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Housing and the Business Cycle Revisited

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Housing and the Business Cycle Revisited

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Abstract

In this paper, I present a multi-sectoral DSGE-model with housing, real rigidities and variable capital utilization that generates aggregate and sectoral co-movements due to sector specific shocks. Furthermore, the model accounts for two puzzles: First, residential investment correlates positively with house prices, and second, GDP residential and business investment tend toward the empirically observed lead-lag pattern. I show that, except for relative prices, all co-movements and the lead-lag pattern of different investment types are endogenous in the calibrated model and independent of the properties of the shock. In a second step, I estimate the these properties with Bayesian techniques. As it turns out, shocks to sectors with similar elasticities in the final good sectors play a role related to aggregated shocks. In contradiction to a standard assumption in the literature, shocks to the construction sector seem to be lower than others.

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1 Introduction

Aggregate and sectoral co-movements are central features of business cycles. With respect to the housing sector Davis and Heathcote (2005) (DH hereafter) point out three stylized facts: (i) gross domestic product (GDP), private consumption expenditure (PCE), business and residential investment, aggregate hours, and house prices are positively correlated. (ii) residential investment is more than twice as volatile as business investment. (iii) Business investment lags GDP while residential investment leads GDP. The data and facts presented by Kydland et al. (2016), Davis and Nieuwerburgh (2015), Iacoviello and Neri (2010), and Iacoviello (2010) corroborate these findings. Table 1 gives a more detailed account of the stylized facts. It reports the estimates of second moments of time series from DH as well as my own estimates from data extended to 2015 for the U.S. My estimates support the conclusion that the stylized facts (i)-(iii) still characterize the cyclical properties of aggregate and sectoral co-movements. In addition, Figure 1 documents the lead-lag structure (iii) as well as the different volatilities (ii).

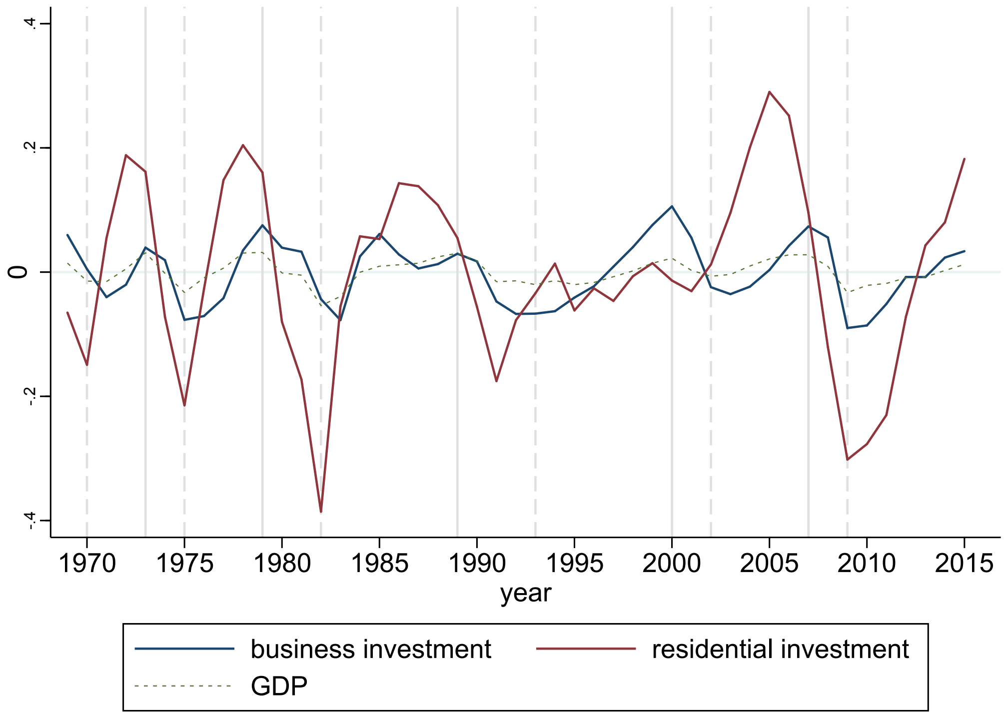
Jaimovich and Rebelo (2009) designate the ability of a model to reproduce co-movements between sectoral and aggregate economic quantities as a litmus test. However, most models do not pass this test. Early attempts by Benhabib et al. (1991), Greenwood and Hercowitz (1991) and Fisher (1997) examine the co-movement problem in models with market and home production. They find that investment in capital used at home and in capital rented to firms correlate negatively. The reason is that a positive shock to the home production technology increases the marginal product of capital used at home relative to the marginal productivity of market capital. More generally, sector specific shocks trigger factor movements to the favored sector which are reinforced by price induced demand effects. As a consequence, they introduce negative correlations between sectoral outputs. The internal propagation mechanism of the model and the properties of other driving processes may weaken or even reverse these correlations. The key to the understanding of a model of aggregate and sectoral fluctuations, thus, should be to separate the relative contribution of both kinds of effects from each other. Investigations of the propagations of sectoral shocks on aggregated fluctuations have been done e.g. by Horvath (1998) and more recent by Caliendo et al. (2017).

Table 1: Empirical second moments

SD	1969-2015 (USA)			DH		
GDP		2.00			2.26	
% SD to GDP						
PCE		0.96			0.78	
Hours worked		1.24			1.01	
Business investment (Busi)		2.51			2.30	
Residential investment (Resi)		7.46			5.04	
House prices (p_h)		2.63*			1.37**	
Output by sector***	x_b	x_m	x_s	x_b	x_m	x_s
	3.75	1.93	1.18	2.72	1.85	0.85
Hours by sector***	N_b	N_m	N_s	N_b	N_m	N_s
≤2000	2.92	1.59	0.78	2.32	1.53	0.66
≥2000	4.17	2.19	0.96			
Correlations						
GDP, PCE		0.91			0.80	
p_h, GDP		0.66*			0.65**	
PCE, Busi		0.57			0.61	
PCE, Resi		0.80			0.66	
Resi, Busi		0.4			0.25	
p_h, Resi		0.66*			0.34**	
Output by sector***	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s
	0.72	0.72	0.9	0.61	0.71	0.82
Hours by sector***	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s
≤2000	0.79	0.90	0.82	0.75	0.86	0.79
≥2000	0.71	0.87	0.95			
Lead-lag correlations	i=1	i=0	i=-1	i=1	i=0	i=-1
Busi_{t-i}, GDP_t	0.22	0.77	0.59	0.25	0.75	0.48
Resi_{t-i}, GDP_t	0.78	0.75	0.15	0.52	0.47	-0.22
Busi_{t-i}, Resi_t	-0.09	0.41	0.65	-0.37	0.25	0.53

Moments from annual per capita, logged, HP-filtered data with filter weight 100, Appendix A gives an detailed overview. Data DH 1948-2001; * only since 1970 available; ** 1970-2001, ***. b stands for construction, m for manufacturing, and s for service sector.

Figure 1: Cyclical behavior of different investment types and GDP



Cyclical component from per capita logged hp-filtered data with filter weight 100. Straight lines indicates a peak in GDP within min. ±2 years, dashed lines indicates a minimum in GDP within min. ±2 years

In this paper, I consider the role of sector specific shocks in order to explain the facts (i)-(iii). My starting point is the model of DH which has had a lasting impact on the housing literature over the last decade. This model is able to explain the positive co-movement of aggregate and sectoral quantities. However, it fails to predict the positive correlation between house prices and residential investment as well as the lead-lag pattern of residential and business investment.

In the DH model there are three intermediary sectors of production: construction, manufacturing, and services. Labor augmenting technical progress in each sector includes a sector specific trend and a sector specific stationary stochastic component. DH model the stochastic part as a three-dimensional, first-order vector-autoregressive model with correlated innovations and non-zero off-diagonal elements in the matrix of autoregressive effects (VAR(1)). Correlated innovations may be seen as a nesting of aggregate and sector specific shocks. As illustrated by an example in Appendix D, uncorrelated innovations lead to purely sectoral shocks, while perfectly correlated innovations give raise to an aggregate shock only. Correlations between zero and one, thus nest aggregate and sector specific shocks. Furthermore, non-zero off-diagonal elements in the autoregressive matrix can lead to some exogenous propagations, which seems implausible as a technology process, but supports the fitness of the model.

As explained by Iacoviello and Neri (2010), large shocks to the construction sector's technology are needed to explain fact (ii), the empirically observed relative volatility of residential investment. However, the induced sectoral reallocation and price effects also induce a negative correlation between house prices and residential investment, contrary to fact (i).[1]

Thus, I am asking is, what is the role played by correlated innovations in the DH model? I find that the model with sectoral independent innovations and zero off-diagonal elements in the matrix of autoregressive effects (3xAR(1)) is unable to reproduce the co-movement between residential and business investment and that the co-movements between private consumption, residential investment, and the sectoral outputs are very weak. Thus, it seems that the properties of the shock process and not the model's internal propagation mechanism drives the results.

My second and main contribution is to present an extended model with variable capacity utilization as in Jaimovich and Rebelo (2009), adjustment costs to the accumulation of business capital as in Christiano et al. (2005) (CEE adjustment costs hereafter), and sectoral frictions in the allocation of capital as in Boldrin et al. (2001), which is able to account for the stylized facts (i) to (iii) without having to resort to correlated innovations. I also also evidence that the housing convex adjustment costs due to the fixed factor land are greater than assumed by DH.

Additionally, I estimate the parameters of the exogenous shocks processes with Bayesian techniques. The extended model as well as the benchmark model are estimated with VAR(1)- and 3x AR(1)-processes. This highlights which parts of the processes are endogenized by the extensions: Mainly, with correlated shocks, the contemporaneous link between the construction and manufacturing sector. In the 3xAR(1) framework, shocks to the manufacturing sector act similar to aggregated shocks. Estimation gives evidence that they are smaller in the extended model. The extended model strengthens co-movements based on sectoral shocks. In contradiction to the calibrated model as well as in comparison to the priors, shocks to the construction

[1]Since residential investment is very intensive in construction goods and new houses are very intensive in residential investment a positive shock to the construction good technology raise the amount of residential investment and decline house prices and visa versa for adverse shocks. See also Davis and Nieuwerburgh (2015) for further discussion.

sector are not the heaviest one, rather the weakest. Odd comparison provides decisive evidence for the mentioned extensions compared to the benchmark with the same kind of exogenous process.

There are meaningful reasons for this extensions. First, in the DH model new houses are faced by adjustment costs due to new land, while business investment faces no adjustment costs at all. As mentioned by Gomme et al. (2001) and Kydland et al. (2016) in the U.S. business investment takes longer to be built up than residential investment. In general one could argue, business capital is more complex than houses which becomes apparent in this longer time span. From this point of view, the choice of adjustment costs for new houses (due to new land) and new capital (with CEE adjustment costs) is reasonable.[2]. Second, while variable capital utilization is an uncontroversial tool for an efficient capital usability, it is hard to imagine this for per capita housing units.

Limitations in sectoral capital mobility are also plausible for assessing the reality. Furthermore, Iacoviello and Neri (2010) guess that limited mobility supports co-movements, when there is only uncertainty in the productivity. This extension should help to validate this guess.

To this end, new business capital and new houses face convex adjustment costs, but they differ in their nature. Furthermore, in contrast to business capital, houses have on the one hand no variable utilization, but on the other lower depreciation rates.

These differences in the investment types helps to account for the stylized facts (i) to (iii) without having to resort to correlated innovations. Higher housing and the introduction of capital adjustment costs enhances co-movements, because they reduce the substitutability between different investment goods as well as consumption. Variable capacity interacts with capital adjustment costs and hence, strengthens co-movements especially when capital adjustment costs are included. Effects based on limited capital mobility are marginal. This contradicts the guess by Iacoviello and Neri (2010) that limited mobility strengthens co-movements at all.

In addition to the mentioned literature, there are two papers related to this approach. Fisher (2007) investigates the puzzle of leading home capital investments in a home production framework. He solves this puzzle by modeling home capital as a complement of market production. Hence, he also reduces the substitutability. My approach differs in the propagation channel. Here, housing and productive capital are

[2]E.g. on microfoundation Lucca (2007) provides an equivalent to CEE adjustment costs. If firms invest in many projects with uncertain time to build and if these projects have complementaries, investment is according to CEE adjustment costs (CEE adjustment costs). This equivalent is valid on a first order approximation.

not complements, but the substitution is disabled by adjustment costs. Hence, there is an implicit limitation in the mobility from business to housing capital. This approach is more in line with limited capital mobility by Boldrin et al. (2001).

Dorofeenko et al. (2014) also introduce CEE adjustment costs as well as default risk in the DH framework. Nevertheless, they do not distinguish between adjustment costs for business and residential investment. Furthermore, they adopt the exogenous process with correlated shocks. To this end, the model tends to the opposite direction of the lead-lag pattern as in the data and it is not clear which parts are driven endogenously. Hence, my paper also contributes to Dorofeenko et al. (2014).

In general, Kydland et al. (2016) investigate also the puzzle of the leadership of residential investment. Their approach rests on nominal frictions not on real ones. In their model, residential investment leads the business cycle. Albeit, the nominal interest rate, which is the driving force behind the leadership, is linked with a lead to the business cycle exogenously.

A comprehensive literature overview about housing and business cycles is provided by Davis and Nieuwerburgh (2015).

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the results in form of second moments and impulse responses. The section presents also robustness checks. As a byproduct of robustness checks, a profound discussion of internal propagation mechanisms is accrued. Section 4 presents the Bayesian estimation of the exogenous shock processes as well as a posterior odd model comparison. The Appendix contains additional material, in particular, it presents the system of equations which determines the model's dynamics, derives the model's balanced growth path and describes the data and the Monte-Carlo-algorithms.

2 The Model

The extended model is a stripped-down version of the DH model from which I borrow the nomenclature. The economy consists of a representative household and three representative firms, one in the intermediary goods sector, one in the production of investment and consumption goods, and one in the production of new homes. Different from DH, there is no government sector and no population growth. All quantities are in per capita terms. Time t is discrete and one period is equal to one year.

2.1 Analytical Framework

Intermediary goods. Consider first the intermediary stage of production. The representative firm rents capital and labor from the household to produce three kinds of goods X_{it}, where $i = b, m, s$ denotes the construction good, the manufacturing good, and the service good, respectively. The production function for each good is Cobb-Douglas with constant returns to scale:

$$X_{it} = (u_{it} K_{it})^{\theta_i} (A_{it} N_{it})^{1-\theta_i}, \quad \theta_i \in (0, 1), \tag{1}$$

where u_{it} is the utilization rate of capital K_{it} in the production of good i, N_{it} is raw labor and A_{it} its efficiency factor. The efficiency of labor is specific to the production of good i and involves a deterministic trend and a stationary stochastic component:

$$\ln(A_{it}) = \ln(A_{i0}) + t \ln(g_{A_i}) + z_{it}, \tag{2}$$

$$z_{it} = \rho_i z_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \text{ iid } \mathcal{N}(0, \sigma_i^2). \tag{3}$$

The innovations ϵ_{it} are uncorrelated in time and between the different technologies $i \in \{b, m, s\}$.

Let P_{it}, r_{it}, and W_t denote, respectively, the price of good i, the rental rate of capital subject to the good specific utilization rate, and the real wage. The firm chooses $u_{it} K_{it}$ and N_{it} to maximize profits Π_{It}, given by

$$\Pi_{It} := \sum_{i \in \{b,m,s\}} \left[P_{it} X_{it} - r_{it} u_{it} K_{it} - W_t N_{it} \right]$$

subject to the production functions (1).

