximum boundary price (17), this leads to a total profit given in (A10).

$$-(\eta N_1 Z + (1 - \eta) N_1 V)(1 - \delta) + [(\varepsilon + \gamma) N_1 Z + (1 - \gamma) N_1 V](1 - \delta) + (\bar{P}_c - Z) \varepsilon N_1 + (1 + \varepsilon) N_1 (\Omega - \Phi) = (\eta - \varepsilon) N_1 (Z - V) + (2 + \varepsilon) N_1 (\Omega - \Phi) \quad (\text{A10})$$

Summing up the results of A (20), the lucky (A10) and the unlucky service provider (A9) reveals an aggregate result given in (A11).

$$-(1 - \varepsilon) N_1 V (1 - \rho) + (3 + \varepsilon) N_1 \Omega \quad (\text{A11})$$

This is exactly the aggregated return of both periods of all correctly operated assets diminished by the wasted value of the unlucky service provider's low value assets serving as collateral for A. As the aggregate payoff of the three market participants equals the loss caused by the limited pledgeability, the calculations up to now are verified.

4.2 Liquidity

If the lucky service provider B is liquid, a prudent choice of δ^{18} ensures a full payment of all funds and interest for A, independently from the effort of B's management in the second period. Gains from interest given in (A12) are yielded.

$$4N_1\Phi \quad (\text{A12})$$

The unlucky service provider B has an equity payout in $t=0$ and receives proceeds from the nontradable low value assets in $t=2$. Those proceeds are diminished by the factor σ due to the mismanagement. The proceeds are sufficient to repay all debt capital and interest to A. The total payoff, which is zero in the worst case, is given in (A13)

$$-(\eta N_1 Z + (1 - \eta) N_1 V)(1 - \delta) + (1 - \varepsilon) N_1 (V - \sigma) - (1 - \varepsilon) N_1 V \delta - (1 - \varepsilon) N_1 \Phi = -(\eta N_1 Z - (\eta - \varepsilon) N_1 V)(1 - \delta) - (1 - \varepsilon) N_1 (\sigma + \Phi) \quad (\text{A13})$$

¹⁸ Refer to Appendix 5.

For the lucky service provider C things are similar as in the insolvency/illiquidity case. The only difference is that C now buys the high value assets at a lower price, the minimum boundary price of B. Using (16) and (17), the total gain of C is shown in (A14).

$$\begin{aligned}
& -(\eta N_1 Z + (1-\eta)N_1 V)(1-\delta) + [(\varepsilon + \gamma)N_1 Z + (1-\gamma)N_1 V](1-\delta) + (\bar{P}_C - \underline{P}_B)\varepsilon N_1 + \\
& + (1+\varepsilon)N_1(\Omega - \Phi) = (\eta N_1 Z - (\eta - \varepsilon)N_1 V)(1-\delta) + (3 + \varepsilon)N_1(\Omega - \Phi)
\end{aligned} \tag{A14}$$

Summing up the results of A (A12), the lucky (A14) and the unlucky service provider (A13) reveals an aggregate result given in (A15).

$$-(1-\varepsilon)N_1\sigma + (3 + \varepsilon)N_1\Omega \tag{A15}$$

This is again exactly the aggregated return of both periods of all correctly operated assets diminished by the wasted value because of the mismanagement of the unlucky service provider in period two.

5. Derivation of δ

The value of δ is to be chosen in order to make the bad management irrelevant for A. The minimum possible payoffs which B will generate in $t=2$ must be enough to fully repay A including interest. The left hand side of (A16) is the value which the unlucky service provider will generate in $t=2$ from all of its assets, given that the management misbehaves. The right hand side represents debt capital which A needs to provide for B's assets in $t=2$ and the interest which A wants to receive for that assets.

$$N_{2,B}(P)(Z - \sigma) + (1-\varepsilon)N_1(V - \sigma) = [N_{2,B}(P)Z + (1-\varepsilon)N_1V]\delta + [N_{2,B}(P) + (1-\varepsilon)N_1]\Phi \tag{A16}$$

In the worst case, B has to sell all of its high value assets, i.e. $N_{2,B}(P)=0$ then. For that case, (A16) can be solved for that δ , which guarantees a full payment for A even if the management of B acts badly and additionally the unluckiest scenario turns into reality (A17).

$$\delta = \frac{V - \sigma - \Phi}{V} \tag{A17}$$

For $\Phi < V - \sigma$ this takes a positive and therefore reasonable value.

