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The return on everything and the business cycle in production economies

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Abstract

The risk premium puzzle is even worse than previously reported if housing is also taken into consideration next to equity. While housing premia are only moderately smaller than equity premia, they are significantly less volatile and the Sharpe ratio of housing is significantly larger. Hence, three question arise: i) are existing approaches to explain the equity premium puzzle also capable of explaining even larger Sharpe ratios than previously required, ii) can return rates and volatilities of various assets be differentiated, and iii) can different Sharpe ratios between the two risky assets be matched.

We analyze these questions, next to business cycle statistics, by including housing into seminal approaches to solve the risk premium puzzle in production economies. Non-disaster economies with habit formation, capital adjustment costs and limited factor mobility fail to generate a Sharpe ratio of housing of the empirically observed size and do not explain co-moving economic activity. A basic model with time-varying disaster risk can reproduce the large Sharpe ratio of housing. Moreover, the model can explain different means and volatilities of the risky assets, economic activity co-moves and the model explains the volatility ratio of business investments, residential investments and house prices. However, the model does not allow to disentangle the Sharpe ratios of the risky assets and premia on equity remain too involatile.

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1 INTRODUCTION

The seminal publications of Mehra and Prescott (1985) and Weil (1989) have issued a challenge to macroeconomic models: explaining the historically observed sizeable equity premium (excess of the return on a stock market index over the return of a relatively riskless security) together with the low risk-free rate for a reasonable degree of risk aversion. While standard real business cycle (RBC) models are successful in accounting for important stylized facts of the business cycle, they typically fail to reproduce the empirically observed characteristics of asset returns. Over the past years, different approaches have been suggested by the literature in order to solve the puzzle. To name but a few, Jermann (1998) combines modifications to the standard preference structure with frictions in the adjustment of input factors, Boldrin et al. (2001) add frictions in the allocation of input factors, and Gourio (2012) introduces a risk for rare but severe economic disasters. While these models are able to replicate the empirical risk premium on stocks, they commonly ignore an asset which, according to Jordà, Schularick, and Taylor (2019) (JST), forms roughly 50% of an advanced economy's total wealth, namely housing. Looking at it the other way round, the RBC literature which focuses on housing generally does not consider implications for asset returns either. In the present paper, we therefore aim to combine the two strands of the literature with the objective to mutually explain key asset pricing and business cycle statistics including housing.

The new database built by Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) (JKKST) covers long term data on the return on equity, on the return on housing and on the return on total wealth as well as data on government bills and bonds for 16 advanced economies. Based on this new data, JST re-measure the return rates on a representative investor's total portfolio and find that the risk premium puzzle by Mehra and Prescott (1985) in fact further worsens if attention is not restricted to stocks only: the Sharpe ratio of housing is even larger than the Sharpe ratio of equity. Their result raises three questions. First, are existing approaches capable of explaining even larger Sharpe ratios than previously required for risky assets, second, can return rates and volatilities of various assets be differentiated, and third, can different Sharpe ratios between the two risky assets be matched. They show that several popular approaches which were previously shown to be successful in reproducing the return rates on stocks, turn out less successful once the return on housing and the return on total wealth are also taken into consideration. While the study of JST considers various different approaches including habit formation as in Abel (1990) and Campbell and Cochrane (1999) or disaster risk with and without recursive utility as in Rietz (1988), Barro (2006), and Bansal and Yaron (2004), they focus on endowment economies throughout.

We think that studying more general asset pricing statistics also in production economies is important for various reasons. First, the analysis of multiple assets asks for an explanation of the empirically observed differences in the mean return rates, volatilities and Sharpe ratios. For example, in the Lucas (1978) framework for asset prices, different Sharpe ratios of assets can only be realized if the correlations of the assets' returns with the model's stochastic discount factor differ. While different volatilities of returns and different correlations between return rates and consumption growth are introduced exogenously in endowment economies, the explanation of these features becomes an important exercise in production economies. Second, as argued by Cochrane and Hansen (1992), any friction which can help to reproduce asset pricing statistics may on the other hand have empirically counterfactual implications for business cycle statistics. Hence, the effects of such frictions on both, asset prices and business cycles, should be analyzed simultaneously within the framework of RBC models. Third, many mechanisms which can explain risk premia in endowment economies may fail in general equilibrium since the household can alter his plans to smooth consumption and thereby insure himself. Fourth, the business cycle is potentially the macroeconomic phenomenon with the largest effects on asset returns. Hence, explaining the key facts of asset return rates and the business cycle in the same internally consistent model is important in order to gain insights into this relationship. Fifth, RBC models are the backbone for a broader class of Dynamic Stochastic General Equilibrium (DSGE) models used for stabilization policy analysis. For this purpose, an unsatisfactory performance with regard to asset pricing statistics may constitute a significant shortcoming of these models. For example, high risk premia may diminish investment activities even if the riskless interest rate is low.

In our analysis we simultaneously focus on partly puzzling stylized facts of asset prices and the business cycle. These stylized facts are identified as common features from historic data which are valid for several developed countries over long time periods. Among the stylized facts which characterize asset returns are: i) a stable risk-free rate smaller than 2.25 percent, ii) return rates on equity moderately larger than returns on housing, iii) risk premia on equity, on housing and on total risk larger than 3 percent, iv) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and iv) a Sharpe ratio of housing significantly larger than the Sharpe ratio of equity. Turning to business cycle statistics, they reveal the following important characteristics: i) residential investments are at least moderately more volatile than business investments, ii) house prices are at least twice as volatile as Gross Domestic Product (GDP), and iii) house prices, business investments as well as GDP are positively correlated with residential investments, and the correlation between house prices and GDP is also positive. Jaimovich and Rebelo (2009) designate the ability to generate correlated movements of subaggregates as a litmus test for RBC models.

The starting point of our analysis is a variation of the Jermann (1998) model with exogenous labor but extended by a separate housing stock. Following Davis and Heathcote (2005), the stock of houses differs from productive capital in two aspects. First, it enters the household's utility function whereas productive capital enters the production function and, second, houses depreciate at a lower rate. While we assume the same capital adjustment costs in line with the 'q-theory' for business investments as in Jermann (1998), convex adjustment costs for housing arise from the fact that new houses require that residential structures must be linked to land. Moreover, we first assume that business investments, residential investments and the consumption good are homogenous goods. This assumption together with the fact that the elasticity of housing in the household's consumption bundle as well as the depreciation rate of houses are both small, allows the household to conveniently smooth his consumption bundle across different states of nature through optimal adjustment of residential investments in response to technology shocks. In consequence, risk premia in the Jermann (1998) model with housing vanish even when large habits in consumption and housing are assumed. Nevertheless, the model can predict a pro-cyclical demand effect for residential investments. House prices fluctuate more than GDP and the model reproduces the observed co-movements from the data.[1]

In a second step we restrain the household's option to smooth his consumption bundle after the shock's realization. We consider a two sector model where residential investments are produced in one sector whereas production of business investments and of the consumption good takes place in a second sector. The productive capital stock is sector-specific and immobile, and subject to adjustment costs in both sectors as in Fehrle (2019). This model can be interpreted as a stripped down version of the multi-sectoral model by Davis and Heathcote (2005) and is similar to the model of Nguyen (2018). Sticking at first to the assumption of exogenous labor supply, the model can produce moderate risk premia. The model's ability to explain sizeable risk premia is lost once labour supply is determined endogenously, but can be recovered if labor mobility between the sectors is limited similar to Boldrin et al. (2001). However, the model performs worse with respect to the residential business cycle statistics and in particular fails to generate co-moving economic activity between the two sectors. In consequence, we conclude that the model cannot explain sizeable risk premia and the observed co-moving economic activity simultaneously. Further, return rates turn out far too volatile in the model. The standard deviation of the risk-free rate exceeds its empirical counterpart by a factor of 8 while the return rates on housing and on the total portfolio are more than 4 and more than 2 times, respectively, as volatile as in the data. Moreover, the model cannot explain any of the empirically observed differences between equity and housing.

Including housing into disaster economies turns out more promising. We consider an otherwise standard RBC model with housing where economic disasters are introduced through large negative shocks which reduce total factor productivity and also destroy productive capital and residential structures to the same extent. Moreover, the model features time-varying disaster risk and recursive preferences of the class introduced by Epstein and Zin (1989). Different elasticities of Tobin's q and of house prices help to explain differences in the mean and in the volatilities between returns on unlevered equity and on housing while leverage additionally helps to differentiate the effect. Keeping the coefficient of relative risk aversion to a moderate level of 5.5, the model can explain a low return rate on government bonds of 1.31 percent on average (1.57 percent in the US data) and is able to replicate an equity premium of 6.56 percent (5.88 in the US data). In accordance with the data, return rates on housing turn out moderately lower than on equity and the housing premium in the model is 3.00 percent (compared to 4.45 percent in the US data). The total risk premium in the model turns out to be 4.98 percent and closely matches the value from the data (5.27 in the US data). Next to mean return rates and premia, the model can also match the low volatility of government bonds fairly well. Time-varying disaster risk helps to increase the volatility of the risky assets' returns and allows to closely reproduce the standard deviations of returns and premia on housing as well as on total risk.

[1] Note that in a benchmark one sector model co-moving business and residential investments are a puzzle because the household intends to increase productive capital first. See also Kydland et al. (2016).

However, the standard deviations of returns and premia on equity remain too small. The model can closely replicate the Sharpe ratio of housing and the Sharpe ratio of the total portfolio from the data but does not match the significantly smaller Sharpe ratio of equity. Although the premia and their volatilities differ between the two risky assets, the model cannot generate different Sharpe ratios.

The disaster model is able to generate relative volatilities of business investments, residential investments and house prices which all fit the data. Business investments are almost 3 times as volatile as GDP, residential investments are more than twice as volatile as business investments and house prices are almost twice as volatile as GDP. In line with Dorofeenko et al. (2014), we find that time-varying uncertainty is important for the latter result. The model further reproduces the empirically observed correlation between GDP and residential investments and between GDP and house prices. The correlations between residential investments and house prices and between residential investments and business investments match the data in sign but are—at odds to the data—close to one.