Consumption and investment goods. At the final stage of production a firm employs the intermediary goods to produce two goods $j = c, d$. The good with label $j = d$ are residential investments and the good labeled $j = c$ is used for consumption and business investment. The latter serves as numéraire, while the relative price of good $j = d$ is given by P_{dt}. The production function of each good is again Cobb-Douglas with constant returns to scale:

$$Y_{jt} = X_{bjt}^{B_j} X_{mjt}^{M_j} X_{sjt}^{S_j}, \quad B_j + M_j + S_j = 1, \tag{4}$$

where X_{ijt} is the amount of intermediary good i employed in the production of the final good j. The firm's profits are given by:

$$\Pi_{Ft} := Y_{ct} + P_{dt}Y_{dt} - \sum_{i \in \{b,m,s\}} P_{it}(X_{ict} + X_{idt}).$$

The firm chooses X_{ict} and X_{idt} to maximize this expression subject to the production technologies (4).

Housing. At the final stage of production there is also a firm which combines land l_t and housing investment goods Y_{dt} to produce new homes Y_{ht} according to

$$Y_{ht} = Y_{dt}^{1-\phi} l_t^{\phi}, \quad \phi \in (0,1), \tag{5}$$

while the accumulation of houses follows:

$$H_{t+1} = (1-\delta_h)H_t + Y_{ht}. \tag{6}$$

Homes depreciate with the rate δ_h, and land is in fixed supply $l_t = 1$ with price P_{lt} by the household. As I show in detail below, the technology (5) introduces convex costs of adjustment in the accumulation of homes. With a price of new homes P_{ht} the firm solves

$$\max_{Y_{dt}, l_t} \quad \Pi_{Ht} := P_{ht}Y_{ht} - P_{dt}Y_{dt} - P_{lt}l_t$$

subject to the production function (5).

Household. The household maximizes expected life-time utility given by:

$$U_t := \mathbb{E} \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, H_{t+s}, 1 - N_{t+s}).$$

His current-period utility u depends on consumption C_t, the stock of houses H_t, and leisure $1 - N_t$ and is parameterized as in DH:

$$u(C_t, H_t, 1-N_t) := \frac{1}{1-\sigma}\left[C_t^{\mu_c} H_t^{\mu_h} (1-N_t)^{1-\mu_c-\mu_h}\right]^{1-\sigma}.$$

The household faces costs of capital accumulation given by:

$$\sum_{i\in\{b,m,s\}} K_{it+1} = I_t \left(1-\varphi\left(\frac{I_t}{I_{t-1}}\right)\right) + \sum_{i\in\{b,m,s\}} (1-\delta(u_{it}))K_{it} \qquad (7)$$

The function $\varphi(x_t)$ has the properties proposed by Christiano et al. (2005) and Jaimovich and Rebelo (2009), namely: $\varphi(x) = 0$, $\varphi'(x) = 0$, and $\varphi''(x) > 0$, where x is the growth factor of investment on the balanced growth path. Thus, the replacement of capital on this path is costless. The rates of capital depreciation δ_{it} depend on the degree of capital utilization u_{it}. As in Jaimovich and Rebelo (2009), the functions δ satisfy $\delta'(u_{it}) > 0$, $\delta''(u_{it}) \geq 0$, with the elasticity of $\delta'(u_{it})$ being constant.

The household must choose his effective supply of capital to sector $i \in \{b,m,s\}$ before the sectoral shocks are revealed while he is able to determine his supply of labor after the realization of the shocks. Thus, there is a friction in the allocation of capital but not in the allocation of labor. Besides the law of capital accumulation (7) and the accumulation of homes (6), the household's decision must satisfy his budget constraint:

$$C_t + I_t + P_{ht}[H_{t+1} - (1-\delta_h)H_t] \leq P_{lt}l_t + \sum_{i\in\{b,m,s\}} \left[r_{it}u_{it}K_{it} + W_tN_{it}\right]. \qquad (8)$$

The left-hand side represents the household's expenditures on consumption, business investment, and new homes, while the right-hand side gives his income from labor, renting capital and selling land to the producers of intermediary goods and new homes.

National accounts. DH implement a hypothetical rental rate for housing denoted Q_t to define consumption and GDP consistently with the National Income and Product Accounts (NIPA). This rate equals the marginal rate of substitution between consumption and housing. The equivalent to the NIPA PCE in the model is the sum of consumption C_t and the rents for housing Q_tH_t. The following holds for GDP: $Y_t = PCE_t + I_t + P_{dt}Y_{dt}$.

2.2 Calibration

The assumptions on the adjustment cost function $\varphi(I_t/I_{t-1})$ ensure that these costs bear no influence on the model's balanced growth path. In addition, identical relations between the degree of capital utilization and the rate of capital depreciation $\delta(u_{it})$ imply that the household will choose the same degree of capital utilization for all three kinds of capital usage. I normalize $u = u_i$ to one. As a consequence, the model's balanced growth path is the same as the one of the DH model (except for the government's share in output). In order to compare both models I will employ the parameter values of DH wherever possible.[3]

Table 2: Parameter values

Risk aversion: σ	2				
Discount factor: β	0.9688*				
k	C	H	N		
k's share in utility: μ_k	0.3*	0.04*	0.66*		
i	b	m	s		
Autoregressive coefficients in i: ρ_i	See table 3				
Std. dev. of innovations in i: σ_i	See table 3				
Trend growth rates in i: g_{A_i}	-0.27%	3.1%	2.37%		
Capital share in i: θ_i	0.132	0.309	0.37		
j	c		d		
construction good share in j: B_j	0.031		0.47		
manufacturing good share in j: M_j	0.27		0.24		
service good share in j: S_j	0.7		0.29		
Land share in new houses: ϕ:	0.2				
Depreciation rate for houses: δ_h	0.0127**				
Capital depreciation elasticity and u=1	$\eta_{\delta';u_{it}}$ =0.62*		$\delta(1)$ =0.089*		
exogenous steady state values:	K/Y	P_hH/Y	$r-\delta(1)$	N	u
	1.52	1.56**	0.06	0.3	1

* endogenous by the model; ** based on the stock of residential structures S ($P_dS/K = 1$ $\delta_s = 0.157$ from DH), Appendix 2.4 provides more information.

Table 3: Estimation of exogenous shocks

	b	m	s
ρ_i	0.693	0.855	0.924
(S.E.)	(0.087)	(0.075)	(0.042)
$\sigma_{\epsilon,i}$	0.041	0.037	0.018

Own calculations, based on data from Davis and Heathcote (2002)

For a given net real interest rate of capital (see Table 2), the normalization of $u = u_i = 1$ determines $\delta(1) = 0.0895$ in the steady state. Furthermore, it determines the constant elasticity of $\delta'(u_{it})$. The respective value is given by $\delta''(u_{it})u_{it}/\delta'(u_{it}) = 0.67$.

I estimate the parameters ρ_i and σ_i of (3) from the detrended Solow residuals obtained from Davis and Heathcote (2002). Table 3 presents the results. The persistence

[3] The stock and the depreciation rate of houses is based on residential structures. I choose the values of residential structures as with DH. Since I choose another value for the land share of new houses, the depreciation rate and the stock of housing to GDP rate differ from DH. See Appendix 2.4 for more information.

parameters ρ_i are close to the estimates of the diagonal elements of the transition matrix reported by DH. This also holds for the standard deviations of the innovations σ_i. But keep in mind that my model does not allow for spillover effects and restricts the off-diagonal elements of their covariance matrix to zero.

Key parameters of the model are $\varphi''(x)$ and ϕ. The former determines the adjustment costs of productive capital and the latter the adjustment costs in the accumulation of homes. For both, there is little guidance in the literature.

Davis and Heathcote (2007) present evidence for a considerable volatility and a large increase in the share of land's value of the value of existing houses. This share increased between 1985 and 2006 from 30-35 percent to 40-45 percent with an average of 36 percent. These results are in line with more recent explorations by Knoll et al. (2017).

In the long run an analytical link between the housing adjustment cost parameter, which is also interpretable as the share of raw land's value in the value of new houses, and the land's share in the value of existing houses exists. This link is presented in the Appendix 2.4. In order to match the observed land share in existing houses ($= 0.36$), I increase the DH value of $\phi = 0.106$ to $\phi = 0.2$.

My target for the choice of $\varphi''(x)$ is to match the empirically observed standard deviation of business investment relative to GDP. I achieve this for $\varphi''(x) = 0.4$. In addition, I check the sensitivity of my results with respect to choice of the adjustment costs parameters $\varphi''(x)$ and ϕ as well as of the extensions, individually.

2.3 Convex adjustment costs

As mentioned above, new houses as well as new business capital face adjustment costs. Since they are the key drivers of the model, I discuss them in detail. Residential investment is tied to the fix factor land. Following this, new houses are an increasing strict monotonic concave function of residential investment. Due to Jensen's inequality fluctuations in residential investment leads to loses in the amount of new houses. The adjustment costs in business investment arise due to changes in the amount of the investment in comparison to the previous period.

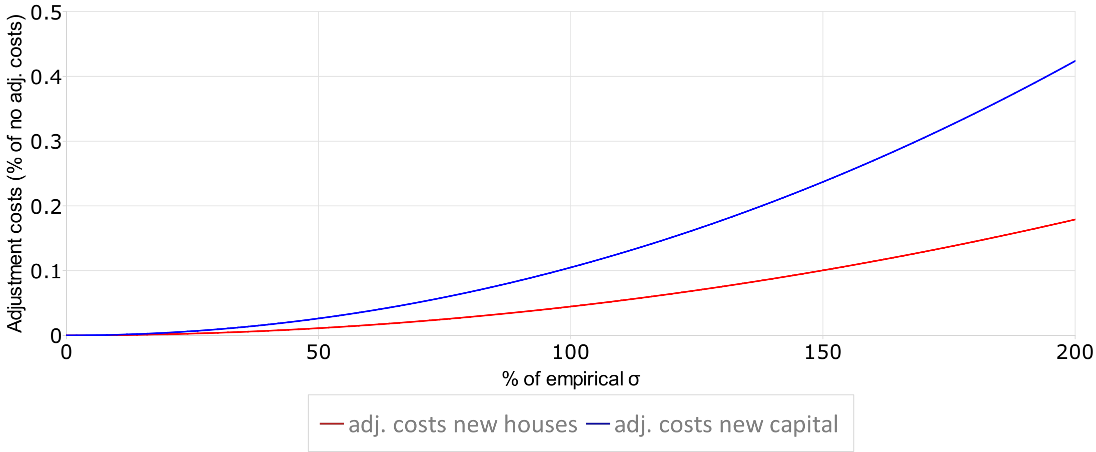
Figure 2 represents a numerical computation of the different adjustment costs. For this, I take for both investment types x:

$$x_1 + x_2 = 2x^*; \quad x_1 \in [x^*, x^*(1 + 2\sigma_x)]; \quad x_2 \in [x^*, x^*(1 - 2\sigma_x)];$$

where x^* is the amount of the investment in the steady state and σ_x is the empirical percentage standard deviation. From this I derive the adjustment costs relative to zero adjustment costs:

$$C_{Y_d} = 1 - \frac{1}{2}\frac{Y_{d1}^{1-\phi} + Y_{d2}^{1-\phi}}{Y_d^{*1-\phi}}$$
$$C_I = \frac{1}{2I^*}\left(I_1\varphi\left(\frac{I_1}{I_2}\right) + I_2\varphi\left(\frac{I_2}{I_1}\right)\right)$$

Figure 2: Adjustment costs



Adjustment costs computed for the presented calibration and a constant fluctuation around the steady state. Costs are related to zero fluctuations, which is interpretable as zero adjustment costs.

The figure shows, adjustment costs to new capital are higher than those to new houses. For alternating investments with the empirical observed volatility, new capital faces about 0.1 percent and new houses about 0.043 percent losses relative to zero fluctuation output. This difference depends on the parameters ϕ and φ, not on the type of adjustment costs. Furthermore, both adjustment costs are convex subject to the volatility. Hence, both investment types face, convex adjustment costs.

The main difference of these types is intertemporal. The decision on the amount of residential investment is a static one. The decision about business investment is subject to the amount in the previous period. Hence, the optimal decision today internalizes changes in adjustment costs tomorrow. It turns out that smooth adjustments lower the losses.

3 Results

This section presents the results from simulations of the model and its ability to reproduce the stylized facts of the data. A detailed analysis of the impulse responses and the following robustness checks to the various shocks will uncover the internal propagation mechanisms.

3.1 Second Moments

Table 4 presents results from simulations of various versions of the model. Second moments of HP-filtered data are averages over 1000 simulations each with 250 periods of observation. The filter weight is 100.

The second column of the Table displays the results from the extended DH model presented in Section 2, the third column presents second moments from the stripped-down DH model with independent technology shocks, and column four reports second moments from the DH model with correlated technology shocks as in employed by DH.

In the interest of readability, hereinafter, I call my model "extended model", the stripped-down DH model with independent shocks "DH-AR model" and the DH model with correlated shocks "DH-VAR model".

Consider first the standard deviations of major economic variables relative to the standard deviation of GDP. They are quite similar in all versions of the model and capture the fact that the standard deviation of residential investment is about more than twice as large as business investment. Additionally, output and hours worked are most volatile in the construction and less so in the service sector. The PCE in the extended model fits the data best. Among the three variants, the extended model predicts the largest relative standard deviation of house prices, which is still smaller than empirically observed.[4] All models also underestimate the volatility of hours worked and the extended model in particular.

[4] Dorofeenko et al. (2014) solve this problem by adding a credit channel and time varying uncertainty.

Table 4: Simulated second moments

SD	Extended model			DH-AR model			DH-VAR model		
GDP		1.42			1.40			1.80	
% SD to GDP									
PCE		0.73			0.60			0.58	
Hours worked		0.25			0.34			0.36	
Business investment (Busi)		2.34			3.03			2.92	
Residential investment (Resi)		5.04			8.21			6.34	
House prices (p_h)		0.68			0.48			0.44	
Output by sector	x_b	x_m	x_s	x_b	x_m	x_s	x_b	x_m	x_s
	3.48	1.87	1.08	4.50	1.88	1.10	3.52	1.52	0.95
Hours by sector	N_b	N_m	N_s	N_b	N_m	N_s	N_b	N_m	N_s
	1.24	0.25	0.23	2.16	0.34	0.33	1.71	0.35	0.32
Correlations									
GDP, PCE		0.98**			0.96**			0.96**	
p_h, GDP		0.92**			0.72**			0.78**	
PCE, Busi		0.85**			0.87**			0.88**	
PCE, Resi		0.37**			0.11			0.36**	
Resi, Busi		0.27**			-0.03			0.31**	
p_h, Resi		0.22**			-0.31			-0.01	
Output by sector	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s
	0.17*	0.12*	0.24**	0.12*	0.00	0.24**	0.24**	0.33**	0.69**
Hours by sector	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s
	0.53**	0.34**	0.98**	0.34**	0.06	0.96**	0.58**	0.40**	0.98**
Lead-lag correlations	i=1	i=0	i=-1	i=1	i=0	i=-1	i=1	i=0	i=-1
Busi_{t-i}, GDP_t	0.48**	0.93**	0.76**	0.53**	0.94**	0.28**	0.53**	0.96**	0.32**
Resi_{t-i}, GDP_t	0.23**	0.44**	0.10	0.05	0.23**	0.21**	0.22**	0.47**	0.23**
Busi_{t-i}, Resi_t	0.03	0.27**	0.26**	0.10	-0.03	-0.07	0.19*	0.31**	0.10*

Averages from 1000 simulations with 250 periods each. Moments are from (per capita) logged HP-filtered values with filter weight 100. All computations are done with the same standard normal distributed random numbers. All variables are stationary. Quadratic policy functions were used. * >90% of simulations r_{xy} is greater than zero; ** >99% of simulations r_{xy} is greater than zero

Consider next the co-movements. With respect to GDP, PCE, house prices, and business investment all three versions of the model are in line with the data and predict positive correlations between these variables. The extended model as well as the DH-VAR model match the positive correlations of GDP, PCE and business investment with residential investment. The DH-AR model cannot reproduce this pattern. Both versions of the DH model also fail to mimic the positive correlation between house prices and residential investment. The extended model only predicts the correct sign but underestimates the empirically observed magnitude. The DH-AR model is also unable to explain the positive correlation between output of the construction and the service sector.