6. Outside Liquidity

In order to circumvent the illiquidity of B in $t=1$, the government needs to provide the missing liquidity. Without outside liquidity, B receives only a price of Z for its εN_1 high value assets instead of the required price given in (16). The government has to pay the difference, as given in (A18).

$$-(P_B - Z)\varepsilon N_1 = -\eta\delta N_1(Z - V) - \varepsilon\delta N_1V + \varepsilon N_1Z + N_1(\Omega - \Phi) \quad (\text{A18})$$

As a consequence, B isn't illiquid and can operate the remaining low value assets until $t=2$, however using a bad management. After paying debt and interest to A, B receives proceeds amounting to (A19), which can be used to reimburse the government.

$$(1 - \varepsilon)N_1(V - \sigma) - (1 - \varepsilon)N_1V\delta - (1 - \varepsilon)N_1\Phi \quad (\text{A19})$$

In total, (A18) and (A19) yield losses of (A20) for the government that can be rolled over to A.

$$-\eta\delta N_1(Z - V) + \varepsilon N_1(Z - V) - N_1V(1 - \delta) - (1 - \varepsilon)N_1\sigma - (2 - \varepsilon)N_1\Phi + N_1\Omega \quad (\text{A20})$$

If the government doesn't intervene, the losses for A are given in (20). Comparing (20) with (A20) it can be shown that governmental provision of outside liquidity might mitigate the problem of spreading illiquidity if condition (A21) is fulfilled.

$$V(1 - \rho) - \sigma - 4\Phi > \varepsilon(V(1 - \rho) - \sigma) \quad (\text{A21})$$

7. Generalization of Results

All results have been derived for an example with one debt capital provider A and two minor players B and C. Qualitatively, the results also hold if there is more competition in each of the sectors. This is discussed in this section

7.1 *More Capital Providers*

Fulfilment of condition (22) is crucial for the participation of the commercial bank A. An interest Φ has to be chosen so that positive or at least zero expected profits are guaranteed. If there are more debt capital providers, they will compete for costumers by setting this interest rate. While one debt provider has the possibility to offer an interest rate which allows for positive expected profits, this will become impossible if there are more identical providers. Condition (22) will still be fulfilled for all of them but the charged interest rate will shrink to a value Φ^* which exactly allows expected profits of zero. If one single debt capital provider deviated from that interest rate, it would either generate expected losses and not participate at all ($\Phi < \Phi^*$), or it would lose all costumers to the cheaper competitors ($\Phi > \Phi^*$).

7.2 *More Financial Service Providers*

If there are more financial service providers, the results would only change quantitatively. Without loss of generality, an example with three minor institutions is considered. If there are three identical service providers, they would split the available assets into three equal parts. In $t=1$, it could turn out that one invested above average and two below average, or that two invested above average and one below average.

In the first case, there is again no competition for the sale of high value assets. The lucky service provider could charge a price derived from the same scheme as discussed above and the unlucky ones could turn out to be insolvent, illiquid or liquid. The results only change quantitatively, because this time one or two service providers might turn insolvent/illiquid.

In the second case, two service providers compete for the high value assets of the unlucky one. The aggregate surplus liquidity of the two lucky ones is still greater than the demand of the unlucky one. As long as the unlucky service provider is liquid and there are only two service providers, it receives its minimum boundary price for its high value assets, so the provider of liquidity charges a liquidity premium. This changes if there is competition, because the two lucky service providers could compete for the assets by offering higher prices until the fair value Z is reached. So the unlucky service provider is better off, the two lucky ones worse and for the commercial bank nothing changes. However, if the unlucky service provider is insolvent/illiquid, all assets are already traded with a liquidity premium of zero even in the case with two service providers. If there is competition, the price will nevertheless not exceed the fair value Z . Consequently, competition can't solve this problem qualitatively.

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