An earlier analysis of risk premia in a production economy with housing, habits, and adjustments costs, which is similar to our extension of the Jermann (1998) model is presented in Jaccard (2011). However, different from our work and in contrast to JKKST and Flavin and Yamashita (2002), Jaccard (2011) considers data where the return on housing is markedly smaller than the return on equity. His empirical targets are based on Piazzesi et al. (2007) who assume that the house price index grows with the price index of residential investments, whereas Davis and Heathcote (2007) and Knoll et al. (2017) show that the main driver for increasing house prices are land prices. Moreover, different from the present paper Jaccard (2011) does not focus on returns on total risk. Lastly, Jaccard (2011) models superficial habits which have no intratemporal effect and the habit parameter is close to one.[2] The economic plausibility of both assumptions is questionable.

To the best of our knowledge, more general risk premia have not been investigated in production economies with disaster risk up to this date.

Favilukis et al. (2017) study a two-sector production economy with aggregated and idiosyncratic income risk and use this framework in order to explain the boom-bust cycle in the first decade of this century. In their model, incomplete markets produce sizable risk premia for returns on equity and housing. While the model can match the Sharpe ratio of equity, the mean and the standard deviation of the return on equity turn out too small. Moreover, the return on housing is twice as large as the return on equity, which contrasts the data. Due to heterogeneity, there is no comparable measure for the volatility of returns on housing.

The risk usually associated with housing wealth is potentially of a more idiosyncratic nature than the risk from equity.[3] In the present paper, we do not consider such differ-

[2] Jaccard (2011) sets the habit parameter implicitly to one and only calibrates the habit persistence. With stationary variables the value of the habit parameter equals the reciprocal of the growth factor (=0.995).

[3] While JKKST report a standard deviation of 3.38 percent for the aggregated return on housing in the US data, Flavin and Yamashita (2002) as well as Landvoigt et al. (2015) find a standard deviation of the individual's return on housing of 14 percent. Hence, one potential approach to explain the different Sharpe ratios between equity and housing found in the aggregated data may be the idiosyncratic nature of the risk associated with housing.

ences in the typical nature of risks. Similarly, the models abstract from other asset specific characteristics such as liquidity, transaction costs and search and matching frictions. Instead, we choose to face the aggregated data from JKKST throughout with a representative agent framework with complete markets. In this regard we understand our study as a first exploration of i) the asset pricing and business cycle characteristics which can already be explained within an elementary representative agent framework with complete markets, of ii) the characteristics for which such a framework becomes insufficient, and of iii) the reasons why a more sophisticated framework which helps to further differentiate between the assets is required for the characteristics in ii). Concerning i) we find that the model with disaster risk allows us to generate a Sharpe ratio which is substantially larger than the value previously confronted with for equity and which is close to the Sharpe ratio that is observed for housing. Moreover, the model can explain different means and volatilities of the risky assets while it still maintains a good fit to business cycle statistics. However, in regard to ii) the main shortcoming of the framework is that it cannot generate different Sharpe ratios of the risky assets. Different Sharpe ratios require different correlations between premia and the stochastic discount factor. Yet, in all of the models considered in the present paper, the return rates of the two risky assets are far too strongly correlated. We conclude, that further adjustments which help to disentangle this correlation are necessary.

From here on the papers reads as follows. In section 2 we first present the stylized facts on which we focus in the remainder of the paper. Section 3 presents and discusses the non-disaster economies, and section 4 introduces and discusses the economies with disaster risk. The paper concludes with section 5 and more detailed derivations are collected in the appendix.

2 Stylized facts

We start with the presentation of stylized facts which characterize historical data on business cycles, housing and asset prices and which the literature has identified as key facts that are commonly valid for most countries over longer time periods (see e.g. JKKST for asset prices and Davis and Nieuwerburgh (2015) for housing and business cycles). In Tables 1 and 2 we provide a summary of these stylized facts for the US (1970-2015), the UK (1969-2015), France (1980-2015), and Japan (1963-2015) while Appendix A provides the results for additional countries. Asset price statistics were computed from annual data from the JKKST database while business cycle statistics are shown for quarterly data from the OECD.stats library.1.

First, the upper part of Table 1 displays the mean return rates on bills, on equity, on housing and on total risk, and the standard deviations of the return rates are found in the lower part of the table. We observe a low risk-free return rate between 0.98 percent in Japan and 2.24 percent in France together with a low standard deviation (2.3-3.7). Note, however, that bills are not totally risk-free and, hence, only provide an upper bound proxy for the *true* risk-free return rate. The return on equity is between 5.86 percent in Japan and 9.61 percent in France and leads to equity premia between 4.88 percent and 7.37 percent. In all countries, the average return on housing turns out moderately smaller than

Table 1: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
UK	8.00	7.00	7.47	1.56	6.44	5.44	5.91	0.27	0.61	0.69
FRA	9.61	5.78	6.61	2.24	7.37	3.54	4.37	0.31	0.57	0.59
JPA	5.86	5.54	6.19	0.98	4.88	4.56	5.21	0.24	0.7	0.65
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$			
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00			
UK	23.41	9.64	8.44	3.73	24.27	8.88	8.62			
FRA	24.11	5.52	6.95	2.55	23.98	6.18	7.39			
JPA	20.15	6.53	8.10	2.53	19.94	6.47	8.03			

Notes: Mean percentage returns on equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP). The corresponding standard deviations $\sigma(X)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). Periods: USA 1970-2015, United Kingdom 1969-2015, France 1980-2015, and Japan 1963-2015. Data from JKKST, own calculations.

Table 2: Empirical business cycle statistics

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64
UK	1.58	2.68	5.56	4.85	0.51	0.16	0.69	0.71
FRA	0.95	2.75	3.17	3.19	0.65	0.64	0.81	0.48
JPA	1.59	2.41	3.84	2.70	0.31	0.27	0.45	0.55

Notes: Business cycle statistics are from quarterly logged per capita hp-filtered (1600) data. σ_x is the standard deviation of x, r_y^x the correlation between x and y. RESI=residential investment, BUSI=non-residential investment, P_h house prices. Periods: USA: 1970-2015, , United Kingdom 1969-2015, France 1980-2015 Japan 1963-2015. Data: See Appendix A, own calculations.

the average return on equity, and housing premia between 3.54 percent in France and 5.44 percent in the UK can be observed. The difference between the two risky returns/premia is the smallest in Japan with just 0.32 percentage-points and the largest in France with 3.83 percentage-points. For the US and the UK the differences are 1.43 percentage-points and 1.00 percentage-points, respectively.[4] Moreover, in the US and the UK the return on total risk is approximately the average of the two risky return rates. In France the return on total risk is close to the smaller return on housing while in Japan the return on total risk exceeds the decomposed return rates on both risky assets.

While equity shows moderately larger returns than housing, on the downside the return rates on equity are two to four times as volatile as the return rates on housing. Both risky returns are least volatile in the US with standard deviations of 16.7 and 3.78, respectively, while the largest standard deviations are observed in France (24.11) for equity and in the

[4]The difference between the return rates in France is closer to the value in the other countries in the time periods chosen by JST (1963-2015 and 1870-2015). Our French data set starts in 1980 due to missing data for the business cycle statistics.

UK (9.65) for housing. In all countries, the standard deviation of returns on total risk is also significantly lower than the standard deviation of returns on equity, and premia are almost identically as volatile as return rates. Finally, in all countries the Sharpe ratio of housing exceeds the Sharpe ratio of equity significantly, and the Sharpe ratio of total risk is close to the Sharpe ratio of housing. Summing up, we observe the following characteristics for return rates: i) a risk-free rate in the range of 1-2.2 percent together with a low volatility, ii) return rates on equity moderately larger than returns on housing, iii) premia on risky returns over 3 percent, iii) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and iv) a Sharpe ratio of housing significantly larger than the Sharpe ratio of equity and similar to the Sharpe ratio of total risk.

Second, Table 2 shows the stylized facts from the housing and the business cycle literature. We observe that GDP has a standard deviation of approximately 1.5-1.6 percent in the US, the UK and Japan while its standard deviation is slightly below 1 percent in France. In the US and the UK residential investments are twice as volatile as business investments while the difference between the two volatilities is moderately smaller in Japan and significantly smaller in France.[5] In all four countries the standard deviation of business investment lies between 2.4 and 2.9 percent and house prices are pro-cyclical. GDP, house prices, residential and business investment co-move and the lowest correlation is observed between business and residential investments. In short, sub-aggregates and house prices co-move pro-cyclically. Usually the literature additionally considers lagged cross-correlations with residential investments since residential investments lead the business cycle in the US. However, Kydland et al. (2016) show that this fact is unique to the US and Canada which is why we omit lead-lag-patterns here.

Next to the four countries discussed in this section, Appendix A shows that we also observe the same stylized facts in most other countries.

3 Economies with non-disaster risk

In this section, we add housing to influential approaches to explain the equity premium puzzle in production economies. We start with an adaption of the Jermann (1998) model with habit formation and capital adjustment costs in line with the 'q' theory (model A). We then extend the model by housing (model B). In a next step, we separate the production of residential investments from the production of the consumption good and business investments. The two sectors are subject to limited sectoral capital mobility similar to Boldrin et al. (2001) and Fehrle (2019). We consider the cases of exogenous labor (model C), endogenous and fully mobile labor (model D), and endogenous labor subject to limited sectoral labor mobility (model E).

[5]For most continental European countries we observe the same relation as in France.

3.1 Housing with Jermann (1998)

Model A: Our study starts with the seminal work of Jermann (1998) with habits in utility, adjustment costs in capital and exogenous labor decisions. Our variation of the model deviates from its original treatment only in that we consider exogenous habits that are out of the household's control.