A more detailed investigation of the distribution of the correlation coefficients reveals that in the extended model all aggregates and house prices co-move with a probability higher than 99 percent and that sectoral outputs co-move with a probability higher than 90 percent. This is neither the case in the DH-AR nor DH-VAR model. Summarizing, the extended model is the only one which accounts for all co-movements.[5]

Finally, consider the lead-lag structure of residential and business investment in the extended model. GDP, business and residential investment tend toward the empirical observed pattern. Residential investment leads more than it lags GDP and vice versa for business investment and GDP. In addition, the correlation coefficient between contemporaneous residential investment and one year ahead business investment is almost the same size as the contemporaneous correlation between both variables. Accordingly, the extended model achieves partially success accounting for the empirically observed lead-lag pattern. The correlations reported in Table 4 show that both DH models are unable to reproduce this pattern.

3.2 Impulse responses

To gain insight into the extended model's propagation mechanism, Figure 3, 5 and 7 present impulse responses of the model's variables due to a shock in the construction, manufacturing and service sector, respectively. Figure 4, 6 and 8 display the corresponding information for the DH-AR model. The size of the shock is equal to $\sigma = 1.44$ percent and it's persistence is equal to $\rho = 0.66$. This corresponds to $\sigma = 0.0072$ and

[5] The original model by DH (with government and population growth) reproduces a weaker correlation between the two investment types as well as a stronger negative correlation between residential investments and house prices. Further, with independent sectoral shocks all negative correlations are slightly stronger than in the stripped-down version.

$\rho = 0.9$ on a quarterly frequency, values often employed in the literature (see e.g. Heer and Maussner (2009)).

Figure 3 and 4 present impulse responses to a shock in the construction sector. In the extended model the shock has positive effects except for the price of the construction good and for house prices. In the DH-AR model the shock leads to a decline in the production of service goods and in business investment. PCE are slightly positive, but nearly unchanged. Putting this together the consumption and business investment producing good sector's output declines.

Figure 5 considers the effects of a shock in the manufacturing sector. Except for the price of manufacturing goods, the shock triggers a positive co-movement between sectoral outputs and aggregate economic activity, as measured by GDP, PCE, and investment. The same pattern emerges in the DH-AR model as illustrated in Figure 6. However, while business investment peaks in the first period in the DH-AR model, the maximum impact on this variable occurs in the second period in the extended model, quite in line with the lead-lag structure observed in the second moments of the simulated time series.

Figure 3: Shock to construction productivity (extended model)

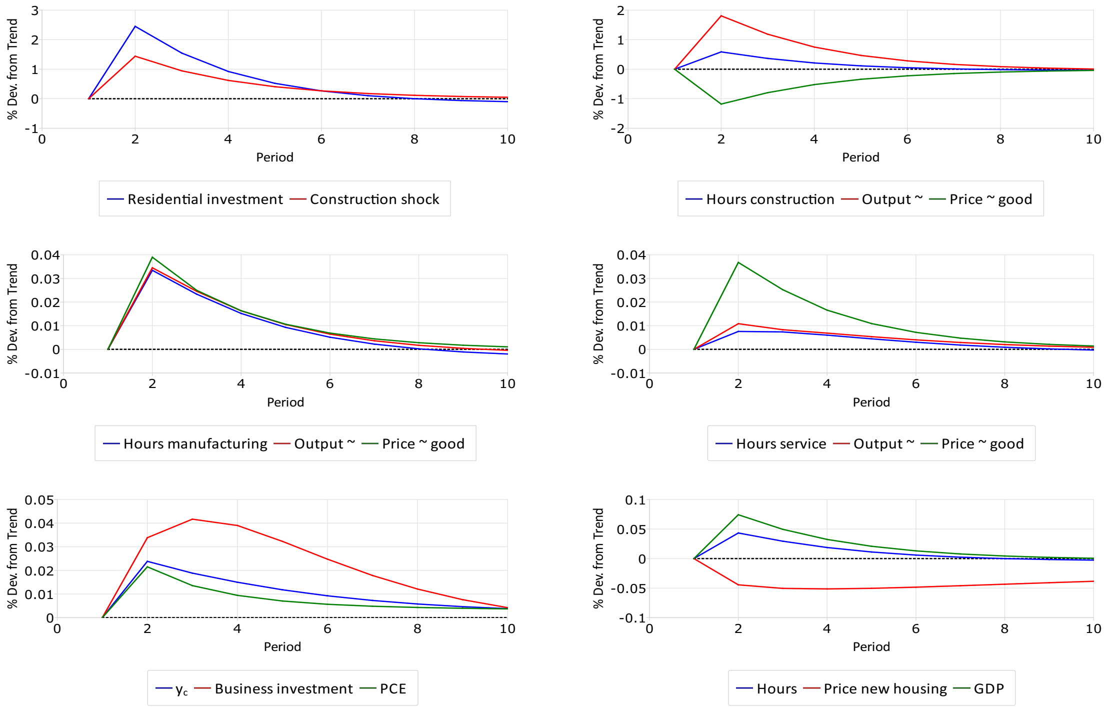


Figure 4: Shock to construction productivity (DH-AR model)

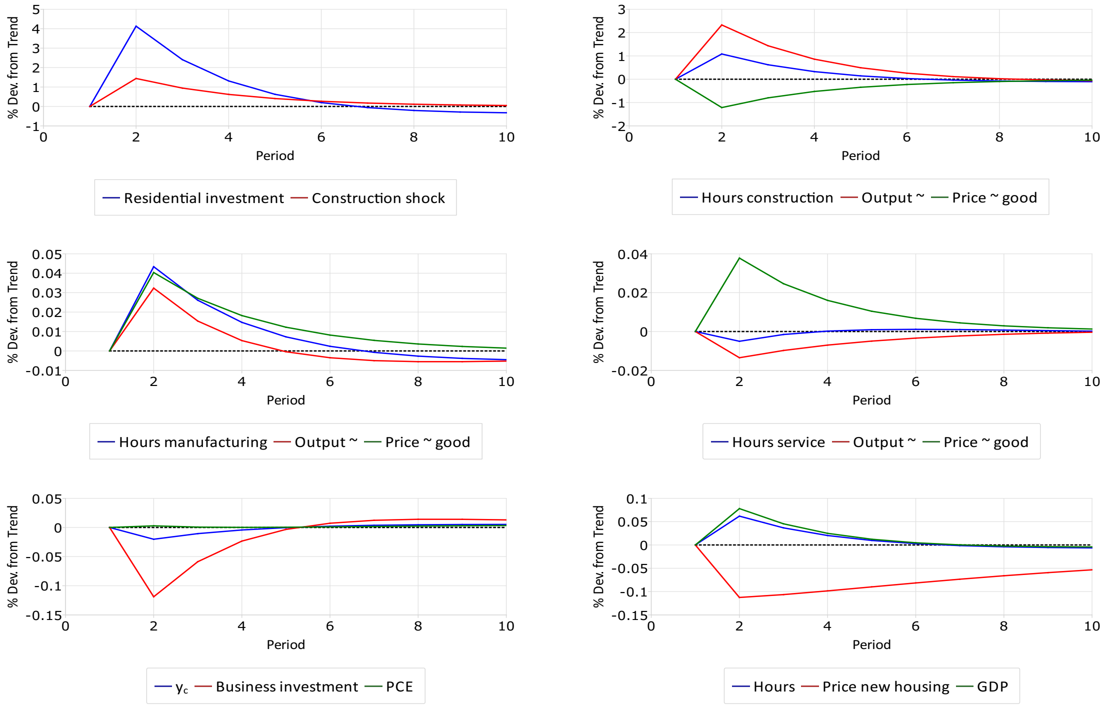


Figure 5: Shock to manufacturing productivity (extended model)

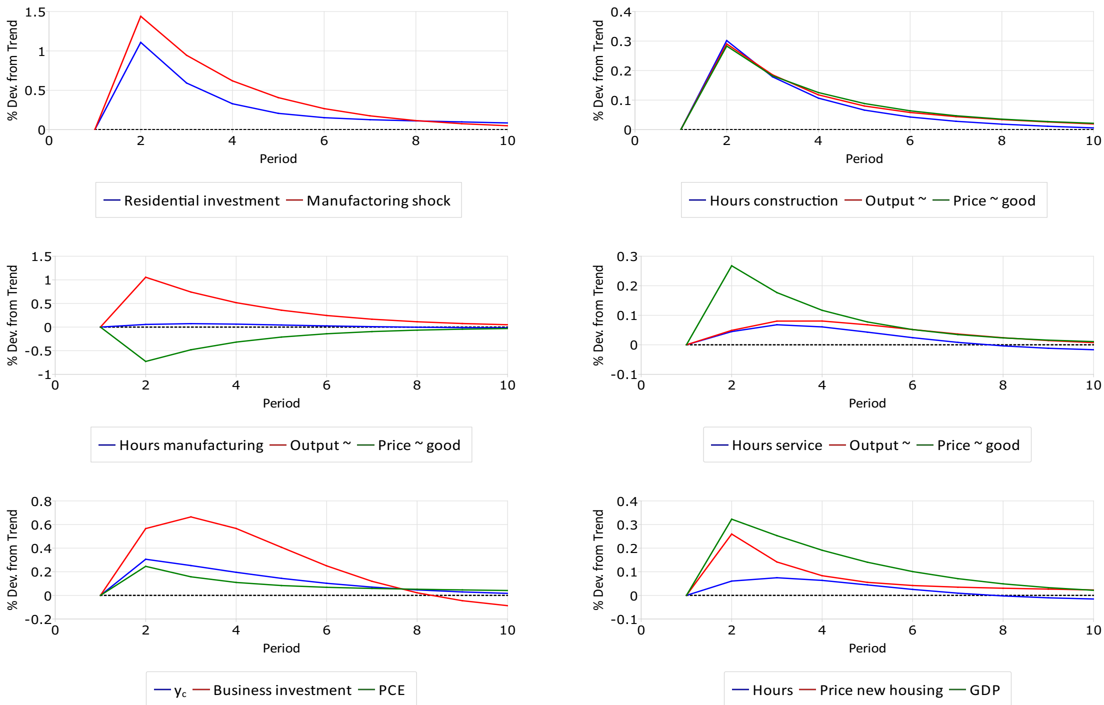


Figure 6: Shock to manufacturing productivity (DH-AR model)

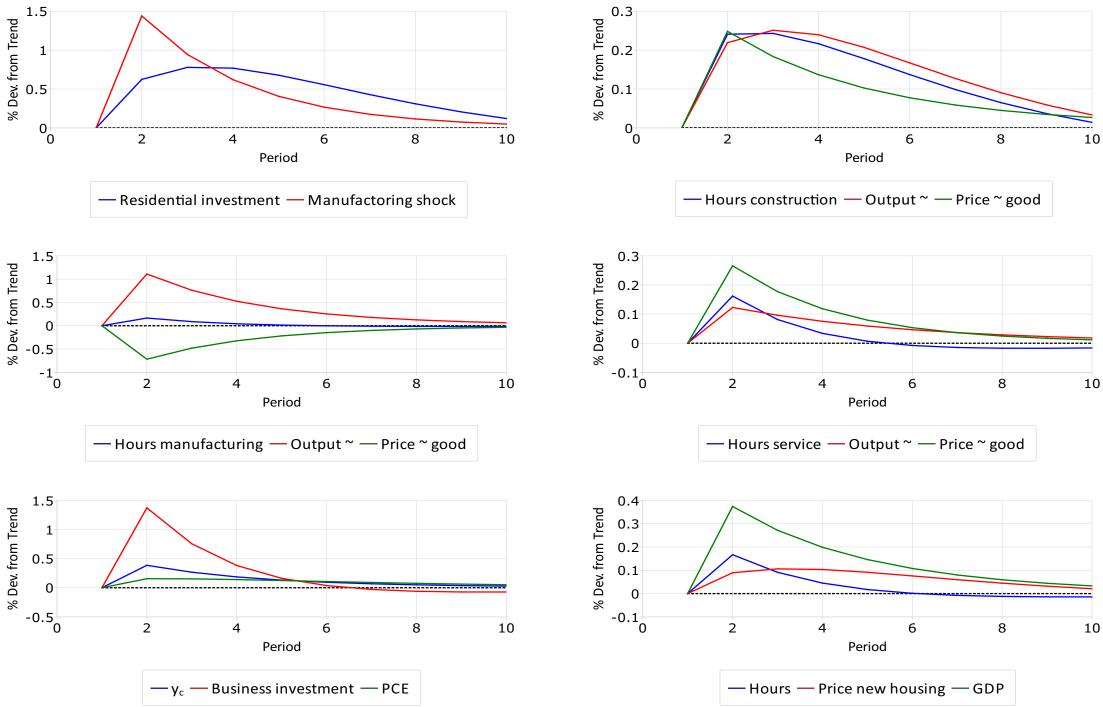


Figure 7: Shock to service productivity (extended model)

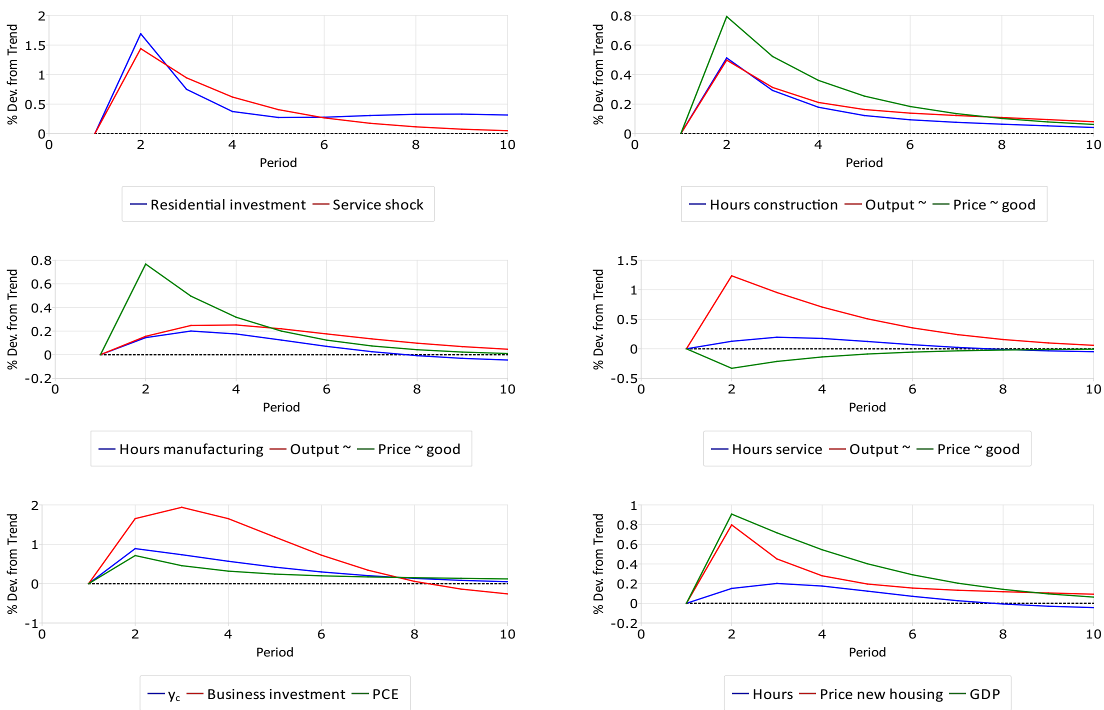


Figure 8: Shock to service productivity (DH-AR model)

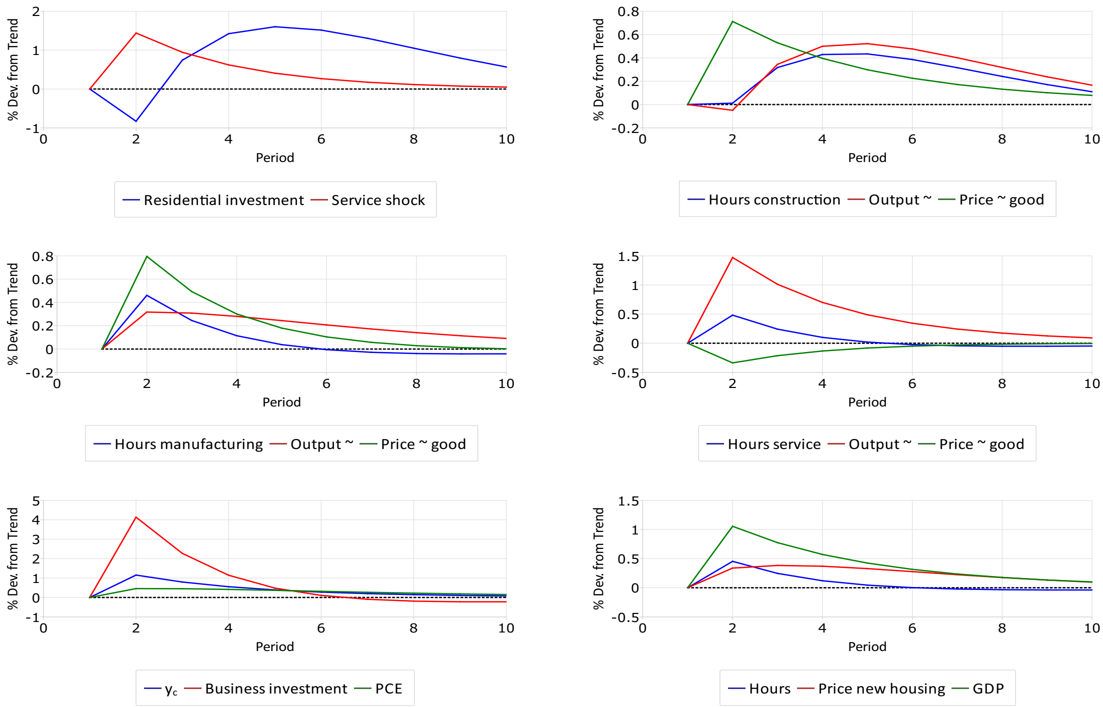


Figure 7 and 8 display the impulse responses to a shock in the service sector. Again, in the extended model there are positive effects on sectoral and aggregate variables, except for the price of the service good. In contrast, the DH-AR model implies short-term negative effects in the construction sector (output in this sector decreases) and a decline in residential investment.[6] Furthermore, as in the case of the construction sector shock, the extended model predicts that business investment peaks in the second period.

In the extended model the response to any shock is positive correlated with any quantity, of course except relative prices. Hence, these co-movements are determined by the model and the corresponding calibration.

While in the DH-AR model only imperfect substitution and adjustment costs in the production of new houses are at work, in the extended version also capacity utilization, adjustment costs of capital and limited sectoral mobility of capital determine the transmission of the shocks.

[6] Although hours increase, the output falls. Since in the DH-AR-model the intersectoral capital mobility is not limited, this is possible. This seems also plausible, because the construction production is relatively intensive in labor but not in capital.

Consider first imperfect substitution. Each shock triggers both, an income and a substitution effect. As long as the income effect dominates, the demand for all final goods will move in the same direction as the shock. The same holds true for the production of intermediary goods. A positive shock in one sector increases the production in all other sectors, if its impact on the demand for the final goods is positive. As Figure 6 shows, this effect is sufficient to generate positive co-movements even in the DH-AR model. The reason is that the production elasticities of manufacturing goods are quite similar in the production of both final goods. Hence, price changes, and, in turn, substitution effects are small. Figures 4 and 8 reveal that the substitution effects dominate in the DH-AR model in the case of shocks to the construction and the service sector, respectively.

Positive correlated shocks that increase the productivity not only in one sector reduce the size of substitution effects and increase the size of the income effect, and, thus explain the co-movement in the DH-VAR model.

Consider second the effect of adjustment costs on the propagation of shocks.

It is straightforward to show that asset prices, Tobin's marginal q for capital , Tq_t and house prices P_{ht} are determined by :[7]

$$\begin{aligned} Tq_t &= \mathbb{E}_t M_{t+1} \left[\theta_i \frac{p_{it+1} X_{it+1}}{K_{it+1}} + (1 - \delta(u_{it+1})) Tq_{t+1} \right] \\ P_{ht} &= \mathbb{E}_t M_{t+1} \left(Q_{t+1} + (1 - \delta_h) P_{ht+1} \right) \end{aligned}$$

respectively, where M_{t+1} is the stochastic discount factor. Asset prices equate the expected discounted return (in terms of utility) of an additional unit of investment with the current marginal utility of consumption. Adjustment costs for capital reduce the return of business investment and increase the demand for consumption goods and for investment in residential structures. Analogously, adjustment costs in the production of new homes due to a given supply of land increase consumption and favor the demand for business investment. I will call this effect a restricted intertemporal substitution.

Adjustment costs of capital are responsible for the hump-shaped pattern of the impulse response of business investment (compare Figures 3, 5, and 7). As mentioned

[7] I derivate these expressions in Appendix 2.2.

before, history matters and an increase in business investments today lowers the losses of higher investment tomorrow. Hence, it is optimal to invest in a hump-shaped form. This leads to the lag of business investment.[8]

Adjustment costs and capacity utilization interact in the following way: increases in business investment lower the future costs of replacing capital so that current increases in capacity utilization become less costly. This strengthens the co-movements on the intermediate stage of production as can be seen from Figures 9-11. This interaction is also responsible for the hump-shaped impulse responses of capacity utilization in the manufacturing and service sector. In addition, the increasing co-movements in production due to increasing co-movements in capacity utilization enhance the income effects, but not the substitution effect. This strengthens the co-movements of aggregated economic activity.

Figure 9: Variation in capital utilization due to a construction shock

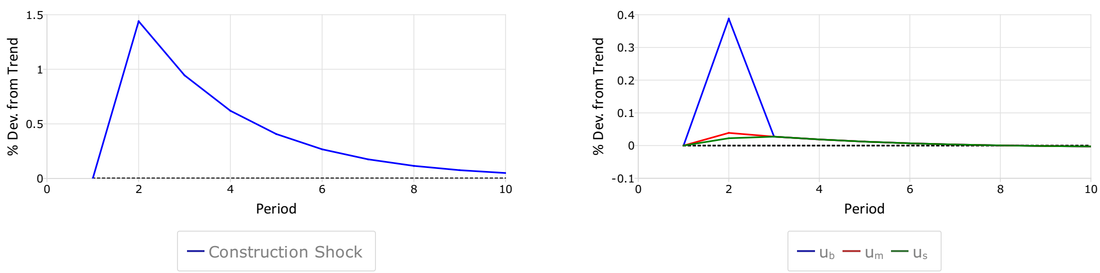
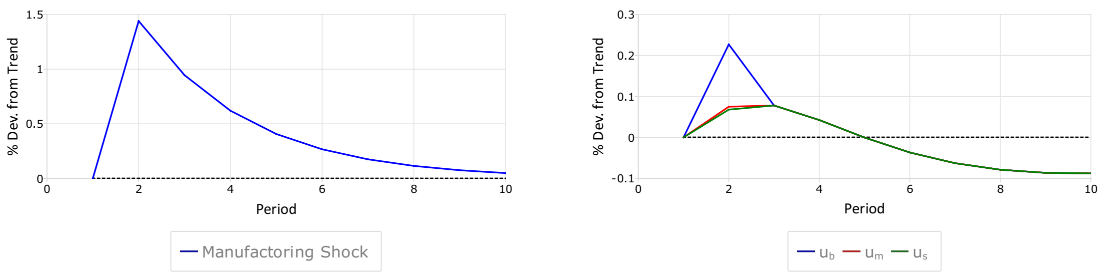
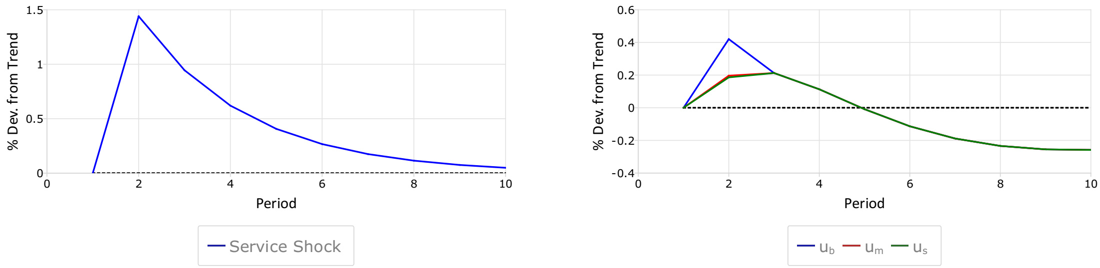


Figure 10: Variation in capital utilization due to a manufacturing shock



[8] Since business investment lags and business investment is part of the GDP residential investment leads GDP.

Figure 11: Variation in capital utilization due to a service shock



The effect of limited capital mobility is minor. This and the other mentioned effects are considered individually in the following robustness analysis section.

3.3 Robustness analysis

The following robustness analysis works out the sensitivity of the key parameters ϕ and φ as well as the impact of the particular extensions. Due to this, the effects of these extensions becomes more clear.

Adjustment costs in housing: First consider the higher land share in housing. This enhances the concavity of new houses with respect to residential investment and, as shown above, increases adjustment costs in housing. The second column of table 5 presents second moments for the DH-AR Model with an increased land share in housing. The higher adjustment costs lower the volatility of residential investment to 40 percent and increase the volatility of business investment slightly. To this end, the residential investment is less than twice as volatile then business investment. Changes of other standard deviations are minor. All contemporaneous correlation coefficients related to residential investment increase and tend towards to the data. This is due to the so-called restricted intertemporal substitution. Only the correlation with house prices is still negative. Changes in cross-correlations are minor.

Figure 12 shows the correlation between house prices and residential investments for different model specifications. These correlations are increasing functions of the land share ϕ on the interval $[0.1, 0.3]$. The slope is similar in all specifications. The land share employed by DH ($\phi = 0.106$) is not sufficient to introduce a positive correlation in any specification. The full extended model accounts for a positive correlation for $\phi > 0.11$: At the land share employed in my simulations ($\phi = 0.2$) the correlation is already positive in all model specifications, besides the DH-AR-Model.

Table 5: Simulated second moments

SD	DH-AR $\phi = 0.2$			+ adj. costs: $\varphi'' = 0.4$			+limited capital mobility		
GDP		1.36			1.20			1.20	
% SD to GDP									
PCE		0.73			0.77			0.77	
Hours worked		0.22			0.22			0.22	
Business investment (Busi)		3.05			2.07			2.08	
Residential investment (Resi)		5.96			5.78			5.66	
House prices (p_h)		0.49			0.70			0.70	
Output by sector	x_b	x_m	x_s	x_b	x_m	x_s	x_b	x_m	x_s
	3.59	1.89	1.10	4.08	2.71	1.11	3.91	2.31	1.13
Hours by sector	N_b	N_m	N_s	N_b	N_m	N_s	N_b	N_m	N_s
	1.21	0.33	0.33	1.40	0.21	0.19	1.34	0.21	0.19
Correlations									
GDP, PCE		0.97			0.98			0.98	
p_h, GDP		0.96			0.92			0.92	
PCE, Busi		0.88			0.89			0.89	
PCE, Resi		0.18			0.30			0.31	
Resi, Busi		0.11			0.24			0.25	
p_h, Resi		-0.06			0.11			0.12	
Output by sector	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s
	0.11	0.04	0.24	0.14	0.05	0.10	0.13	0.06	0.10
Hours by sector	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s
	0.38	0.24	0.99	0.57	0.31	0.96	0.57	0.32	0.96
Lead-lag correlations	i=1	i=0	i=-1	i=1	i=0	i=-1	i=1	i=0	i=-1
Busi_{t-i}, GDP_t	0.50	0.96	0.27	0.44	0.94	0.68	0.44	0.94	0.68
Resi_{t-i}, GDP_t	0.06	0.25	0.22	0.17	0.40	0.10	0.17	0.40	0.11
Busi_{t-i}, Resi_t	0.16	0.11	-0.03	0.05	0.24	0.19	0.06	0.25	0.19

Averages from 500 simulations with 54 periods each. Moments are from (per capita) logged HP-filtered values with filter weight 100. All computations are done with the same standard normal distributed random numbers. All variables are stationary. Linear policy functions were used.

Table 6: Simulated second moments

SD	Extended model $\varphi = 0$			CEE cap. utilization			Extended model (VAR)		
GDP		1.81			1.22			1.82	
% SD to GDP									
PCE		0.51			0.81			0.70	
Hours worked		0.40			0.26			0.27	
Business investment (Busi)		3.56			1.96			2.42	
Residential investment (Resi)		3.75			5.57			4.51	
House prices (p_h)		0.39			0.74			0.66	
Output by sector	x_b	x_m	x_s	x_b	x_m	x_s	x_b	x_m	x_s
	2.72	1.67	1.07	3.90	2.10	1.19	2.84	1.52	0.95
Hours by sector	N_b	N_m	N_s	N_b	N_m	N_s	N_b	N_m	N_s
	0.93	0.40	0.40	1.37	0.25	0.23	1.12	0.26	0.25
Correlations									
GDP, PCE		0.97			0.99			0.98	
p_h, GDP		0.88			0.93			0.94	
PCE, Busi		0.91			0.88			0.85	
PCE, Resi		0.20			0.35			0.62	
Resi, Busi		0.17			0.27			0.42	
p_h, Resi		-0.03			0.18			0.53	
Output by sector	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s
	0.16	0.13	0.47	0.16	0.10	0.21	0.32	0.40	0.68
Hours by sector	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s
	0.46	0.37	1.00	0.59	0.40	0.97	0.60	0.45	1.00
Lead-lag correlations	i=1	i=0	i=-1	i=1	i=0	i=-1	i=1	i=0	i=-1
Busi_{t-i}, GDP_t	0.50	0.98	0.34	0.41	0.93	0.71	0.48	0.93	0.77
Resi_{t-i}, GDP_t	0.07	0.26	0.19	0.19	0.42	0.11	0.38	0.62	0.09
Busi_{t-i}, Resi_t	0.15	0.17	0.00	0.05	0.27	0.23	-0.01	0.42	0.46

Averages from 500 simulations with 54 periods each. Moments are from (per capita) logged HP-filtered values with filter weight 100. All computations are done with the same standard normal distributed random numbers. All variables are stationary. Linear policy functions were used.