Model B: We proceed to extend the Jermann (1998) model by housing. The household draws utility from housing H_t and consumption C_t, and both are subject to habit formation. Habits X_{ht}, $X \in \{C, H\}$, are exogenous and evolve according to $X_{ht} = \chi_X X_{t-1}$. Labour supply remains exogenous. Output Y_t is produced with capital K_t and is subject to labor augmenting technical progress growing at the rate a_y in the long run, and to productivity shocks Z_t following an AR(1)-process, $\ln Z_{t+1} = \rho_y \ln Z_t + \epsilon_{t+1}$, $\epsilon_t \sim \text{iidN}(0, \sigma_y^2)$. Consumption, business investment I_t, and residential investment D_t are homogeneous goods. We stick to the assumption of capital adjustment costs as in Jermann (1998).[6] A fixed factor normalized to one, namely land, affects the transformation from residential investment to new houses. The planner's problem in a centralized economy therefore reads as follows:[7]

$$
\begin{aligned}
\max_{C_t, I_t, D_t, K_{t+1}, H_{t+1}} \quad & U_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{((C_t - C_{ht})^{\mu_c} (H_t - H_{ht})^{\mu_h})^{1-\eta} - 1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = a_y^{(1-\alpha_y)t} Z_t K_t^{\alpha_y}, \\
& Y_t = C_t + I_t + D_t, \\
& K_{t+1} = (1-\delta_k) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t, \\
& H_{t+1} = (1-\delta_h) H_t + D_t^{1-\phi},
\end{aligned}
\tag{1}
$$

where $\eta, \alpha_y, a_y, \mu_c, \mu_h > 0$, $\mu_c + \mu_h = 1$, $0 < \beta, \delta_k, \delta_h < 1$, and $\Phi(x) := \frac{\varphi_1}{1-\kappa} x^{1-\kappa} + \varphi_2$, $\varphi_1 > 0$, $\varphi_2 \in \mathbb{R}$.

We follow Davis and Heathcote (2005) and define GDP by $\text{GDP}_t = Y_t + MRS_t^{H,C} H_t$, where $MRS_t^{H,C} = (\mu_h/\mu_c)(C_t - C_{ht})/(H_t - H_{ht})$ denotes the marginal rate of substitution between housing and consumption so that its product with the current housing stock yields the implicit rent from housing. Finally, the return rate $R_{E,t+1}$ on investment in productive capital, the return rate $R_{H,t+1}$ on housing, the return rate $R_{T,t+1}$ on total risk, and the risk-

[6] In an earlier version of the paper we assumed investment adjustment costs as in Christiano et al. (2005). Fehrle (2019) shows for the Davis and Heathcote (2005) framework that these adjustment costs account better for the lag pattern of business investment. However, to remain in line with Jermann (1998), Gourio (2012) and our disaster risk framework, we changed to capital adjustment costs. Besides the lead-lag structure, which is beyond the scope, changes are minor.

[7] See Appendix C for details.

free rate $R_{f,t}$ are given by

$$
\begin{aligned}
1+R_{E,t+1} &= \frac{r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1-\delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} = \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \\
1+R_{H,t+1} &= \frac{MRS^{H,C}_{t+1} + (1-\delta_h)P_{ht+1}}{P_{ht}}, \\
1+R_{T,t+1} &= \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2} + (MRS^{H,C}_{t+1} + (1-\delta_h)P_{ht+1})H_{t+1}}{q_t K_{t+1} + P_{ht+1}H_{t+1}}, \\
1+R_{f,t} &= \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}},
\end{aligned}
$$

where $q_t = 1/\Phi'(I_t/K_t)$ is Tobin's q, $r_t = \alpha_y(Y_t/K_t)$ is the rental rate of capital which equals the marginal product of capital, $\Lambda_t = \mu_c(C_t - C_{ht})^{\mu_c(1-\eta)-1}(H_t - H_{ht})^{\mu_h(1-\eta)}$ is the marginal utility of consumption and $P_{ht} = D_t^{\phi}/(1-\phi)$ denotes house prices which equal the reciprocal of the residential investment's marginal rate of production of new houses.

Calibration A: We identify one period in the model with one quarter in the data and closely follow the calibration in Jermann (1998). More precisely, we set the coefficient of relative risk aversion to $\eta = 5$, the elasticity of capital in the production function to $\alpha_y = 0.36$ and the quarterly trend growth rate to $a_y = 1.005$ as in Jermann (1998). We slightly deviate from the value of $\delta_k = 0.025$ used in Jermann (1998) and, in foresight of model B, instead adjust the depreciation rate of capital from Nguyen (2018), who strips down the Davis and Heathcote (2005) model, to quarterly data which yields $\delta_k = 0.022$. The autocorrelation parameter and the conditional standard deviation of the AR(1)-process governing productivity are pinned down to ρ_y=0.95 and σ_y=0.01. In line with Jermann (1998), we choose the remaining parameters of the model, i.e. the household's time preference β, the habit parameter χ_c, and the parameter κ controlling the elasticity of the investment capital ratio with respect to Tobin's q, in such way to closely replicate the risk-free rate, the equity premium and the relative volatility of business investment to GDP from the US data. We minimize the (unweighted) sum of squared deviations between the targets in the model and the values in the data over a grid covering $\beta^\star := \beta a_y^{1-\eta} \in [0.99; 0.999]$, $\chi_c \in [0; 0.95]$ and $\kappa \in [0; 6.25]$ where the number of gridpoint is 10, 10 and 50, respectively. The resulting values are summarized in Column A of Table 3.

Calibration B: In order to keep the different variations of the model comparable and in order to emphasize the effects of introducing housing into the Jermann (1998) model, all parameters from model A also remain at the same values in model B.[8] In particular, we do not re-optimize the previously "free" parameters for model B in order to match the

[8] Note however, that η now is the coefficient of relative risk aversion with respect to the composite good and no longer with respect to consumption only. Moreover, now $\beta^\star := \beta a^{(\mu_c + (1-\phi)\mu_h)(1-\eta)}$.

Table 3: Calibration

	A	B	C	D / E	Description
$\beta^{\star}$	0.994	0.994	0.999	0.999	discount factor
η	5	5	5	5	coefficient of relative risk aversion
$\mu_c^{*)}$	–	0.81	0.81	0.53	weight of consumption in composite good
$\mu_h^{*)}$	–	0.19	0.19	0.12	weight of housing in composite good
χ_c	0.95	0.95	0.825	0.825	habit parameter of consumption
χ_h	–	0.95	0.825	0.825	habit parameter of housing
χ_n	–	–	–	0.95	habit parameter of leisure
a_y	1.005	1.005	1.005	1.005	growth rate (y sector)
a_d	–	–	1.002	1.002	growth rate (d sector)
ϕ	–	0.106	0.106	0.106	share of land in housing
α_y	0.36	0.36	0.25	0.25	capital share in production (y sector)
α_d	–	–	0.20	0.20	capital share in production (d sector)
κ_y	4.05	4.05	6.25	6.25	elasticity of Tobin's q (y sector)
κ_d	–	–	1.25	1.25	elasticity of Tobin's q (d sector)
δ_k	0.022	0.022	0.022	0.022	rate of capital depreciation (y sector)
δ_h	–	0.009	0.009	0.009	rate of housing depreciation (d sector)
σ_y	0.01	0.01	0.0094	0.0094	conditional standard deviation of log TFP (y sector)
ρ_y	0.95	0.95	0.966	0.966	autocorrelation of log TFP (y sector)
σ_d	–	–	0.0172	0.0172	conditional standard deviation of log TFP (d sector)
ρ_d	–	–	0.923	0.923	autocorrelation of log TFP (d sector)

Notes: $^{*)}$ Endogenous by the model. **A:** Jermann (1998) adaption. **B.** A + Housing in utility. **C.** B + two sectors. **D**: C + endogenous labor. **E**: D + limited sectoral labor mobility.

(additionally available) targets. However, re-optimizing would not change the following main results.

We calibrate the additional parameters from housing in model B as follows. First, we also borrow the depreciation rate of housing $\delta_h = 0.009$ from the same source as we did δ_k. Second, we follow Grossmann et al. (2019) and pin down the weights μ_c and μ_h of consumption and of housing in the consumption bundle such way that the ratio of expenditures on housing to total consumption is 19 percent on the balanced growth path and so that $\mu_c + \mu_h = 1$ holds. Third, the habit parameter for housing is set to the same high value of $\chi_h = 0.95$ as for consumption. Finally, we take the value of the land parameter $\phi = 0.106$ from Davis and Heathcote (2005).

Results: The return rates as well as the business cycle statistics for our variation of the Jermann (1998) model (row A) and for the model extended by housing (row B) are summarized in tables 4 and 5. We compute the annualized mean return rates and the annualized standard deviation of return rates from a simulation of 100,000 periods. The second moments of the business cycle are reported as the average outcome from 100 repeated simulations of HP-filtered time series of the model's equilibrium outcomes, each for 180 periods. The model solution is obtained from a second-order perturbation method.

First, as shown by Jermann (1998), model A is able to generate a sizeable equity premium and a risk-free rate which are close to the values observed in the data. Moreover, we

Table 4: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
					Model					
A	7.66	–	–	1.55	6.05	–	–	0.25	–	–
B	4.61	4.39	4.52	4.26	0.34	0.13	0.26	0.07	0.07	0.07
C	4.71	4.45	4.56	0.38	4.31	4.06	4.16	0.21	0.21	0.21
D	2.57	2.47	2.51	2.05	0.51	0.42	0.46	0.07	0.07	0.07
E	4.88	4.61	4.72	1.50	3.34	3.07	3.18	0.18	0.18	0.18
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$			
USA	16.71	3.78	6.9	2.31	16.47	4.41	7.00			
					Model					
A	25.17	–	–	6.19	24.22	–	–			
B	5.01	2.07	3.79	0.73	4.95	1.93	3.71			
C	21.83	20.73	21.19	8.08	20.26	19.06	19.57			
D	7.18	5.95	6.44	1.47	7.03	5.76	6.27			
E	22.60	21.45	21.92	13.39	18.19	16.72	17.33			

Notes: Mean percentage returns on equity (R_E), housing (R_H), total risk (R_T) and bills (R_f), as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP). The corresponding standard deviations $\sigma(X)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). We employ a second order perturbation and simulated time series with 100,000 periods. **A**: Jermann (1998) adaption. **B**: A + Housing. **C**: B + two sectors. **D**: C + endogenous labor. **E**: D + limited sectoral labor mobility.

can also closely replicate the volatility of business investments relative to the volatility of GDP. On the other hand, the return rates, especially the risk-free rate, turn out too volatile in the model.

However, once housing is introduced into the model, all risk premia—on equity, housing and total risk—turn out close to zero, and the volatility of return rates is reduced drastically. Introducing housing into the model provides the household with an option to better insure against fluctuations in his marginal utility in the same way as discussed by Uhlig (2007) for endogenous labor decisions. Since consumption and residential investment are homogeneous, the household is now able to reduce residential investments in favor of consumption in response to negative productivity shocks. The relatively small elasticity ($\mu_h = 0.14$) combined with a small depreciation rate of housing ($\delta_h = 0.009$) favour the household's possibilities to smooth his consumption bundle across states with different realizations of the shock. In consequence, the stochastic discount factor becomes far less volatile so that risk premia almost disappear. Moreover, the household's efforts to smooth his consumption bundle by adequately adjusting residential investment and consumption also show up in the second moments of the business cycle. The volatility of residential investment in the model is twice as large as in the data and the demand of residential goods moves procyclical. Further, residential investments are positively correlated with house prices and the other variables considered.

Table 5: Simulated business cycle statistics I

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
				Data				
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64
				Model				
A	1.25	2.91	–	–	–	–	–	–
B	0.96	0.74	11.63	1.16	0.98	0.98	0.98	0.96
C	2.27	1.04	0.96	4.42	−0.03	0.00	0.08	0.97
D	1.45	0.69	4.25	2.37	0.65	0.77	0.85	0.90
E	1.96	0.96	2.10	3.97	−0.06	0.08	0.36	0.89

Notes: σ_x is the standard deviation of x, r_y^x the correlation between x and y. Business cycle statistics from HP-filtered (1600) times series. We employ a second order perturbation and report the average outcomes from repeated simulations with 180 periods. **A**: Jermann (1998) adaption. **B**: A + Housing. **C**: B + two sectors. **D**: C + endogenous labor. **E**: D + limited sectoral labor mobility.

3.2 Moving to Boldrin et al. (2001)

Sectoral frictions with exogenous labor supply: In the previous subsection, the household can nearly perfectly hedge against consumption fluctuations since the marginal rate of transformation between residential investment and consumption was one. We restrict this option in the following by moving to a two-sector model—separating production of the residential good from production of the consumption good—with frictions in factor mobility. In particular, capital is assumed immobile between the two sectors. The resulting model is similar to Nguyen (2018). We start with exogenous labor supply before discussing the effects of endogenous labor supply and labor supply which is contracted sector-specifically one period ahead as proposed by Boldrin et al. (2001).

Model C: The household's utility and the law of motion of the housing stock remain the same as in (1). The model economy consists of two sectors indexed by y and d. The sector y produces a homogeneous consumption and business investment good while the residential investment good is produced in sector d. Sector-specific technical progress grows at the rate a_x in sector x, $x \in \{y, d\}$, and both sectors are subject to sector-specific and uncorrelated productivity shocks Z_{xt} governed by AR(1)-processes, $\ln Z_{x,t+1} = \rho_x \ln Z_{x,t} + \epsilon_{x,t+1}$, $\epsilon_{x,t} \sim \text{iidN}(0, \sigma_x^2)$. The household is confronted with capital adjustment costs in both sectors and, once installed, capital is totally immobile. The household's problem in a centralized economy reads as follows:

$$
\begin{aligned}
\max_{C_t,D_t,I_{yt},I_{dt},K_{yt+1},K_{dt+1},H_{t+1}} \quad & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{((C_t - C_{ht})^{\mu_c}(H_t - H_{ht})^{\mu_h})^{1-\eta} - 1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = a_y{}^{(1-\alpha_y)t} Z_{yt} K_{yt}^{\alpha_y}, \\
& Y_t = C_t + I_t, \\
& D_t = a_d{}^{(1-\alpha_d)t} Z_{dt} K_{dt}^{\alpha_d}, \\
& I_t = I_{dt} + I_{yt}, \\
& K_{yt+1} = (1-\delta_k)K_{yt} + \Phi_y\left(\frac{I_{yt}}{K_{yt}}\right)K_{yt}, \\
& K_{dt+1} = (1-\delta_k)K_{dt} + \Phi_d\left(\frac{I_{dt}}{K_{dt}}\right)K_{dt}, \\
& H_{t+1} = (1-\delta_h)H_t + D_t^{1-\phi}.
\end{aligned} \tag{2}
$$

where $\eta, \alpha_y, \alpha_d, a_y, a_d, \mu_c, \mu_h > 0$, $\mu_c + \mu_h = 1$, $\beta, \delta_k, \delta_h \in (0,1)$, $\Phi_y(x) = \varphi_{y,2} + \frac{\varphi_{y,1}}{1-\kappa_y}x^{1-\kappa_y}$, and Φ_d analogously.

GDP is now defined by $\text{GDP}_t = Y_t + P_{dt}D_t + MRS_t^{H,C}H_t$ where P_{dt} is the relative price of residential investment goods. The return on housing remains the same as before but with $P_{ht} = P_{dt}D_t^{\phi}/(1-\phi)$, while the return on equity is the weighted sum of the return on capital in the two sectors, i.e. with the obvious adaption of notation from the one-sector model

$$
1 + R_{E,t+1} = \frac{\alpha_y Y_{t+1} - I_{y,t+1} + q_{y,t+1}K_{y,t+2} + \alpha_d P_{d,t+1}D_{t+1} - I_{d,t+1} + q_{d,t+1}K_{d,t+2}}{q_{y,t}K_{y,t+1} + q_{d,t}K_{d,t+1}}.
$$

The return on total risk is adjusted in an analogous way.

Calibration: The parameters η, ϕ, δ_k and δ_h remain at the same values they were previously set to in model B. Likewise, the weights μ_c and μ_h of consumption and housing in the household's utility are still pinned down by imposing that the ratio of expenditures on housing to total consumption is 19 percent on the balanced path. In order to take the two sector framework into account, we assume the same capital shares, α_y and α_d, as in Nguyen (2018). Moreover, we also take the autocorrelation parameters ρ_y and ρ_d of shocks to productivity in both sectors from Nguyen (2018). The standard deviations of innovations are chosen in such way that their ratio is kept the same as in Nguyen (2018) while the level is adjusted to reproduce a standard deviation of GDP comparable to models A and B and to the data. As already noted, we abstract from technology spillovers.[9] While Nguyen (2018) does not consider long-run growth, we choose $a_y = 1.005$ and $a_d = 1.002$ to match the annual output growth rates in the two sectors as reported by Davis and Heath-

[9]Spillovers in Nguyen (2018) are negligible. Fehrle (2019) makes the case for preventing spill-overs and correlated shocks.

cote (2005).

The remaining parameters are set again in such way that the (unweighted) sum of squared deviations between our targets in the model and in the data is minimized. The list of targets now includes the risk-free rate, the equity premium, the housing premium as well as the relative standard deviations and the correlations from the business cycle statistics in Table 2, all for US data. Our grid covers $\beta^\star := \beta a_y^{(\mu_c+(1-\phi)\mu_h\alpha_d)(1-\eta)} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)} \in [0.99, 0.999]$, $\chi_c, \chi_h \in [0.7, 0.95]$, and $\kappa_y, \kappa_d \in [0.625, 6.25]$ and is built-up from $10 \times 5 \times 5 \times 10 \times 10$ grid-points. A summary of the model's calibration is given in column C of Table 3.

Results: The return rates and business cycle statistics in the two-sector model are shown in row C of tables 4 and 5. First, restricting the household's option to smooth his consumption bundle by switching from residential investments to consumption has the desired effect on asset prices. Compared to model B, risk premia in the model again increase substantially. The model generates an equity premium of 4.31 percent which is about one and a half percentage-points below the value found in the data while the premium on housing in the model is moderately lower at 4.06 percent and is a half percentage-point below its empirical counterpart. Similar to the empirical findings, the model yields a premium of total risk in between the two premia of equity and housing. The model also reproduces a low risk-free rate but fails to explain the observed volatilities of asset prices. The standard deviation of the risk-free rate exceeds its empirical value by a factor of four, return rates on housing turn out too volatile by a factor of almost six, and the volatility of returns on total risk is too large by a factor of almost three. The model, hence, cannot explain a Sharpe ratio of housing which is markedly larger than that of equity.

The restriction of the household's option in the allocation between consumption and residential investments has a negative effect on the business cycle statistics. While in model B the household's preference to smooth the consumption bundle induces procyclical co-movement in the demand of residential goods, the positive correlation between house prices and residential investment now disappears. Moreover, the assumption of uncorrelated shocks in the two sectors prevents co-movements between residential and business investment. Since consumption and business investments account for the largest part of GDP, residential investments and GDP fluctuate almost uncorrelatedly.

Endogenous labor supply: Allowing the household to adjust labor supply in response to productivity shocks, again opens a channel which admits to smooth the consumption bundle more evenly across different states of shocks. As pointed out by Uhlig (2007), risk premia in the model should suffer.

Model D: Hours worked in the two sectors, N_{yt} and N_{dt}, augment the production functions and aggregated hours $N_t = N_{dt} + N_{yt}$ cannot exceed the time endowment of the household which is normalized to one. Accordingly, leisure $(1 - N_t)$ is added to the household's utility function which is parameterized as in Davis and Heathcote (2005) but extended by habit formation in leisure equivalent to consumption and housing, i.e. $N_{ht} := 1 - \chi_n(1 - N_{t-1})$. The changes to the household's problem from (2) are as follows

$$
\begin{aligned}
\max_{\ldots,N_{yt},N_{dt}} \; & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{((C_t - C_{ht})^{\mu_c}(H_t - H_{ht})^{\mu_h}((1-N_t)-(1-N_{ht}))^{\mu_n})^{1-\eta}-1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = Z_{yt}K_{yt}^{\alpha_y}(a_y^t N_{yt})^{1-\alpha_y}, \\
& Y_t = C_t + I_t, \\
& D_t = Z_{dt}K_{dt}^{\alpha_d}(a_d^t N_{dt})^{1-\alpha_d}, \\
& N_t = N_{dt} + N_{yt}, \\
& N_t \leq 1 \\
& \ldots
\end{aligned}
\tag{3}
$$

where now $\mu_c, \mu_h, \mu_n > 0, \; \mu_c + \mu_h + \mu_n = 1$.

Calibration: Again, in order to place emphasis on the effects of introducing endogenous labor decisions to the model, all parameters from model C remain at the same values as before. We only adjust the weights μ_c, μ_h and μ_n in the household's utility in such way that i) the housing expenditures remain at 19 percent of total consumption expenditures and ii) he works one third of his time endowment on average. Moreover, the habit parameter χ_n is set to its upper bound 0.95 of plausible values. Column D/E of Table 3 outlines the calibration.