Figure 12: Correlation of house prices and residential investment subject to land share in new houses

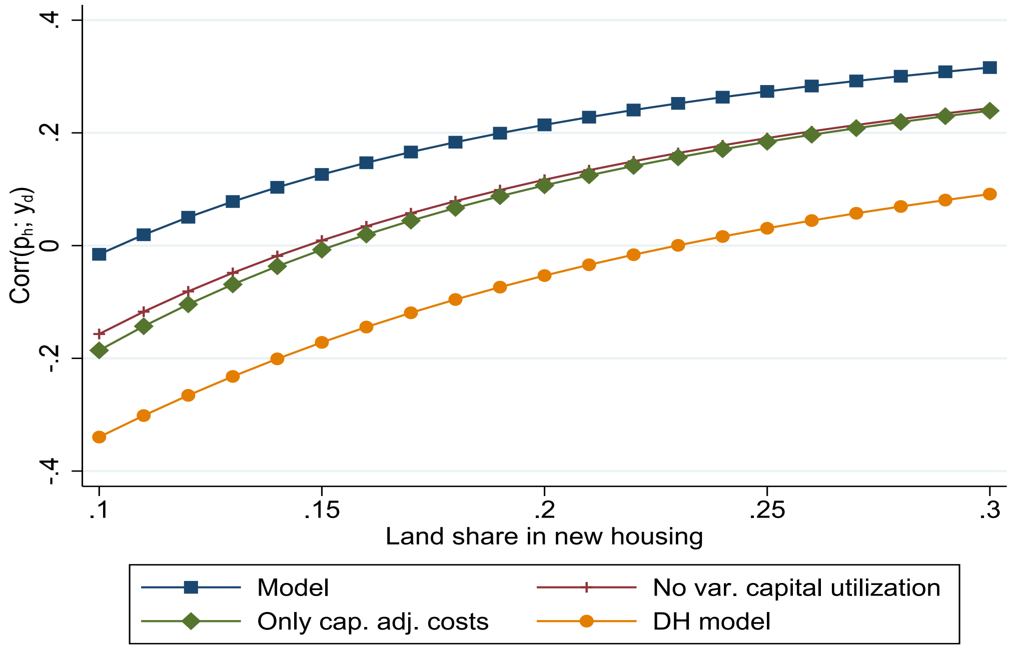
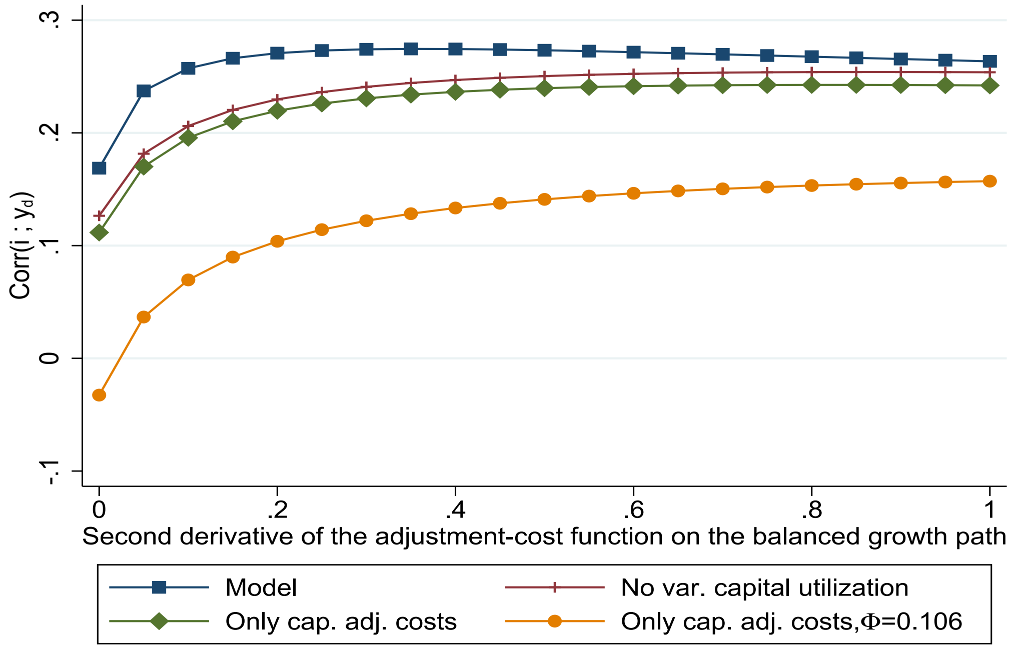


Figure 13 plots the correlation between business and residential investment as a function of the adjustment cost parameter φ'' on the interval $[0, 1]$. The vertical distance between the line marked with dots and the line marked with diamonds depicts the increased correlation if the land share increases from $\phi = 0.106$ (the value employed by DH) to $\phi = 0.2$. The effect of the higher land share is slightly larger for lower business adjustment costs.

Figure 14 plots the cross-correlation of the current period's business investment and the prior period's residential investment also as a function of φ''. The vertical distance between the line marked with dots and the line marked with diamonds depicts again the increased cross-correlation if the land share increases from $\phi = 0.106$ (the value employed by DH) to $\phi = 0.2$. The effect of the higher land share is slightly smaller for low business adjustment costs.

Figure 12 -13 gives evidence for the robustness of the increased land share to the correlation of residential investment with house prices, residential investment and lagged residential investment. It seems that the effect of a higher land share is constant and not very sensitive to some model specifications.

Figure 13: Correlation of business and residential investment subject to investment adjustment costs



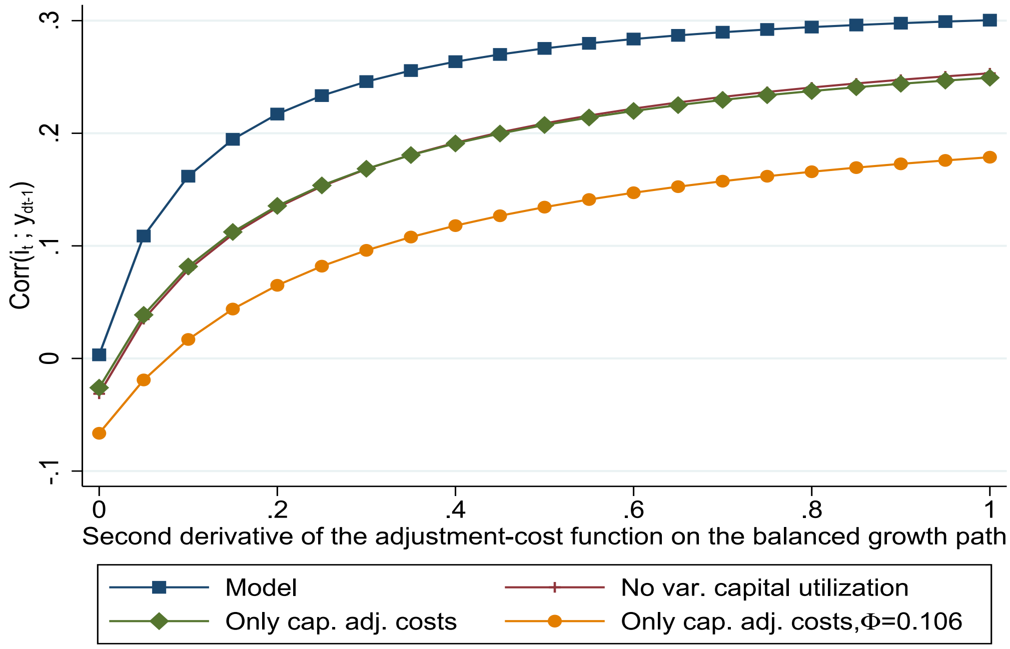
Adjustment costs in business capital: Column three of Table 8 presents the introduction of the employed business investment costs compared to column two. The higher adjustment costs lower the volatility of business investment by one third and increase the volatility of residential investment. The volatility ratio between the two investment types is higher than two. The volatility of house prices increases by nearly 50 percent, but does not exceed the volatility of the business cycle. Due to the lower intertemporal substitutability between the investment types, all correlations related with investment increase. As already mentioned, with CEE adjustment costs it is optimal to invest hump-shaped. Following this the skewness of the cross-correlogram tends towards the empirical one.

In Figure 12 the calibrated adjustment costs in the accumulation of capital shifts the function upward to the one marked with diamonds. At the land share employed in my simulations ($\phi = 0.2$) the correlation is already positive.

All specifications of the model in Figure 13 increase the correlation between both investment types markedly in the interval $[0, 0.3]$, while further increases of this parameter have only marginal effects. The differences between different specification decrease with business investment adjustment costs higher than $\varphi'' \approx 0.3$.

Figure 14 shows without capital adjustment costs, no model specification accounts

Figure 14: Cross-correlation with one lag of business and residential investment subject to investment adjustment costs



for the lead-lag structure between the two types of investment. Increasing capital adjustment costs, increases the cross-correlation of business investment and the prior period's residential investment. Again, this shows the effect of an optimal smooth business investment adjustment and the related restricted intertemporal substitution. In all specifications the slope is decreasing.

All model specifications are faced with large changes due to changes in business adjustment costs in the interval of $[0, 0.3]$, afterwards the model seems robust for higher values of business adjustment costs.

Limited capital mobility: The last column of Table 8 presents the effect of limited capital mobility compared to the specification of the previous column. Effects are marginal. The same is shown in Figure 12-14, where the tiny distance between the lines marked with diamonds and the lines with crosses illustrate the effects of limited capital mobility. The small effects of limited sectoral decreases with variable utilization of capital since this reduces the friction.

Variable capital utilization: For a detailed analysis of the variable capital utilization, column 2 of Table 6 presents second moments from the extended model without capital adjustment costs. Since limited capital mobility has marginal effects, the differences to the second column of Table 5 are mainly due to variable capital utilization. The volatility of business investment increases and that of residential decreases. They are nearly equal. This is due to a more flexible production, which also leads to stronger sectoral co-movements. These co-movements increase all reported correlation coefficients slightly. Effects on cross-correlations are marginal.

Differences between second moments of the extended model (column 2 Table 4) and the last column of Table 5 illustrates effects of variable capital utilization, by the presence of capital adjustment costs. The volatility of business investment increases and that of residential decreases again. In contrast to the absence of capital adjustment costs, residential investment is more than twice as volatile than business investment. Despite house prices, the effects on co-movements associated with residential investment are lower with the calibrated capital adjustment costs. Variable capital utilization enhances the effect of CEE adjustment costs on the lead-lag pattern of GDP, business and residential investment.

The distance between the line with crosses and the line with squares in Figure 14 reflects the larger effect on the cross-correlation between business and residential investment when CEE adjustment costs interacts with variable capital utilization. The effect increases until $\varphi'' \approx 0.4$ and afterwards slowly decreases.

The distance between the line with crosses and the line with squares in Figure 13 reflects a similar pattern of the effect of variable capital utilization on the contemporaneous correlation between residential and business investment. The effect peaks at $\varphi'' \approx 0.2$.

The positive effect on co-movements between house prices and residential investment decreases slowly with higher land share in housing on the presented interval in Figure 12. This is the distance between the line with crosses and the line with squares.

Capital utilization modeled as by Jaimovich and Rebelo (2009) has two different effects in the extended model. On the one hand, production is more flexible, which lowers substitution effects. On the other hand, higher business investments lower the future costs of replacing capital. Current increases in capacity utilization become less costly. I separate these effects by modeling variable capital utilization as by Christiano et al. (2005). Here, a higher utilization rate is costly in terms of the consumption/business investment good instead in terms of capital. An additional unit of business investment does not interact with the costs of higher capital utilization. As in the benchmark, the value of the elasticity of capital utilization costs is determined endogenously by the steady state. The second column of Table 6 presents the second moments. The volatility of business investment is slightly lower and that of residential investment is slightly higher. The coefficient which measures the leadership of business investment related to the business cycle decreases. All other cross-correlations are more in line with the data in the benchmark extended model. Co-movements are similar or slightly lower as with capital utilization costs modeled by Jaimovich and Rebelo (2009). In general, the interaction of capital utilization and adjustment costs propagate co-movements and the observed lead-lag pattern slightly. Nevertheless, the model reacts robust to alternative costs of higher capital utilization.

VAR-Schocks: As a last check, I combine the extensions with the exogenous VAR(1)-process. Second moments of this combination are presented in the last column of Table 6. Besides residential investment, the relative standard deviations are similar to the DH-VAR- and the extended model. Residential investment is less volatile, especially compared to the DH-VAR-model. The ratio of the investment's volatilities is slightly below two. Economic activity co-moves in this variation similar to the extended and DH-VAR-model or more. The same applies to the leadership of residential investment and the lagging business investment. Especially, co-movements and the lead-lag pattern related to residential investment accounts well for the presented stylized facts in Table 1.

This check gives evidence that the extensions and the VAR-process are not complements, neither perfect substitutes or equivalents, respectively.

4 Estimation

4.1 Methods

I apply Bayesian estimation to the models' underlying exogenous process. I estimate the DH- and extended model, each with three independent AR(1)- and one VAR(1)-process, respectively. I estimate only the parameters of the processes and no further parameters. This helps to identify the explanatory power of the extensions. Furthermore, I apply a posterior odd comparison.

I solve the model with relative deviations around the linear balanced growth path for estimation exercises. As recommended by DeJong and Dave (2011), I take the residuals from a log linear regression instead of HP-filtered data to be consistent to the

Table 7: Prior Distributions

Parameter	Domain	Density	Mean	Std. Deviation
AR				
ρ_b	[0,1)	Beta	0.693	0.087
ρ_m	[0,1)	Beta	0.855	0.075
ρ_s	[0,1)	Beta	0.924	0.042
$\sigma_{\epsilon,b}$	$\mathbb{R}^+$	InvGamma	0.041	0.04
$\sigma_{\epsilon,m}$	$\mathbb{R}^+$	InvGamma	0.037	0.04
$\sigma_{\epsilon,s}$	$\mathbb{R}^+$	InvGamma	0.018	0.04
VAR				
ρ_b	[0,1)	Beta	0.708	0.089
ρ_{bm}	(-1,1)	TruncNorm.	0.009	0.083
ρ_{bs}	(-1,1)	TruncNorm.	-0.092	0.098
ρ_{mb}	(-1,1)	TruncNorm.	-0.006	0.078
ρ_m	[0,1)	Beta	0.871	0.073
ρ_{ms}	(-1,1)	TruncNorm.	-0.15	0.087
ρ_{sb}	(-1,1)	TruncNorm.	0.003	0.038
ρ_{sm}	(-1,1)	TruncNorm.	0.027	0.036
ρ_s	[0,1)	Beta	0.92	0.42
Σ		InvWishart		4 d.f.*
$\sigma_{\epsilon,b}$	$\mathbb{R}^+$		0.041	
$\sigma_{\epsilon,m}$	$\mathbb{R}^+$		0.036	
$\sigma_{\epsilon,s}$	$\mathbb{R}^+$		0.018	
$100Cov(\epsilon_b, \epsilon_m)$	$\mathbb{R}$		0.013	
$100Cov(\epsilon_b, \epsilon_s)$	$\mathbb{R}$		0.022	
$100Cov(\epsilon_m, \epsilon_s)$	$\mathbb{R}$		0.031	

*degrees of freedom

models solution. I use time series of house prices, business and residential investment from 1970 to 2015.[9]

Priors are chosen differently for different kinds of exogenous processes, but are the same for identical ones. For the three times AR(1)-process my choice for the autoregressive parameters is the beta distribution and for the variances of the innovations the inverse gamma. The means equal the calibrated values. The standard deviations of the beta distributions equal those of the estimation of calibrated autoregressive coefficients and 0.04 for innovations as the annualized standard in the literature (see e.g. Smets and Wouters (2007).