Results: Row D of Table 4 confirms the already expected consequences of endogenous labor supply for the return rates in the model. Compared to model C, the return rates on equity, housing, and total risk, decrease and become significantly less volatile. In consequence, risk premia drastically fall by a magnitude of order.

Endogenous labor supply reintroduces the possibility for adjustments in the allocation of the consumption bundle after the shock's realization. The household is able to adjust his working hours intersectorally and can shift conveniently between consumption and residential investments. In consequence, the discussed demand effect for residential investment recurs as can be seen in row D of Table 5. The model can explain the volatilities of residential investment and house prices fairly well while business investment remains too involatile. Moreover, the model can also generate the positive correlations between house prices, residential investment, and GDP found in the data. Yet, residential and business investment are correlated too strongly.

Limited labor mobility: Two well-known extensions that help to revive risk premia when labor decisions are endogenous are limited sectoral mobility as described by Boldrin et al. (2001) and wage rigidities as proposed by Uhlig (2007). To keep in line with the present framework of limited factor mobility, we focus on the former.

Model E: The household is now unable to adapt his labor supply in response to technology shocks but is committed to working hours that are contracted sector-specifically one period ahead. Nothing else changes so that the household's problem remains as in (3) with the exception that he now optimizes with regard to N_{yt+1} and N_{dt+1} while taking N_{yt} and N_{dt} as given in any period t.

Calibration: We stick to the calibration in column D/E of Table 3 from the previous model D with a frictionless labor market.

Results: Return rates from the two-sector model with limited labor mobility are summarized in row E of Table 4. Limited labor mobility provides a mixture of the two previous cases with exogenous labor supply in model C and with endogenous and frictionless labor supply in model D. Hence, risk premia increase significantly compared to model D but remain below the values from model C. Moreover, the standard deviation of the risk-free rate turns out too large by a factor of almost six and the return rates on housing and on total risk are more than three times too volatile.

Table 5 shows in its row E that the model can generate positive correlations between business and residential investment, between GDP and residential investment and between GDP and house prices which are all close to the values in the data. However, the attempt to explain risk premia by shutting down the channel that enables the household to smooth his consumption bundle comes at the cost of too involatile residential investment which is no longer positively correlated with house prices.

3.3 Summary and discussion

In the classic Jermann (1998) model, habits increase the household's desire to smooth consumption of the composite good. However, if the model is extended by housing in a one sector framework, optimal adjustment of the allocation of output to consumption and residential investment enables the household to insure himself more conveniently against fluctuations in the consumption bundle. A small elasticity of housing in the consumption bundle and the rather small depreciation rate of housing favor the behavior. A similar argument holds in a multi-sector framework with perfect labor markets where the household can adapt the allocation of hours worked in each sector in response to productivity shocks. While this option implies that the marginal utility does not fluctuate enough between different realizations of the shock and therefore yields risk premia close to zero, it induces, on the other hand, a demand effect which results in positive correlations between residential investment and house prices and in standard deviations in business cycle statistics that are close to the data.

Risk premia can be increased through sectoral frictions as e.g. limited capital and labor mobility. Yet, this comes at the cost of losing the empirical co-movements of residential investment. Therefore, we conclude that the present framework cannot simultaneously reproduce asset pricing statistics and business cycle statistics as observed in the data. Moreover, the models fail to explain the different volatilities and Sharpe ratios between the two

Table 6: Risk premia components

	$\frac{2\sqrt{\text{Var}[M_{t+1}]}}{(\mathbb{E}[M_{t+1}])^4}$	SR_E	SR_H	$r_{M,\text{EP}}$	$r_{M,\text{HP}}$	$r_{\text{EP,HP}}$
			Data			
USA	–	0.36	1.01	–	–	0.19
			Model			
B	0.07	0.07	0.07	−0.99	−0.99	1.00
C	0.23	0.21	0.21	−0.92	−0.92	1.00
D	0.08	0.07	0.07	−0.98	−0.96	0.99
E	0.23	0.18	0.18	−0.80	−0.80	1.00

Notes: SR_E : annualized Sharpe ratio of equity, SR_H : annualized Sharpe ratio of housing, $r_{X,Y}$: correlation between variables X and Y where M_{t+1} : stochastic discount factor, $EP_{t+1} := R_{E,t+1} - R_{f,t}$: ex-post equity premium, $HP_{t+1} := R_{H,t+1} - R_{f,t}$: ex-post housing premium.

risky assets throughout. In all models and contrary to the data, the mean as well as the standard deviation of returns on total risk are the weighted averages from the returns on equity and housing.

In order to provide some additional reasoning for the models' failures in regard to asset price statistics, note that the models' Euler equations together with the definition of the risk-free rate imply

$$\mathbb{E}_t\left[M_{t+1}(R_{E,t+1} - R_{f,t})\right] = \mathbb{E}_t\left[M_{t+1}(R_{H,t+1} - R_{f,t})\right] = 0,$$

where M_{t+1} denotes the models' respective stochastic discount factor. Taking unconditional expectations, the equality also holds unconditionally for the models' stationary distributions. Hence, for both assets $X \in \{E, H\}$,

$$\begin{aligned}\mathbb{E}\left[M_{t+1}\right]\mathbb{E}\left[R_{X,t+1} - R_{f,t}\right] &= -\text{Cov}\left[M_{t+1}, R_{X,t+1} - R_{f,t}\right] = \\ &= -\text{Corr}\left[M_{t+1}, R_{X,t+1} - R_{f,t}\right]\sqrt{\text{Var}\left[R_{X,t+1} - R_{f,t}\right]}\sqrt{\text{Var}\left[M_{t+1}\right]},\end{aligned}$$

or equivalently for the Sharpe ratio

$$\text{SR}_X := \frac{\mathbb{E}\left[R_{X,t+1} - R_{f,t}\right]}{\sqrt{\text{Var}\left[R_{X,t+1} - R_{f,t}\right]}} = -\frac{\sqrt{\text{Var}\left[M_{t+1}\right]}}{\mathbb{E}\left[M_{t+1}\right]}\text{Corr}\left[M_{t+1}, R_{X,t+1} - R_{f,t}\right]. \tag{4}$$

The first factor on the right hand side defines an upper bound and is common to both assets, while different correlations between risk premia and the stochastic discount factor between the two assets are necessary in order to explain different Sharpe ratios. More precisely, in order to match the different Sharpe ratios observed in the data, the correlation between premia on housing and the stochastic discount factor must be (in absolute value) approximately 3 times as large as the correlation between premia on equity and the stochastic discount.

We summarize the decomposition of the Sharpe ratios provided by equation (4) in Table 6. First, we observe that in all models the two risky return rates are almost perfectly correlated and, hence, the correlations with the stochastic discount factor are nearly identical. By (4), the Sharpe ratios of the two risky assets must also coincide. Although models C and E can replicate the empirically observed Sharpe ratio of equity fairly well, we can conclude that they achieve this result in an unfitting way. In order to leave room for a significantly larger Sharpe ratio of housing, the correlation between the stochastic discount factor and the premia on equity would have to be substantially smaller (in absolute value). The smaller correlation $r_{M,\mathrm{EP}}$ would then have to be offset with a larger standard deviation of the stochastic discount factor to keep the equity premium and its volatility the same, and less volatile premia on housing would be necessary in order to still match their mean. The models B and D, which fail to generate sizeable risk premia and Sharpe ratios, suffer from a too low volatility of the stochastic discount factor—the agent can adjust his decisions sufficiently well in response to shocks in order to keep fluctuations in his marginal utility small.

Moreover, the nearly perfect correlation between the return rates of the two risky assets also implies that the mean *and* the standard deviation of the total portfolio are the weighted averages of the two assets. Contrary to the observations from the stylized facts, the Sharpe ratio of total risk must coincide with the Sharpe ratios of the two risky assets.[10]

Fehrle (2019) discusses the implications of a larger share of land in the production of new houses. He shows that in the in the Davis and Heathcote (2005) framework the ability to account for co-moving economic activity, especially for the correlation between residential investment and house prices, can be improved. We follow Fehrle (2019) and repeat our computations for $\phi = 0.3$ which is the upper bound considered by Fehrle (2019). The results are summarized in Appendix B.[11] We find that improvements in the business cycle statistics are only marginal and effects on asset return statistics are ambiguous.

4 Housing with disaster risk

In this section we move to another popular approach to explain risk premia. We combine an otherwise standard RBC model with housing and with key elements from the literature on economic disasters. The model is based on Gourio (2012). It is extended by housing and features a time-varying risk for disasters which reduce productivity and which also partly destroy the stocks of productive capital and residential structures. We choose to keep the model as simple as possible and provide easily traceable insights of the model's mechanisms instead of a richer framework that would supply more degrees of freedom to

[10] For any non-stochastic share $w \in (0,1)$ of equity in total wealth, if $r_{EP,HP} = 1$, then

$$\sigma(TP) = (w^2\sigma^2(EP) + (1-w)^2\sigma^2(HP) + 2w(1-w)r_{EP,HP}\sigma(EP)\sigma(HP))^{\frac{1}{2}} = w\sigma(EP) + (1-w)\sigma(HP),$$

and, if additionally $SR_E = SR_H$, then also

$$SR_T = \frac{w\sigma(EP)}{w\sigma(EP) + (1-w)\sigma(HP)} SR_E + \frac{(1-w)\sigma(HP)}{w\sigma(EP) + (1-w)\sigma(HP)} SR_H = SR_E = SR_H.$$

[11] Favilukis et al. (2017) proceed similarly by setting the land share equal 0.25.

match the data. A more detailed presentation of the model, including our solution method, is delegated to Appendix D.

4.1 Model

The basic framework of the model follows the one-sector model from the previous section. The household derives utility from a composite good $\tilde{C}_t$ that is represented by a Cobb-Douglas aggregate consisting of consumption C_t, housing H_t and leisure $1-N_t$, i.e.

$$\tilde{C}_t := C_t^{\mu_c} H_t^{\mu_h} (1-N_t)^{1-\mu_c-\mu_h}.$$

We assume that the household's preferences over streams of the composite good are described by a recursive utility function, as introduced by Epstein and Zin (1989) and Weil (1989), of the form

$$\tilde{V}_t = \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t \tilde{V}_{t+1}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ is the household's elasticity of intertemporal substitution (EIS) and γ is the coefficient of relative risk aversion (RRA). Note however that γ and ψ describe the household's RRA and EIS with respect to the composite good $\tilde{C}$. Since the composite good aggregator is of the Cobb-Douglas type, the consumption-based RRA is given by $\mu_c\gamma$ and the consumption-based EIS reads $\frac{1}{1-\mu_c(1-1/\psi)}$.[12] For easier notation we define $V_t := \tilde{V}_t^{1-1/\psi}$ which satisfies the recursion

$$V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}},$$

where we use, similar to Caldara et al. (2012), the notation

$$\theta := 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

In the case where $\theta = 0$, the RRA equals the reciprocal of the EIS and the household's utility reduces to the 'classical' expected discounted sum of within period CRRA utilities. Hence, θ can also be interpreted as the deviation from this 'classic' case.

Output Y_t is produced with the help of capital K_t and labor N_t according to the Cobb-Douglas production function $Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}$ where A_t denotes labor augmenting technological progress which grows stochastically as will be outlined below. We stick to the assumption that investments in the productive capital stock are met with adjustment costs as in Jermann (1998). Output is allocated between the homogenous goods consumption, business investments, and investments in residential structures. Residential structures must be combined with land, which acts as adjustment costs to residential investments, before entering the stock of houses.

[12] See also Swanson (2012) and Heiberger and Ruf (2019).

Additionally, the economy faces a great disaster risk. Disasters are introduced through an exogenous shock in form of a binary variable b_t which indicates disasters in case of $b_t = 1$ while $b_t = 0$ in normal times. Following Gourio (2012), disasters appear with time-varying probability and size. More specifically, we assume that

$$P(b_{t+1} = 1|b_t = 0) = \min\{p_t, 1\},\ P(b_{t+1} = 0|b_t = 0) = 1 - \min\{p_t, 1\}$$

where the log of p_t follows an AR(1)-process

$$\ln p_{t+1} = (1 - \rho_p) \ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1},\ \epsilon_{p,t} \sim \text{iidN}(0, \sigma_p^2). \tag{5}$$

Additionally, disasters remain persistent with probability no less than $q \in (0, 1)$ so that

$$P(b_{t+1} = 1|b_t = 1) = \max\{q, \min\{p_t, 1\}\},\ P(b_{t+1} = 0|b_t = 1) = 1 - \max\{q, \min\{p_t, 1\}\}.$$

On the one hand, disasters result in a decline of productivity by the factor $1 - e^{\omega_{t+1}}$ so that technology grows stochastically according to

$$\begin{aligned} A_{t+1} &= A_t a e^{z_{t+1} + \omega_{t+1} b_{t+1}}, \\ z_{t+1} &= \rho_z z_t + \epsilon_{z,t+1},\ \epsilon_{z,t} \sim \text{iidN}(0, \sigma_z^2). \end{aligned}$$

On the other hand, disasters also result in the destruction of a fraction $1 - e^{\omega_{t+1}}$ of the stocks of capital and residential structures, i.e.

$$\begin{aligned} H_{t+1} &= e^{\omega_{t+1} b_{t+1}(1-\phi)} \underbrace{\left((1 - \delta_h) H_t + D_t^{1-\phi}\right)}_{=: H^\star_{t+1}}, \\ K_{t+1} &= e^{\omega_{t+1} b_{t+1}} \underbrace{\left((1 - \delta_k) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t\right)}_{=: K^\star_{t+1}}, \text{where } \Phi(x) := \frac{\varphi_1}{1 - \kappa} x^{1-\kappa} + \varphi_2. \end{aligned}$$

Finally, the disaster size $1 - e^{\omega_{t+1}}$ also evolves stochastically according to

$$\begin{aligned} \omega_t &:= \bar{\omega} e^{\hat{\omega}_t}, \\ \hat{\omega}_{t+1} &= \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1},\ \epsilon_{\omega,t} \sim \text{iidN}\left(0, \sigma_\omega^2\right), \end{aligned} \tag{6}$$

where $\bar{\omega} < 0$. We slightly deviate from the treatment in Gourio (2012) in the specification of the process governing the disaster size and allow autocorrelation but restrict outcomes to $\omega_t < 0$ so that disasters always have negative effects. The specification is similar to Fernández-Villaverde and Levintal (2018).[13]

[13] Gourio (2012) additionally considers a transitory component of disasters. We checked the effects of a transitory shock component as well. Since we find that the effects for our targets are marginal, we omit the transitory component for the sake of simplicity.

Summing up, the household's problem in a centralized economy reads as follows

$$
\begin{aligned}
\max_{C_t, I_t, D_t, N_t, K^*_{t+1}, H^*_{t+1}} / \min \quad & V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \\
\text{s.t.} \quad & Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}, \\
& Y_t = C_t + I_t + D_t, \\
& H_{t+1} = e^{\omega_{t+1} b_{t+1}(1-\phi)} \left(D_t^{1-\phi} + (1-\delta_h)H_t\right), \\
& K_{t+1} = e^{\omega_{t+1} b_{t+1}} \left((1-\delta_k)K_t + \Phi\Big(\frac{I_t}{K_t}\Big)K_t\right), \\
& D_t, I_t \geq 0,
\end{aligned}
\tag{7}
$$

where K^*_{t+1} and H^*_{t+1} is the size of the stocks before b_{t+1} realizes and Φ remains defined as before. We define GDP again as the sum of consumption, both investment types and the implicit rent from housing.

Return Rates and Leverage The return rates on equity, housing and total risk are defined by (see also Gourio (2012) and Heiberger (2018)):

$$
\begin{aligned}
1 + R_{E,t+1} &= e^{b_{t+1}\omega_{t+1}} \frac{r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} = \frac{Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K^\star_{t+2}}{q_t K^\star_{t+1}}, \\
1 + R_{H,t+1} &= e^{(1-\phi)b_{t+1}\omega_{t+1}} \frac{MRS^{H,C}_{t+1} + (1-\delta_h)P_{ht+1}}{P_{ht}} = \frac{\frac{\mu_h}{\mu_c}C_{t+1} - P_{h,t+1}D^{1-\phi}_{t+1} + P_{h,t+1}H^\star_{t+2}}{P_{h,t}H^\star_{t+1}}, \\
1 + R_{T,t+1} &= \frac{\frac{\mu_h}{\mu_c}C_{t+1} - P_{h,t+1}D^{1-\phi}_{t+1} + P_{h,t+1}H^\star_{t+2} + Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K^\star_{t+2}}{P_{h,t}H^\star_{t+1} + q_t K^\star_{t+1}},
\end{aligned}
$$

where $r_t = \alpha(Y_t/K_t), W_t = (1-\alpha)(Y_t/N_t), q_t = 1/\Phi'(I_t/K_t)$ and $MRS^{H,C}_t = (\mu_h/\mu_c)(C_t/H_t)$ are the marginal product of capital, the real wage rate, Tobin's q and the marginal rate of substitution between housing and consumption, respectively. Moreover, the risk-free rate satisfies

$$
1 + R_{f,t} = \frac{1}{\mathbb{E}_t M_{t+1}},
$$

where M_{t+1} denotes the model's stochastic discount factor given by

$$
M_{t+1} := \beta\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta} \quad \text{with} \quad \Lambda_t := \mu_c \frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{C_t}.
$$

However, $R_{f,t}$ is the return rate of a real risk-free government bond which, as already noted, does not have an equivalent empirical counterpart. We therefore follow Barro (2006) and Gourio (2012) and assume that bonds in the model may also default during disasters. More concretely, we consider government (gb), corporate (cb) and housing (hb) bonds which differ by their recovery rates $\Gamma_{x,t}$ during disasters, $x \in \{\text{gb}, \text{cb}, \text{hb}\}$. The price $Q_{x,t}^{(T_x)}$ of such a bond with bond specific maturity T_x then satisfies the recursion

$$Q_{x,t}^{(T_x)} = \mathbb{E}_t[M_{t+1}\left(1 - b_{t+1} + b_{t+1}\Gamma_{x,t+1}\right)Q_{x,t+1}^{(T_x-1)}], \quad \text{where } Q_{x,t+1}^{(0)} \equiv 1.$$

The ex-post return rates from holding bonds with maturity T_x for one period are defined by

$$1 + R_{x,t+1}^{(T_x)} := \frac{\left(1 - b_{t+1} + b_{t+1}\Gamma_{x,t+1}\right)Q_{x,t+1}^{(T_x-1)}}{Q_{x,t}^{(T_x)}}.$$

We assume that the rates at which bonds default during disasters are coupled to the disaster size $1 - e^{\omega_{t+1}}$ via constant fractions $\chi_x \in [0,1]$ so that

$$\Gamma_{x,t+1} = 1 - \chi_x(1 - e^{\omega_{t+1}}).$$

Finally, since the return on equity in the data is calculated from stock returns, it includes leverage. This does not hold for housing returns. To be in line with the data, we also consider leveraged return rates in the model. More precisely, we assume that in each period the constant fraction $m_{cb} \in [0,1)$ of the firm's capital stock is financed by debt through bonds which all have maturity T_{cb}. Since the Modigliani and Miller theorem holds, the leveraged return rate on equity and the leveraged return rate on total risk are then given by

$$R_{E,t+1}^{lev} := \frac{1}{1 - m_{cb}} R_{E,t+1} - \frac{m_{cb}}{1 - m_{cb}} R_{cb,t+1}^{(T_{cb})},$$
$$R_{T,t+1}^{lev} := \frac{P_{h,t}H_{t+1}^{*}}{P_{h,t}H_{t+1}^{*} + q_t K_{t+1}^{\star}} R_{H,t+1} + \frac{q_t K_{t+1}^{\star}}{P_{h,t}H_{t+1}^{*} + q_t K_{t+1}^{\star}} R_{E,t+1}^{lev}.$$

4.2 Calibration

Our analysis considers different variations of the model. We start with a variation of the model, named F, which excludes housing before introducing housing into the model in variation G. In models F and G disaster risk is time-varying in that both the probability p_t and the disaster size ω_t follow stochastic processes (5) and (6), respectively. Model H shuts down the stochastic effect for p_t and model I for ω_t while in model J both effects are shut down.