For the VAR(1)-process, I choose for the diagonal elements of the autoregressive matrix a beta distribution and for the off-diagonals a normal distribution truncated

[9]The Appendix A gives an detailed overview.

at (-1,1). The means equal the calibrated values and the standard deviations of the distributions equal those of the calibrated estimation. The covariance matrix's prior is an inverted Wishart distribution, scaled by the calibrated covariance matrix. I choose four degrees of freedom to ensure the mean of the distribution equals the calibrated covariance matrix. Table 7 gives an overview of all priors.

For the AR(1) specification I apply a Metropolis-Hastings-Gaussian-Random-Walk algorithm as describe by Herbst and Schorfheide (2016) to approximate draws from the posteriors. For the VAR(1) models I use the sequential Monte-Carlo algorithm with likelihood tempering as described by Herbst and Schorfheide (2016), because the posteriors are probably multimodal.[10]

4.2 Results

Table 8: Posterior Distribution AR

		Quantile				
Parameter Value at	Modus	2.5%	5%	50%	95%	97.5%
DH-AR						
ρ_b	0.59	0.48	0.50	0.59	0.66	0.68
ρ_m	1	0.99	0.99	1	1	1
ρ_s	0.99	0.97	0.98	0.99	1	1
$\sigma_{\epsilon,b}$	0.06	0.05	0.05	0.06	0.07	0.08
$\sigma_{\epsilon,m}$	0.36	0.30	0.31	0.37	0.45	0.46
$\sigma_{\epsilon,s}$	0.13	0.11	0.11	0.13	0.15	0.16
Extended (AR)						
ρ_b	0.79	0.67	0.69	0.79	0.87	0.88
ρ_m	1	0.99	0.99	1	1	1
ρ_s	0.75	0.67	0.69	0.75	0.81	0.81
$\sigma_{\epsilon,b}$	0.02	0.02	0.02	0.03	0.03	0.03
$\sigma_{\epsilon,m}$	0.15	0.13	0.13	0.16	0.19	0.19
$\sigma_{\epsilon,s}$	0.04	0.03	0.03	0.04	0.05	0.05

Draws from the posterior are approximated via RWMH with $\theta^i \sim N(\theta^{i-1}, \Sigma)$. Σ is the negative inverse of the Hessian at the posterior's mode. I draw 100 000 times and burn the first 50 000.

Estimated parameters: Table 8 presents the posterior distributions of the exogenous processes' parameters in the DH-AR-Model and the extended (AR) model. First of all, the parameters of the shocks in the manufacturing sector are conspicuous. The autoregressive parameter is nearly one and the variance of the innovations is the largest of all three sectors, by far. Similar to the result in the impulse response consideration,

[10] Appendix C explains the preferences of the algorithm in detail.

this is due to similar elasticities in the production of the consumption/business investment good and the production of the residential investment good. The marginal substitution effect result, that the shock behaves like an aggregated one. The large volatility and the high persistence gives evidence that the business cycle is not only driven by sectoral shocks. The difference in the volatility of the manufacturing sector's innovation reflects the propagation of co-movements due to the extensions.

The service sector's autoregressive coefficients in the DH-AR-model are quite similar than to those in the manufacturing sector. This is not true in the extended model. Here, the persistence of the construction and service shock is similar. The volatility of the corresponding innovations is also similar, while in the DH-AR-model the volatility of the service sector is larger.

While the calibrated innovations as well as the priors have the highest standard deviation in the construction sector in both processes the sector's shock shows the smallest fluctuations at each quantile. Hence, there is evidence that the large fluctuations in residential investments are not due to large technology shocks in the construction sector. In the extended model the innovations show smaller volatilities.

An overview of the parameters posterior distribution of the VAR(1)-process is given in Table 9. The autoregressive matrix with negative parameters is difficult to interpret. There are no great outliers in comparison to the priors. Although, all correlations are positive in the prior, this is not true for the median of the covariances. In the DH-VAR-model only the median of the covariance between the innovations in the construction and manufacturing sector is positive and in the extended model only between the construction and the service sector. The interquartile range is very high. In both models the standard deviation of the manufacturing innovations is the highest. In the DH-VAR-model the particular quantiles of the standard deviations of the construction sector are slightly higher than the innovations in the service sector. Besides the 2.5percent quantile, the reverse is given in the extended model. The posteriors of the exogenous VAR(1)-process is indeed very diffuse.

Table 9: Posterior Distribution VAR

	Quantile						Quantile				
Parameter Value at	2.5%	5%	50%	95%	97.5%	Parameter Value at	2.5%	5%	50%	95%	97.5%
DH-VAR						Extended (VAR)					
ρ_b	0.73	0.75	0.85	0.92	0.93	ρ_b	0.58	0.63	0.75	0.84	0.86
ρ_{bm}	-0.13	0.03	0.15	0.25	0.27	ρ_{bm}	-0.16	-0.14	-0.06	0.06	0.09
ρ_{bs}	-0.32	-0.29	-0.13	0.01	0.03	ρ_{bs}	-0.27	-0.25	-0.12	0.02	0.04
ρ_{mb}	-0.05	-0.01	0.14	0.22	0.24	ρ_{mb}	-0.21	-0.19	0.08	0.07	0.09
ρ_m	0.86	0.87	0.94	0.98	0.98	ρ_m	0.90	0.92	0.97	0.99	0.99
ρ_{ms}	-0.31	-0.29	-0.17	-0.07	-0.06	ρ_{ms}	-0.23	-0.20	-0.07	0.01	0.02
ρ_{sb}	-0.03	0.00	0.05	0.09	0.09	ρ_{sb}	-0.08	-0.08	-0.02	0.04	0.06
ρ_{sm}	-0.05	-0.05	-0.01	0.03	0.07	ρ_{sm}	-0.05	0.02	0.08	0.13	0.14
ρ_s	0.88	0.89	0.93	0.97	0.98	ρ_s	0.89	0.90	0.95	0.98	0.99
$\sigma_{\epsilon,b}$	0.021	0.023	0.032	0.042	0.044	$\sigma_{\epsilon,b}$	0.016	0.016	0.021	0.030	0.033
$\sigma_{\epsilon,m}$	0.037	0.041	0.066	0.11	0.12	$\sigma_{\epsilon,m}$	0.028	0.032	0.056	0.10	0.12
$\sigma_{\epsilon,s}$	0.008	0.009	0.014	0.027	0.033	$\sigma_{\epsilon,s}$	0.014	0.017	0.024	0.036	0.042
$100Cov(\epsilon_b, \epsilon_m)$	-0.129	0.000	0.179	0.304	0.332	$100Cov(\epsilon_b, \epsilon_m)$	-0.244	-0.138	-0.013	0.038	0.048
$100Cov(\epsilon_b, \epsilon_s)$	-0.048	-0.042	-0.017	0.015	0.028	$100Cov(\epsilon_b, \epsilon_s)$	-0.011	-0.005	0.017	0.061	0.084
$100Cov(\epsilon_m, \epsilon_s)$	-0.294	-0.211	-0.052	0.015	0.041	$100Cov(\epsilon_m, \epsilon_s)$	-0.312	-0.202	-0.009	0.042	0.057

Draws from the posterior are approximated via SMC as described by Herbst and Schorfheide (2016). I use 2500 particles, 1500 bridges, one RWMH step and one block.

Posterior odd comparison: Table 10 gives in the in the last column the marginal likelihoods of the models. It turns out that the extended VAR model has the highest probability. According to Jeffreys (1961), there is decisive evidence in favor of the extended models compared to their particular DH-model. The extended AR-model accounts for a large fraction of the gap between the two DH-models.

The second column of Table 10 presents the values of the likelihood function of the calibrated models. This is interpretable as the marginal likelihoods, where the priors consist of independent degenerated distributions with an infinity mass at the calibrated parameters values. Since likelihood functions of such models have large cliffs and peaks, iot is not useful to compare them. It shows the fitness of the particular calibration. The calibration is more suitable for the models with AR-processes.

Table 10: Likelihood and Posterior values

Model	$\ln L(Y\|\theta^{cal})$	$\ln \int L(Y\|\theta)p(\theta)d\theta$
DH-AR	-8441.95	90.56
Extended (AR)	-7770.62	221.44
DH-VAR	-11781.13	280.25
Extended (VAR)	-26177.43	287.86

Marginal likelihood of the AR-models is calculated as weighted harmonic mean with a choice for the weights in line with Geweke (1999). Marginal likelihood of the VAR-models is calculated as the product of the bridges average unnormalized weights

5 Conclusion

This paper explores the role of uncorrelated sector specific technology shocks to induce aggregate economic fluctuations being in line with a number of well-established stylized facts. The facts reported in DH and echoed by several other papers include i) the co-movement of GDP, PCE, business and residential investment, aggregate hours, and house prices, ii) the fact that residential investment is more than twice as volatile as business investment, and iii) that business investment lags GDP while residential investment leads GDP.

DH present a multisectoral model with correlated shocks to sectoral labor augmenting technical progress which is able to explain fact ii) and mostly i) but fails to be in line with fact iii). This model with uncorrelated shocks is unable to generate co-movements in housing or rather in residential investment and the remaining economic activity. Hence, fact i) is mostly driven by the shock's correlation. I introduce two frictions in form of adjustment costs of capital as in Christiano et al. (2005) and limited sectoral mobility of capital as in Boldrin et al. (2001) into their model and increase the adjustment costs to new houses. Furthermore, I introduce variable capacity utilization as in Jaimovich and Rebelo (2009). The extended model is able to replicate facts i)-iii). The main improvement in the empirical plausibility of the model is due to adjustment costs whose effect is enhanced by capacity utilization. The effect of sectoral immobility is small. The results are robust for adjustment costs $\varphi'' > 0.3$.[11]

Impulse responses illustrate that all variables, except prices, co-move with each sectoral shock. Hence, sectoral and aggregate co-movements arise independently of the intensity and persistence of the particular shocks. The same is true for the cross-correlations of the investment types which is caused by different kinds of adjustment costs. Therefore, co-movements and the lead-lag pattern are fully endogenous.

The extended model matches most relative standard deviations quite well. The introduction and enlargement of adjustment costs fits the standard deviations of PCE, business and residential investment best. The volatility of house prices is barely sufficient. The standard deviation of the investment types is via a degree of freedom in φ'', but the value does not seem to be high.

Bayesian estimation suggests despite the internal propagations due to the extensions that the business cycle is not only driven by sectoral shocks. This shows the high volatility of the manufacturing sector. Nevertheless, in the DH-AR-model the quantiles of the manufacturing sector's shock standard deviation are twice as high as in the corresponding extended model. The standard deviations of the construction sector's innovation are low, especially in the extended model. This gives evidence that the high volatility of house prices as well as residential investment are not initiated by large technology shocks in the construction sector, but rather due to aggregated shocks.

Further examination of the triggers of business cycles should incorporate an aggregated shock. Albeit, in distinction to the VAR-models, without any negative sectoral correlations and non-zero off-diagonals on the autoregressive matrix. This enables a better interpretation of the manufacturing sectors exogenous process, but avoids the exogenous process drives results the model should accounts endogenously. This approach makes variance decomposition simple.

A way to examine the source of business cycles without aggregated shocks could be done as in the approach by Ireland (2004). Here, the measurement error of the

[11] Christiano et al. (2005) estimated in their benchmark model $\varphi'' = 2.45$ on a quarterly basis. Jaimovich and Rebelo (2009) choose $\varphi'' = 1.3$ in combination with capital utilization $(\delta''(u_{it})u_{it}/\delta'(u_{it}) = 0.15$ (here ≈ 0.6)) on a quarterly basis.

observation equation in the state space system captures all properties of the data that sectoral shocks could not explain.

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APPENDIX

(not for publication)

Housing and the Business Cycle Revisited

A Data

Pop: Population

SA1 Personal Income Summary: Personal Income, Population, Per Capita Personal Income: line 2. Source: Bureau of Economic Analysis (BEA)

GDP: nominal GDP_{2009}*quantity index $\text{GDP}_t/(100*\text{Pop}_t)$

NIPA nominal: Table 1.1.5 line 1; quantity index: Table 1.1.3 line 2, annual. Source: BEA

PCE: nominal PCE_{2009}*quantity index $\text{PCE}_t/(100*\text{Pop}_t)$

NIPA nominal: Table 1.1.5 line 2; quantity index: Table 1.1.3 line 2, annual. Source: BEA

Labor: Hours worked private $\text{industries}_t/(\text{Pop}_t)$

NIPA: Table 6.9B,C,D line 3. Source. BEA

BUSI: nominal $\text{nonresidential}_{2009}$*quantity index $\text{nonresidential}_t/(100*\text{Pop}_t)$
+nominal government investment_{2009}*quantity index government $\text{investment}_t/(100*\text{Pop}_t)$
−nominal gov. defense Investment_{2009}*quantity index gov. defense $\text{Investment}_t/(100*\text{Pop}_t)$

NIPA nominal: Table 1.1.5 line 9; quantity index: Table 1.1.3 line 9, annual.
NIPA nominal: Table 3.9.5 line 3,19; quantity index: Table 1.1.3 line 3,19, annual.
Source: BEA

RESI: nominal RESI_{2009}*quantity index $\text{RESI}_t/(100*\text{Pop}_t)$

NIPA nominal: Table 1.1.5 line 13; quantity index: Table 1.1.3 line 13, annual. Source: BEA

$\mathbf{x}_b$: nominal $\text{construction}_{2009}$*quantity index $\text{Construction}_t/(100*\text{Pop}_t)$

GDP by Industry nominal: GO line 11; quantity index: ChainQtyIndexes line 11, annual. Source: BEA

$\mathbf{x}_m$: nominal $\text{manufacturing}_{2009}$*quantity index $\text{manufacturing}_t/(100*\text{Pop}_t)$
+nominal mining_{2009}*quantity index $\text{mining}_t/(100*\text{Pop}_t)$
+nominal agriculture, forestry, fishing, and hunting_{2009}*quantity index $\sim_t/(100*\text{Pop}_t)$

GDP by Industry nominal: GO line 12,6,3; quantity index: ChainQtyIndexes line 12,6,3, annual. Source: BEA

$\mathbf{x}_s$: nominal wholesale trade$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal retail trade$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal transportation and warehousing$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal information$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal professional and business service$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal educational service, health care and social assistance$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal arts, entertainment, recreation, accommodation and food service $_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)
+nominal other services, except government$_{2009}$*quantity index $\sim_t$/(100*Pop$_t$)

GDP by Industry nominal: GO line 34,35,40,49,65,74,82,89; quantity index: ChainQtyIndexes line $\sim$, annual. Source: BEA

Output in finance, insurance and real estate is omitted due to consistence to DH. The reasons behind are calibration exercises. A large fraction of real estate value added is imputed from owner-occupied housing. Hence, accounting for real estate services would lead to a biased capital share in the service technology. See DH for further discussion.