First, the share α of capital in production and the depreciation rates δ_k and δ_h of productive capital and housing remain the same as in our variation of the Jermann (1998) model with housing (model B). We increase the coefficient of RRA moderately to $\gamma = 5.5$ and set the now disentangled EIS to $\psi = 2$ following Gourio (2012). We maintain our strategy to set the elasticities in the composite good in such way that the household's expenditures on

Table 7: Calibration

Parameter	Value	Description
β	0.995	discount factor
ψ	2	elasticity of intertemporal substitution
γ	5.5	coefficient of relative risk aversion
$\mu_c^{*)}$	0.30	weight of consumption in composite good
$\mu_h^{*)}$	0.07	weight of housing in composite good
α	0.36	capital share in production
ϕ	0.30	share of land in housing
κ	0.80	elasticity of Tobin's q wrt. investment-capital ratio
δ_k	0.022	rate of capital depreciation
δ_h	0.009	rate of housing depreciation
$\ln a$	0.005	growth rate
ρ_z	0.00	autocorrelation of log technology shock
ρ_ω	0.00	autocorrelation of log disaster size
ρ_p	0.95	autocorrelation of log disaster probability
σ_z	0.01	conditional standard deviation of log technology shock
σ_ω	0.67	conditional standard deviation of log disaster size
$\sigma_p/\sqrt{1-\rho_p^2}$	2.5	unconditional standard deviation of log disaster probability
$\bar{\omega}$	-0.067	disaster size
$\bar{p}/\exp(\frac{\sigma_p^2}{2(1-\rho_p^2)})$	0.0079	mean disaster probability
q	0.93	probability for disaster persistence
χ_{gb}	0.20	default loss of government bonds as fraction of disaster size
χ_{cb}	0.38	default loss of corporate bonds as fraction of disaster size
T_{gb}	1	maturity of the government bond
T_{cb}	10	maturity of the corporate bond
m_{cb}	0.37	corporate's financial leverage

Notes: $^{*)}$ Endogenous by the model.

housing account to 19 percent of his total consumption expenditures and that the household works one third of his time endowment on balanced growth (Model D/E). Moreover, the average growth rate a of technology during normal times is also kept the same as before and corresponds to the value in Gourio (2012). We set $\rho_z = 0$ and $\sigma_z = 0.01$ so that during normal times the stochastic process governing technological progress is identical to the process for the permanent component of productivity in Gourio (2012). The share of land in new houses is set to the upper bound $\phi = 0.3$ from Fehrle (2019) in order to fit the model closer to the data.[14]

The calibration of the 'free' parameters β and κ and of the additional parameters from the introduction of rare disasters is guided by Gourio (2012) and Fernández-Villaverde and Levintal (2018) but moderately adjusted in order to fit the model closer to the data. More precisely, we set $\beta = 0.995$ and $\kappa = 0.8$. Further, we follow Gourio (2012) and assume an iid process for the disaster size, i.e. $\rho_\omega = 0$. We choose $\bar{\omega} = -0.067$ and $\sigma_\omega = 0.67$ which implies a mean disaster size of approximately 8 percent. In comparison, the mean disaster size of the transitory and permanent components combined is approximately 6 percent in Gourio (2012). For the probability to enter a disaster, we choose a moderately larger autocorrelation $\rho_p = 0.95$ and a moderately lower standard deviation $\sigma_p = 2.5\sqrt{1-\rho_p^2}$ instead of $\rho_p = 0.9$ and $\sigma_p = 2.8\sqrt{1-\rho_p^2}$ in Gourio (2012). Finally, $\bar{p}$ is set such way that the average probability of entering a disaster is 0.72 percent—the same value used by Gourio (2012)—while the persistence of disasters is pinned down to $q = 0.93$—moderately above the value of $q = 0.914$ employed by Gourio (2012).

Lastly, we make the following assumptions for asset prices. Consistent with Barro (2006) and Gourio (2012) the default loss of government bonds is 20 percent of the disaster size, i.e. $\chi_{gb} = 0.2$, while the default loss of corporate bonds is set to a higher value of $\chi_{cb} = 0.38$. We consider government bonds with maturity of one period since our empirical counterpart are bills, and the maturity of corporate bonds is set to $T_{cb} = 10$. Gourio (2012) reports financial leverage of approximately 30 percent in the data. However, he interprets leverage in a broader way, i.e. also as operating leverage, and therefore chooses a larger level of leverage of approximately 50 percent for the calibration of his model. Our value of $m_{cb} = 0.37$ lies in between.

4.3 Results

Table 8 presents the asset return statistics for unlevered equity, for housing and for a real risk-free bond while Table 9 shows the return rates for a government bond with partial default in disasters and for leveraged equity. The business cycle statistics are summarized in Table 10. Note that we follow Gourio (2012) and, except for row G*, report statistics which are computed from samples where no disasters appear.

First, comparing rows F and G reveals that the introduction of housing into the model has only negligible effects on the return rates of unlevered and leveraged equity. The model (G) can explain return rates on leveraged equity and on government bonds which

[14] We present and discuss the results of model G with $\phi = 0.106$ in Appendix B. Favilukis et al. (2017) proceed similar by setting the land share equal to 0.1 and 0.25.

Table 8: Simulated returns, premiums and second moments II (no default, no leverage)

	R_E	R_H	R_T	R_f	EP	HP	TP	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
					Model					
F	4.6	–	–	0.88	3.70	–	–	0.90	–	–
G	4.88	4.35	4.64	1.13	3.72	3.19	3.49	0.88	0.94	0.91
G*	3.31	2.8	3.08	−0.15	3.46	2.95	3.24	0.62	0.60	0.62
H	5.34	4.91	5.15	−1.25	6.66	6.21	6.46	5.33	8.51	6.33
I	4.41	4.04	4.25	2.32	2.06	1.69	1.90	0.64	0.71	0.68
J	4.70	4.42	4.58	1.33	3.34	3.06	3.22	2.67	4.43	3.19
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$			
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00			
					Model					
F	3.42	–	–	1.61	4.11	–	–			
G	3.56	2.64	3.15	1.58	4.22	3.41	3.85			
G*	5.82	5.26	5.56	3.55	5.55	4.92	5.26			
H	1.25	0.73	1.03	0.07	1.25	0.73	1.02			
I	2.78	1.86	2.36	1.07	3.20	2.37	2.81			
J	1.25	0.70	1.01	0.07	1.25	0.69	1.01			

Notes: Mean percentage returns of equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity (EP), housing (HP), and the total risk premium (TP). The corresponding standard deviations $\sigma(X)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). We employ projection methods and simulated time series with 100,000 periods. The sample does **not** include disasters except for row G*. **F**: Benchmark rare disaster. **G**: F + Housing (no disaster sample). **G***: F + Housing (disaster sample). **H**: G but constant disaster probability. **I**: G but constant disaster size. **J**: G but constant disaster probability and size.

are close to the data and the model can replicate an equity premium of 6.56 percent. In accordance to the data, the return on housing (4.35%) and the housing premium (3.00%) turn out smaller than the return on equity and the equity premium. Yet, they remain approximately 1.5 percentage points below the values found in the data. Nevertheless, the model can closely match the empirical total risk premium. The model can further generate a low volatility of government bonds and reproduces the empirically observed standard deviations of returns and premia on housing fairly well. The standard deviations of equity returns and premia in the model are less than half of their empirical counterparts. Risk premia in the model are moderately more volatile than the risky return rates. Although the return rates and volatilities differ between the two risky assets, their Sharpe ratios turn out almost identical at approximately 0.9 and also coincide with the Sharpe ratio of total risk. Hence, the model can closely replicate the Sharpe ratio of housing of approximately 1.01 in the data but fails for the Sharpe ratio of equity which is substantially lower at 0.36 in the data.

Turning to business cycle statistics, the volatility of GDP in the model is too small. However, the model is able to generate relative volatilities of business investments (2.78), residential investments (5.56) and house prices (1.67) which all fit the data. Moreover, the model also reproduces the empirically observed correlation between GDP and residential

Table 9: Simulated returns, premiums and second moments II (default and leverage)

	R_E^{lev}	R_H	R_T^{lev}	R_{gb}	EP^{lev}	HP	TP^{lev}	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
					Model					
F	7.64	–	–	1.07	6.52	–	–	0.92	–	–
G	7.93	4.35	6.34	1.31	6.56	3.00	4.98	0.90	0.91	0.91
G*	6.08	2.80	4.63	0.03	6.04	2.76	4.60	0.66	0.61	0.65
H	10.87	4.91	8.25	−0.72	11.65	5.66	9.02	5.80	7.75	6.22
I	6.05	4.04	5.17	2.42	3.57	1.59	2.70	0.66	0.69	0.67
J	7.51	4.42	6.17	1.59	5.86	2.80	4.53	2.93	4.06	3.15
	$\sigma(R_E^{lev})$	$\sigma(R_H)$	$\sigma(R_T^{lev})$	$\sigma(R_{gb})$	$\sigma(EP^{lev})$	$\sigma(HP)$	$\sigma(TP^{lev})$			
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00			
					Model					
F	6.47	–	–	1.43	7.12	–	–			
G	6.67	2.64	4.87	1.39	7.29	3.29	5.50			
G*	9.34	5.26	7.44	3.45	9.13	4.51	7.08			
H	2.01	0.73	1.45	0.07	2.01	0.73	1.45			
I	5.03	1.86	3.62	0.94	5.41	2.29	4.01			
J	2.00	0.70	1.44	0.07	2.00	0.69	1.44			

Notes: Mean percentage returns of leveraged equity (R_E^{lev}), housing (R_H), leveraged total risk (R_T^{lev}) and bills (R_{gb}), as well as the leveraged equity (EP^{lev}), housing (HP), and the leveraged total risk premium (TP^{lev}) and the corresponding standard deviations $\sigma(X)$. We employ projection methods and simulated time series with 100,000 periods. The sample does **not** include disasters except for row G*. **F**: Benchmark rare disaster. **G**: F + Housing (no disaster sample). **G***: F + Housing (disaster sample). **H**: G but constant disaster probability. **I**: G but constant disaster size. **J**: G but constant disaster probability and size.

investments and between GDP and house prices. The model shows almost perfect positive correlations between residential investments and house prices and between residential investments and business investments—and both correlations are also positive in the data.