$\mathbf{N}_b$: hours construction$_t$/(Pop$_t$)

$\leq$ 2000 NIPA: Table 6.9B,C line 8; $\geq$ 2000 NIPA: Table 6.9D line 9.
Source: BEA

$\mathbf{N}_m$: hours manufacturing$_t$/(Pop$_t$)
+hours mining$_t$/(Pop$_t$)
+hours agriculture, forestry, fishing, and hunting/(Pop$_t$)

$\leq$ 2000 NIPA: Table 6.9B,C line 9,7,4; $\geq$ 2000 NIPA: Table 6.9D line 10,7,4.
Source: BEA

$\mathbf{N}_s$: $\leq$ 2000
hours transportation and public utility$_t$/(Pop$_t$)
+hours wholesale trade$_t$/(Pop$_t$)
+hours retail trade$_t$/(Pop$_t$)
+hours Services$_t$/(Pop$_t$)
$\geq$ 2000
hours Utilities$_t$/(Pop$_t$)

+hours wholesale $\text{trade}_t/(\text{Pop}_t)$
+hours retail $\text{trade}_t/(\text{Pop}_t)$
+hours transportation and $\text{warehousing}_t/(\text{Pop}_t)$
+hours $\text{information}_t/(\text{Pop}_t)$
+hours professional and business $\text{service}_t/(\text{Pop}_t)$
+hours educational service, health care and social $\text{assistance}_t/(\text{Pop}_t)$
+hours arts, entertainment, recreation, accommodation and food $\text{service}_t/(\text{Pop}_t)$
+hours other services, except $\text{government}_t/(\text{Pop}_t)$

$\leq$ 2000 NIPA: Table 6.9B,C line 12,16,17,19;
$\geq$ 2000 NIPA: Table 6.9D line 13,14,15,16,18,19,20,21. Source: BEA

Hours in finance, insurance and real estate are omitted due to consistence to DH. The reasons behind are calibration exercises. A large fraction of real estate value added is imputed from owner-occupied housing. Hence, accounting for real estate services would lead to a biased capital share in the service technology. See DH for further discussion.

p_h: real house price index_t, available since 1970

real house price index s.a. U.S. (Seasonally Adjusted, private consumption deflated), annual. Source: OECD.Stat

1.1 Price adjustment

The presented data in real terms are based on chained indices. Since these indices are non-linear, there is a lack of additivity (see e.g. Whelan (2002), Flor (2014)). To examine these errors Table 12 presents three alternative approaches. The first one is in line with Reich (2003) and Balk and Reich (2008). In this approach I deflated all nominal aggregates with the GDP-deflator. The second approach follows Gomme and Rupert (2007) in line with Greenwood et al. (1997) where I deflated all nominal aggregates with the consumer price index (CPI). The third approach gets on without aggregation. Although, this approach omits government nonresidential investment and second moments, for the intermediate sectoral outputs are not available.

Overall, it turns out that co-movements occur in all observed variables with all approaches. Furthermore, it turns out that the correlation between residential and business investment and between the output in the construction and manufacturing sector becomes noticeably smaller with GDP and CPI deflated data. Nevertheless, they co-move obviously. Other changes, especially in the third approach, are not noticeable.

I decide to present the aggregated chained real terms in the paper because the methodology is in line with Davis and Heathcote (2005) and changes in second moments due to more consistent approaches are only slightly. Thus, the error should be small.

B Full model

Figure 15 displays the flow of services and goods between the household sector and the different sectors of production.

2.1 Analytic framework

In this section I present the full dynamic equilibrium of the model. Since my focus is on an interior solution I omit non-negative restrictions.

The firm on the intermediate stage of production has to solve the following maximization problem:

$$\begin{aligned} \max_{u_{it}K_{it},N_{it}} \quad & \pi_{It} = \sum_{i}^{b,m,s} P_{it}X_{it} - W_tN_t - \sum_{i}^{b,m,s} r_{it}u_{it}K_{it} \\ \text{s.t.:} \quad & X_{it} = (u_{it}K_{it})^{\theta_i}(A_{it}N_{it})^{(1-\theta_i)}, \quad \theta_{it} \in (0,1) \\ & \sum N_{it} \leq N_t \end{aligned} \tag{9}$$

The following first order conditions (FOCs) are the solution of the problem:

$$\frac{\partial \pi_{It}}{\partial u_{it}K_{it}} = \theta_i(u_{it}K_{it})^{\theta_i - 1}(A_{it}N_{it})^{(1-\theta_i)} - r_{it} \stackrel{!}{=} 0 \tag{10}$$

$$\frac{\partial \pi_{It}}{\partial N_{it}} = (1-\theta_i)(u_{it}K_{it})^{\theta_i}(A_{it}N_{it})^{(-\theta_i)} - W_t \stackrel{!}{=} 0 \tag{11}$$

The representative firm in the final good sector has to solve the following maximization problem:

$$\begin{aligned} \max_{X_{bjt},X_{mjt},X_{sjt}} \quad & \pi_{Et} = Y_{ct} + P_{dt}Y_{dt} - \sum_{i} P_{it}(X_{ict} + X_{idt}) \\ \text{s.t.:} \quad & Y_{jt} = X_{bjt}^{B_j}X_{mjt}^{M_j}X_{sjt}^{S_j}; \quad X_{ict} + X_{idt} \leq X_{it} \end{aligned}$$

The following FOCs are the solution of the problem:

$$\frac{\partial \pi_{Et}}{\partial X_{bjt}} = \frac{B_j P_{jt} Y_{jt}}{X_{bjt}} - P_{bt} \stackrel{!}{=} 0 \tag{12}$$
$$\frac{\partial \pi_{Et}}{\partial X_{mjt}} = \frac{M_j P_{jt} Y_{jt}}{X_{mjt}} - P_{mt} \stackrel{!}{=} 0 \tag{13}$$
$$\frac{\partial \pi_{Et}}{\partial X_{sjt}} = \frac{S_j P_{jt} Y_{jt}}{X_{sjt}} - P_{st} \stackrel{!}{=} 0 \tag{14}$$

The representative real estate developer has to solve the following maximization problem:

$$\max_{Y_{dt}, l_t} \quad \pi_{REt} = P_{ht} Y_{ht} - P_{lt} l_t - P_{dt} Y_{dt}$$
$$\text{s.t.:} \quad Y_{ht} = Y_{dt}^{1-\phi} l_t^{\phi}$$

The following FOCs are the solution of the problem:

$$\frac{\partial \pi_{It}}{\partial X_{dt}} = \frac{(1-\phi) P_{ht} Y_{ht}}{X_{dt}} - P_{dt} \stackrel{!}{=} 0 \tag{15}$$
$$\frac{\partial \pi_{It}}{\partial l_t} = \frac{\phi P_{ht} Y_{ht}}{l_t} - P_{lt} \stackrel{!}{=} 0 \tag{16}$$

The periodical utility function U_t of the representative household has the Cobb-Douglas form with constant returns to scale and constant relative risk aversion. Explicitly:

$$U_t = \frac{\left(C_t^{\mu_c} H_t^{\mu_h} (1-N_t)^{1-\mu_c-\mu_h}\right)^{1-\sigma}}{1-\sigma} \tag{17}$$

There are investment adjustment costs, limited capital mobility, and the depreciation rate of business capital depends on the state of capital utilization. So the law of capital accumulation reads as follows:

$$\sum_i^{b,m,s} K_{it+1} = I_t \left(1 - \varphi\left(\frac{I_t}{I_{t-1}}\right)\right) + (1 - \delta(u_{it})) \sum_i^{b,m,s} K_{it}; \tag{18}$$

I choose $\varphi\left(\frac{I_t}{I_{t-1}}\right) = \frac{\bar{\varphi}}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2$ for the investment adjustment costs, where g_I is the growth rate of investment on the balance growth path. The function of the capital depreciation rate reads as follows: $\delta(u_{it}) = \bar{\delta}u_{it}^{x},\ x \geq 1$.

The law of motion of housing is:

$$H_{t+1} = Y_{ht} + (1-\delta_h)H_t \tag{19}$$

Therefore the household explicitly faces the following maximization problem:

$$\max_{C_t;N_{it};I_t;u_{it};K_{it+1};H_{t+1}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U_{t+s}$$

$$\text{s.t.:}\quad C_t + I_t + P_{ht}H_{t+1} \leq (1-\delta_h)P_{ht}H_t + P_{lt}l_t + \sum_{i}^{b,m,s} r_{it}u_{it}K_{it} + W_tN_t;$$

$$\sum_{i}^{b,m,s} K_{it+1} = I_t\left(1-\varphi\left(\frac{I_t}{I_{t-1}}\right)\right) + \sum_{i}^{b,m,s}(1-\delta(u_{it}))K_{it};$$

The first auxiliary condition is the budget constraint.

The FOCs of the household reads as follows:

$$\frac{\mu_c}{C_t}\left(C_t^{\mu_c}H_t^{\mu_h}(1-N_t)^{1-\mu_c-\mu_h}\right)^{1-\sigma} = \Lambda_t \tag{20}$$

$$\frac{1-\mu_c-\mu_h}{1-N_t}\left(C_t^{\mu_c}H_t^{\mu_h}(1-N_t)^{1-\mu_c-\mu_h}\right)^{1-\sigma} = \Lambda_t W_t \tag{21}$$

$$\beta\mathbb{E}_t\frac{\mu_h}{H_{t+1}}\left(C_{t+1}^{\mu_c}H_{t+1}^{\mu_h}(1-N_{t+1})^{1-\mu_c-\mu_h}\right)^{1-\sigma} = \Lambda_t P_{ht} - \beta\mathbb{E}_t\Lambda_{t+1}(1-\delta_h)P_{ht+1} \tag{22}$$

$$\Lambda_t r_{it}K_{it} = \Gamma_t x\bar{\delta}u_{it}^{x-1}K_{it} \tag{23}$$

$$\beta\mathbb{E}_t\left(\Lambda_{t+1}r_{it+1}u_{it+1} + \Gamma_{t+1}(1-\bar{\delta}u_{it+1}^{x})\right) = \Gamma_t \tag{24}$$

$$\left(1-\frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}}-g_I\right)^2 - \varphi\left(\frac{I_t}{I_{t-1}}-g_I\right)\frac{I_t}{I_{t-1}}\right) + \mathbb{E}_t\left(\beta\frac{\Gamma_{t+1}}{\Gamma_t}\varphi\left(\frac{I_{t+1}}{I_t}-g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right) = \frac{\Lambda_t}{\Gamma_t} \tag{25}$$

Λ_t is the Lagrange multiplier associated with the budget constraint and Γ_t associated with the law of motion of the capital accumulation.

Since firms maximize their profits, the household maximizes his utility and $U_t'(\cdot) > 0$ all constraints are binding. It is straightforward to show that firms make zero-profits.

Perfect competition and profit maximization implies:

$$X_{it} = X_{idt} + X_{ict} \tag{26}$$

$$N_t = \sum_{i}^{b,m,s} N_{it} \tag{27}$$

I derive the market clearing condition from the binding budget constraint:

$$Y_{ct} = C_t + I_t \tag{28}$$

The firms technologies and equations 10-28 define the model's unique solution.

The choice of the optimal capital utilization is up to the household. This seems implausible. However, with firms as investors this decision could be taken by the firm. Since both solutions are first-best and the first fundamental theorem of welfare holds, the result would be the same. I omit this for the sake of simplicity.

DH implement a hypothetical rental rate for housing denoted Q_t to define consumption and GDP consistently with the NIPA. This rate equals the marginal rate of substitution between consumption and housing. In general, the marginal rate of substitution is the ratio of the prices of the goods, and if one of them is the numéraire it is only the price of the non-numéraire or the reciprocal. The equivalent to the NIPA PCE in the model is the sum of consumption C_t and the rents for housing $Q_t H_t$. Then one has for GDP $Y_t = PCE_t + I_t + P_{dt} Y_{dt}$.

2.2 Asset pricing and the marginal return on investment

In this paper I argue that the product of the stochastic discount factor and the expected gross return on business investment and new houses is one. In this section I will show that this is true for both investment types. It holds from equation 10 and 24:

$$\frac{\Gamma_t}{\Lambda_t} = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[\theta_i \frac{p_{it+1} X_{it+1}}{K_{it+1}} + (1 - \delta u^x_{it+1}) \frac{\Gamma_{t+1}}{\Lambda_{t+1}} \right]$$

where $\frac{\beta \Lambda_{t+1}}{\Lambda_t} := M_{t+1}$ is the stochastic discount factor. The ratio $\frac{\Gamma_t}{\Lambda_t}$ is the value of an additional unit of capital to the price of an additional unit of business investment which is commonly called Tobin's marginal q (Tq_t). Note that with CEE adjustment costs Tobin's marginal q is different from Tobin's average q. See Jaimovich and Rebelo (2009) for further discussion. Tq_t reflects also the price of capital due to CEE adjustment costs via the reciprocal of equation 25 or via asset pricing theory. Hence:

$$1 = \mathbb{E}_t M_{t+1} \left[\frac{\theta_i \frac{p_{it+1} X_{it+1}}{K_{it+1}} + (1 - \delta u_i^x) T q_{t+1}}{T q_t} \right]$$

The term in parentheses is the expected gross return to business capital investment $R_{t+1}^{K_i}$.

From equation 22 I derive:

$$P_{ht} = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[\frac{\frac{\mu_h}{H_{t+1}} \left(C_{t+1}^{\mu_c} H_{t+1}^{\mu_h} (1 - N_{t+1})^{1-\mu_c-\mu_h} \right)^{1-\sigma}}{\Lambda_{t+1}} + (1 - \delta_h) P_{ht+1} \right]$$

The first term in parentheses is the expected marginal rate of substitution between housing and consumption in period $t+1$ which equals the expected implicit rental rate for housing in $t+1$. Thus, it is obvious that P_{ht} is the housing's asset price. It follows with the expected gross return to new houses R_{t+1}^H:

$$1 = \mathbb{E}_t M_{t+1} R_{t+1}^H$$

All things considered, it holds:

$$\Lambda_t = \mathbb{E}_t \beta \Lambda_{+1} R_{t+1}^H = \mathbb{E}_t \beta \Lambda_{+1} R_{t+1}^{K_i}$$

This means the discounted marginal return of new houses equals the discounted marginal return on business investment and also the marginal utility of consumption, which is the standard condition in a basic consumption-saving model.

2.3 Growth rates and stationary variables

Trend growth rates in the extended model are equal to DH. Table 11 illustrates them. I obtain stationary variables due to $x_t = X_t / g_x^t$, $\lambda_t = \Lambda_t / g_h^{t(\mu_h(1-\sigma))} g_k^{t(\mu_c(1-\sigma)-1)}$ and $\gamma_t = \Gamma_t / g_h^{t(\mu_h(1-\sigma))} g_k^{t(\mu_c(1-\sigma)-1)}$. X_t represents any variable except Λ_t and Γ_t, g_x the corresponding growth rate.

2.4 Calibration

As DH,too, I choose the following values of the parameters: 0.06 is the net rate of the return on capital in all sectors less depreciation.[12] σ is set equal two. The parameters of the intermediate and final good technologies are: $\theta_b = 0.132$, $\theta_m = 0.309$, $\theta_s = 0.237$, $B_c = 0.031$, $M_c = 0.27$, $S_c = 0.699$, $B_d = 0.47$, $M_d = 0.24$ and $B_d = 0.29$. The growth rates in the intermediate sectors are: $g_{A_b} = -0.27\%$, $g_{A_m} = 3.1\%$ and $g_{A_s} = 2.37\%$. In the steady state the capital/GDP ratio is 1.52. The amount of hours on the balanced growth path is set to 0.3.

For $\bar{\varphi}$ I choose 0.4. To this end the standard deviation of business investment relative to those of output matches the data from DH.

Since the optimal capital utilization in the steady state is the same across the different technologies, I set the optimal capital utilization to one in all sectors on balanced growth path. The parameters $\bar{\delta}$ and x are endogenous and it follows that the constant elasticity of $\delta'(u_{it})$ $(\delta''(u_{it})u_{it}/\delta'(u_{it}))$ is 0.67 (with $\bar{\delta} = 0.089$ and $x = 1.67$).[13]

Davis and Heathcote (2007) gives evidence for a great volatility and a large increase in the land share of existing houses from 30-35 percent to 40-45 percent between 1975 and 2006. The land share of existing houses is on average 36 percent.

Properly speaking, only the stock of residential structures S_t depreciate over time, not land. Following DH, the accumulation law of housing is technically:

$$\begin{aligned} H_{t+1} =& l_t^{\phi} Y_{dt}^{1-\phi} + l_{t-1}^{\phi} \left((1-\delta_s)Y_{dt-1}\right)^{1-\phi} \\ &+ l_{t-1}^{\phi} \left((1-\delta_s)^2 Y_{dt-1}\right)^{1-\phi} + ... \\ =& l_t^{\phi} Y_{dt}^{1-\phi} + (1-\delta_s)^{1-\phi} H_t \end{aligned}$$

Where δ_s is the pure depreciation of structures. This implies $\delta_h = (1-\delta_s)^{1-\phi} - 1$. Furthermore with the accumulation law of structures $S_{t+1} = Y_{dt} + (1-\delta_s)S_t$, it is straightforward to show the following relationship in the steady state between the value of the stock of structures and of houses:

$$\frac{P_d S}{P_h H} = \frac{g_h(1-\phi)}{g_d} \left(1 - \frac{(1-\delta_s)^{\frac{1}{1-\phi}}}{g_d^{1-\phi}}\right) \left(1 - \frac{(1-\delta_s)}{g_d}\right)^{-1}$$

[12] I will show in Apendix 2.5 that the return on capital and the capital utilization is the same in each sector on balanced growth. Therefore capital depreciation is the same in each sector. Following this the net rate of the return on capital less depreciation in all sectors must be the same.

[13] Jaimovich and Rebelo (2009) choose 0.15 on a quarterly basis.

Where g_x denotes the growth rate of the particular variable. Hence, $1 - \frac{P_d S}{P_h H}$ is the share of the value of the stock of land in the value of the stock of houses. I take from Davis and Heathcote (2005) the depreciation rate of residential structures equal 0.0157 and the stock of residential structure to GDP ratio equal 1. Hence, the only degree of freedom in the equation above is ϕ. I choose this to match the empirical observations of Davis and Heathcote (2007). This is for $\phi = 0.2$.

From this it follows a depreciation rate of houses $\delta_h \approx 0.126$ and a steady state ratio $\frac{P_h H}{Y} = 1.56$.

From this calibration the values of the remaining endogenous parameters reads as follows: $\mu_c \approx 0.3$ and $\mu_h \approx 0.04$. Those are similar to DH. The inverse of the discount rate β is 0.9668.

The parameters of the shocks are listed in table 3 and were discussed in the paper.

2.5 Characteristics of the steady state

In this section I show some explicit characteristics of the steady state. I derived the steady state by paper and pencil. In the steady state all variables are stationary and $z_{it} = 1 \; \forall \; t$. Thus, $x_t = x_{t+1} = x$ holds and therefore the expectation operator $\mathbb{E}_t$ is dropped.

Capital utilization: Keep in mind, there are no business investment adjustment costs on the balanced growth path and consequently not in the steady state and therefore $\lambda = \gamma$. You can also obtain this result via equation 25. From equation 10 and 23 I derive the steady state condition:

$$\begin{aligned} x\bar{\delta}u_i^{x-1}k_i &= \theta_i \frac{p_i x_i}{u_i} \\ \Leftrightarrow x\bar{\delta}u_i^{x} &= \theta_i \frac{p_i x_i}{k_i} \end{aligned} \tag{29}$$

Consider the Euler-equation (equation 24) in the steady state with equation 29, 10 and $\lambda = \gamma$:

$$\begin{aligned} \frac{\lambda}{g_k^{\mu_c(1-\sigma)-1} g_h^{\mu_h(1-\sigma)}} &= \beta\lambda(1 - \bar{\delta}u_i^x + x\bar{\delta}u_i^x) \\ \Leftrightarrow \left(\frac{\beta g_k^{\mu_c(1-\sigma)-1} g_h^{\mu_h(1-\sigma)} - 1}{\beta(1-x)\bar{\delta} g_k^{\mu_c(1-\sigma)-1} g_h^{\mu_h(1-\sigma)}} \right)^{\frac{1}{x}} &= u_i \end{aligned} \tag{30}$$

The left-hand side of equation 30 is independent of any technology specific parameters.

Thus in the steady state capital utilization is equal across all technologies in the steady state.

C Estimation and Monte Carlo algorithms

RWMH I use a Metropolis-Hastings-Gaussian-Random-Walk algorithm as described by Herbst and Schorfheide (2016) to draw from an approximation of the posterior of the AR-models. As covariance matrix I take the negative of the inverse of the Hessian at the mode of the posterior. I do not scale, because with $c = 1$ about 30 percent of the draws are accepted, which seems optimal. I draw 100,000 times and burn the first 50,000. I calculate marginal likelihoods with Geweke (1999) choice. There is no change in the interval of $[0.5, 0.99]$ for the presented accuracy.

SMC For the VAR(1) models I use the sequential Monte Carlo algorithm with likelihood tempering as described by Herbst and Schorfheide (2016), because the posteriors are probably multimodal. In the selection step I resample multinominal when the effective sample size is lower than half of the whole sample. Sampling in the mutation step is via Metropolis-Hastings-Gaussian-Random-Walk with one step and one block for each particle. The variance estimation bases on the previous bridge distribution. The scaling constant is adaptive to accept around 25 percent percent of draws. The tempering follows the square of the number of the bridge over the number of all bridges. The first bridge is drawn from the prior. For the quantiles of the distribution I account for the weights. Marginal likelihood of the VAR-models is calculated as the product of the bridges average unnormalized weights.

D Nested innovations

I will give a simple formal example of how correlated innovations could be thought as nested aggregate and sector specific innovations. Consider two sectoral shocks with the following innovations:

$$\underbrace{\begin{bmatrix}\epsilon_{1t}\\ \epsilon_{2t}\end{bmatrix}}_{=:\boldsymbol{\epsilon}_t} = \underbrace{\begin{bmatrix}(1-\zeta_1)\sigma_{1A} & \zeta_1\sigma_{1t} & 0\\ (1-\zeta_2)\sigma_{2A} & 0 & \zeta_2\sigma_{2S}\end{bmatrix}}_{:=\Omega} \underbrace{\begin{bmatrix}A_t\\ S_{1t}\\ S_{2t}\end{bmatrix}}_{=:\boldsymbol{\eta}_t}, \; \boldsymbol{\eta}_t \sim \mathcal{N}\left(\mathbf{0}_{3\times 1}, I_3\right). \tag{31}$$

Hence:

$$\begin{aligned}\Sigma &:= \mathbb{E}\left(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^T\right) = \mathbb{E}\left(\Omega\boldsymbol{\eta}_t\boldsymbol{\eta}_t^T\Omega^T\right) = \Omega\Omega^T \\ &= \begin{bmatrix} (1-\zeta_1)^2\sigma_{1A}^2 + \zeta_1^2\sigma_{1S}^2 & (1-\zeta_1)(1-\zeta_2)\sigma_{1A}\sigma_{2A} \\ (1-\zeta_1)(1-\zeta_2)\sigma_{1A}\sigma_{2A} & (1-\zeta_2)^2\sigma_{2A}^2 + \zeta_2^2\sigma_{2S}^2 \end{bmatrix}\end{aligned} \tag{32}$$

For $\zeta_1 \vee \zeta_2 = 1$ the covariance is zero, thus innovations are independent of each other. Furthermore, with $\zeta_1 = \zeta_2 = 0$ it follows:

$$\Sigma = \begin{bmatrix} \sigma_{1A}^2 & \sigma_{1A}\sigma_{2A} \\ \sigma_{1A}\sigma_{2A} & \sigma_{2A}^2 \end{bmatrix} \Rightarrow \rho_{12} = \frac{\sigma_{1A}\sigma_{2A}}{\sqrt{\sigma_{1A}^2\sigma_{2A}^2}} = 1$$

and therefrom the shocks are perfectly correlated. In all other cases the correlation is between zero and one, which is the case in the benchmark DH Model. Hence, the shock could be thought as a nest of sectoral and aggregate innovations.

E Tables and Figures

Table 11: Growth rates on the balanced growth path

N_i, N, u_i, l	1
K_i, C, I, Y_c	$g_k = \left[g_{A_b}^{(1-\theta_b)B_c} g_{A_m}^{(1-\theta_m)M_c} g_{A_s}^{(1-\theta_s)S_c}\right]^{(1-\theta_b B_c - \theta_m M_c - \theta_s S_c)^{-1}}$
$P_iX_i, P_iX_{ij}, P_hY_h, P_hH$	g_k
P_dX_d, P_lX_l	g_k
X_{bj}, X_b	$g_b = g_k^{\theta_b} g_{A_b}^{1-\theta_b}$
X_{mj}, X_m	$g_m = g_k^{\theta_m} g_{A_m}^{1-\theta_m}$
X_{sj}, X_s	$g_s = g_k^{\theta_s} g_{A_s}^{1-\theta_s}$
Y_d	$g_{y_d} = g_{B_d}^{B_d} g_{M_d}^{M_d} g_{S_d}^{S_d}$
H, Y_h	$g_h = g_l^{\phi} g_d^{1-\phi}$
Q, P_h	$g_{p_h} = g_k / g_h$

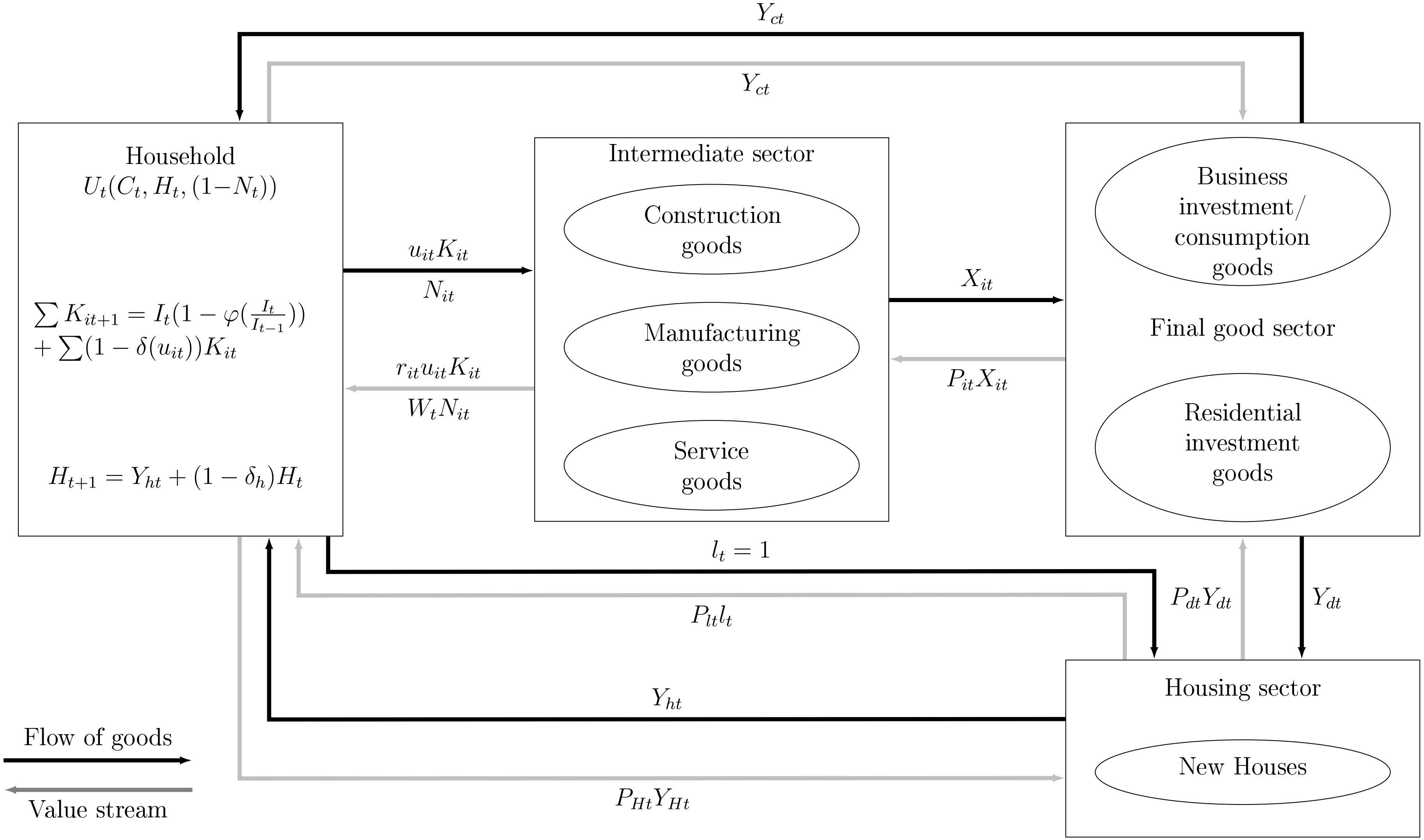
Source: DH, own calculation

Table 12: Different deflated empirical second moments

SD	Aggregated real data			GDP deflated			CPI deflated			No aggregation		
Output		2.00			2.00		2.00			2.00		
% SD to Output												
PCE		0.96			0.89			0.96			0.96	
Hours worked		1.24			1.24			1.24			1.24	
Busi		2.51			2.48			2.56			3.14	
Resi		7.46			8.16			8.24			7.42	
House prices (p_h)		2.63*			2.63*			2.63*			2.63*	
Output by sector	x_b	x_m	x_s	x_b	x_m	x_s	x_b	x_m	x_s	x_b	x_m	x_s
	3.75	1.93	1.18	3.98	2.56	1.24	4.07	2.51	1.23	n/a	n/a	n/a
Hours by sector	N_b	N_m	N_s	N_b	N_m	N_s	N_b	N_m	N_s	N_b	N_m	N_s
≤2000	2.92	1.59	0.78	2.92	1.59	0.78	2.92	1.59	0.78	2.92	1.59	0.78
≥2000	4.17	2.19	0.96	4.17	2.19	0.96	4.17	2.19	0.96	4.17	2.19	0.96
Correlations												
GDP, PCE		0.91			0.94			0.92			0.91	
p$_h$, GDP		0.66*			0.66*			0.66*			0.66*	
PCE, Busi		0.57			0.52			0.49			0.51	
PCE, Resi		0.80			0.77			0.81			0.80	
Resi, Busi		0.41			0.35			0.38			0.41	
p_h, Resi		0.66*			0.75*			0.75*			0.66*	
Output by sector	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s	x_b, x_m	x_b, x_s	x_m, x_s
	0.72	0.72	0.90	0.33	0.69	0.66	0.35	0.75	0.63	n/a	n/a	n/a
Hours by sector	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s	N_b, N_m	N_b, N_s	N_m, N_s
≤2000	0.79	0.90	0.82	0.79	0.90	0.82	0.79	0.90	0.82	0.79	0.90	0.82
≥2000	0.71	0.87	0.95	0.71	0.87	0.95	0.71	0.87	0.95	0.71	0.87	0.95
Lead-lag correlations	i=1	i=0	i=-1	i=1	i=0	i=-1	i=1	i=0	i=-1	i=1	i=0	i=-1
Busi$t-i$, GDP$_t$	0.22	0.77	0.59	0.12	0.68	0.59	0.16	0.69	0.58	0.22	0.71	0.50
Resi$t-i$, GDP$_t$	0.78	0.75	0.15	0.75	0.77	0.23	0.75	0.77	0.23	0.78	0.75	0.15
Busi$t-i$, Resi$_t$	-0.09	0.41	0.65	-0.14	0.35	0.61	-0.10	0.38	0.62	-0.07	0.41	0.61

Moments from per capita, logged, HP-filtered data with filter weight 100. * only since 1970 available, private consumption deflated by the OECD.

Figure 15: Structure of the Model



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