The moments discussed so far from simulations without disasters are driven only by the agents' expectations about disaster whereas the actual occurrence of disasters is shut off. Row G* of the tables shows the moments from simulations which include disasters. With disasters in the sample, the mean return rate of the government bond already falls close to zero. The return on leveraged equity declines by approximately 2 percentage points and the equity premium decreases by 0.5 percentage points towards its empirical target. The return on housing and the housing premium decrease slightly less by approximately 1.5 and 0.25 percentage points, respectively, and we can observe similar effects for the return on total risk. Of course, the most obvious effect of samples with disasters is on the variables' second moments. The sample with disasters helps to increase the volatilities of the risky assets but also implies a counterfactual large standard deviation of the government bond. Moreover, GDP becomes too volatile and the model's fit of the relative standard deviations deteriorates.

The effects of time-varying disaster risk are illustrated in rows H, I, and J, which show the results from the model if the stochastic nature of the disaster probability, of the disaster size, or of both components is shut down. First, row H reveals that a time-varying proba-

Table 10: Simulated business cycle statistics II

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
				Data				
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64
				Model				
F	0.90	2.76	–	–	–	–	–	–
G	0.93	2.78	5.56	1.67	1.00	0.99	0.66	0.66
G*	2.72	1.94	3.55	1.07	1.00	0.96	0.63	0.63
H	0.84	1.12	1.78	0.53	1.00	1.00	1.00	1.00
I	0.89	2.22	4.01	1.20	1.00	0.98	0.69	0.69
J	0.84	1.12	1.70	0.51	1.00	1.00	1.00	1.00

Notes: σ_x is the standard deviation of x, r_y^x the correlation between x and y. Business cycle statistics from HP-filtered (1600) times series. We employ projection methods. Business cycle statistics are the mean outcome of repeated simulations of hp-filtered (1600) times series with 180 periods each. The sample does **not** include disasters except for row G*. **F**: Benchmark rare disaster. **G**: F + Housing (no disaster sample). **G***: F + Housing (disaster sample). **H**: G but constant disaster probability. **I**: G but constant disaster size. **J**: G but constant disaster probability and size.

bility for the economy to be hit by a disaster is essential for the model's dynamics. While the unlevered return rates on the risky assets and on total risk change only moderately if the probability for disasters is held constant, the risk-free rate and the return on bonds—government and corporate—decrease considerably. As a consequence of decreasing return rates on corporate bonds, the return on leveraged equity increases substantially by more than 3 percentage points and the leveraged equity premium now exceeds its empirical value almost by a factor of 2. On the other hand, since the housing premium is unlevered, it is only affected by the decreasing return on government bonds and, hence, rises only moderately above its empirical counterpart. The premium on total risk remains close to the average of the two risky assets. Further, row H shows that a time-varying probability to enter disasters is also the main factor to generate fluctuations in the return rates. In fact, with constant disaster probability the standard deviation of bonds falls close to zero and the standard deviations of returns on the risky assets and on total risk collapse by a factor of 4-5. Similarly, row H of Table 10 also illustrates that a time-varying disaster probability helps the model to generate the relative volatilities of business investments, residential investments and house prices and also helps to disentangle the otherwise perfect correlations between variables.

Since we assumed an uncorrelated shock process ($\rho_\omega = 0$), the model's results depend far less on the stochastic nature of the disaster size. In fact, shocks to the disaster size do not provoke any reactions of the model's variables in non-disaster periods but the effects show only indirect through expectations in the model solution. We can see from row I of tables 8 and 9 that, if the stochastic effect on the disaster size is shut off, the return rates on the risky assets and on total risk decline while the return on bonds increases. Moreover, a time varying disaster size helps to moderately increase the volatilities of all return rates and also helps to increase the relative volatilities of business investments, residential investments and house prices (see row I of Table 10). On the other hand, the variables' correlations remain almost unchanged.

4.4 Summary and discussion:

Figure 1 displays the reaction of the model's variables in response to a one time shock to technology z_t (panels (a)-(c)), to a one time shock to the probability p_t of entering a disaster (panels ((d)-(f)) and to a disaster which lasts for 5 periods (panels (g)-(i)) starting from the stochastic steady state in a non-disaster period.[15] We show percent deviations from the initial balanced growth path.

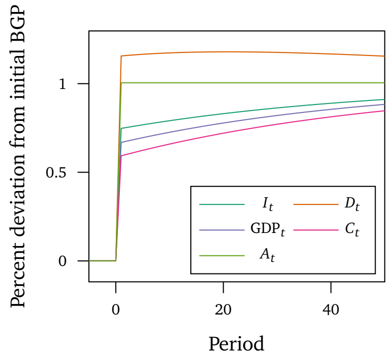
First, the variables' response to a 'classic' technology shock (panels (a)-(c)) is standard, and business investments, residential investments, and consumption increase in the period the shock hits the economy. An increase of business investments implies an increasing Tobin's q, $q_t = (1/\varphi_1)(I_t/K_t)^{\kappa}$, and increasing residential investments imply increasing house prices, $P_{h,t} = (1/(1-\phi))D_t^{\phi}$. Although D_t increases more than I_t, the elasticity κ of Tobin's q exceeds the elasticity ϕ of house prices and Tobin's q expands significantly more than house prices. Moreover, increasing productivity yields an increasing marginal product of capital and increasing consumption implies an increasing marginal rate of substitution between housing and consumption. In consequence, the returns on unlevered equity and on housing increase but—mainly due to the larger elasticity of Tobin's q—the return on unlevered equity dominates. Bonds do not react since the technology process is uncorrelated ($\rho_z = 0$), and debt additionally multiplies the effect for the leveraged return on equity.

On the other hand, an increase of the probability for the economy to enter a disaster has the following effects (see panels (d)-(f)). Positive autocorrelation ($\rho_p > 0$) implies an increased risk for a drop in productivity and for destruction of capital in the next period. In consequence, the representative agent lowers investments in productive capital and in residential structures and increases consumption instead. Decreasing investments entail drops in Tobin's q and in house prices. Although investments in residential structures decline more than investments in productive capital, the different elasticities again imply that the effect on Tobin's q dominates the effect on house prices. Moreover, a reduction of working hours implies a decreasing marginal product of capital r_t whereas increasing consumption implies that the marginal rate of substitution between housing and consumption increases. The more pronounced drop in Tobin's q compared to the drop in house prices combined with an increasing $MRS^{H,C}$ yields a larger contraction of the return on unlevered equity than of the return on housing and the effect is further amplified by leverage. Finally, increased disaster risk increases the stochastic discount factor so that bond prices increase. Yet, the effect is significantly smaller than on the risky assets.

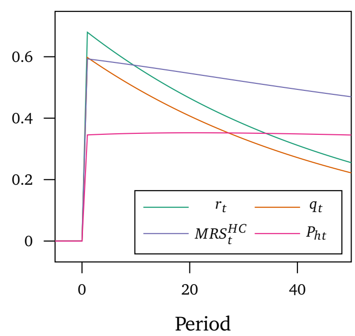
Lastly, an occurrence of a disaster (panels ((g)-(i)) implies that technology A_t drops by the factor $e^{\bar{\omega}}$ as long as the disaster continues. In the period the disaster starts, a second effect appears. The probability that the disaster remains persistent raises to $q = 0.93$ whereas the probability to enter a disaster was initially only $p \approx 0.0072$. The massive increase in probability for continued destruction of technology, capital and residential structures in the

[15] As already noted, the assumption of an iid process for the disaster size ($\rho_\omega = 0$) implies that shocks to the disaster size do not provoke any reaction of the model's remaining variables.

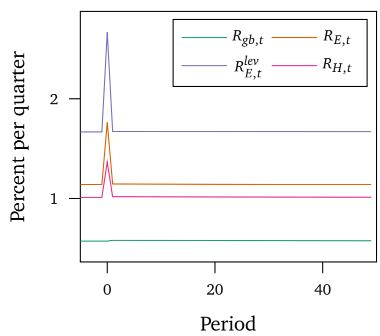
Figure 1: Impulse response functions to productivity and disaster risk shocks



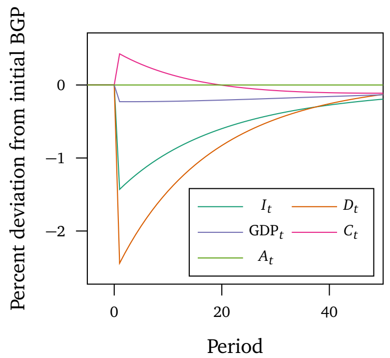
(a) Technology shock to z_t (I)



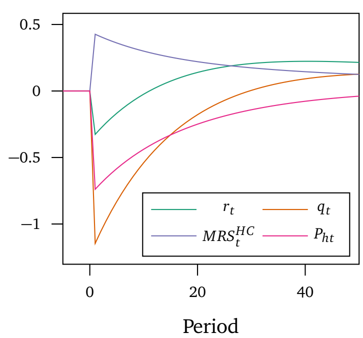
(b) Technology shock to z_t (II)



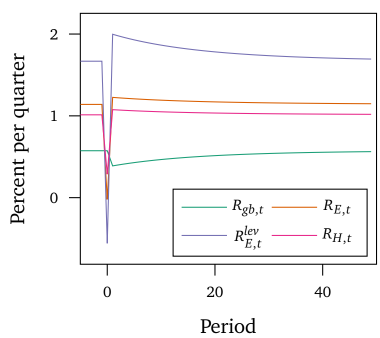
(c) Technology shock to z_t (III)



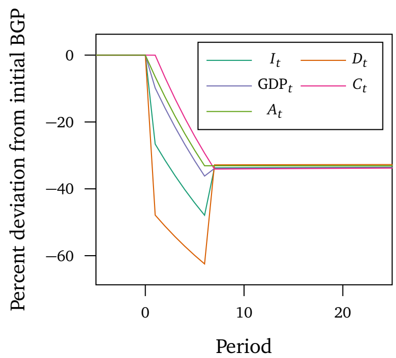
(d) Disaster risk shock to p_t (I)



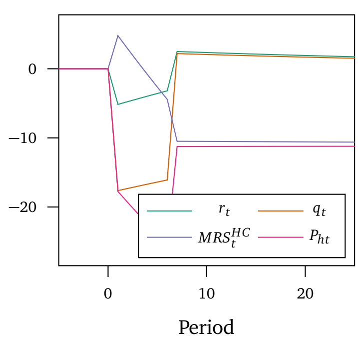
(e) Disaster risk shock to p_t (II)



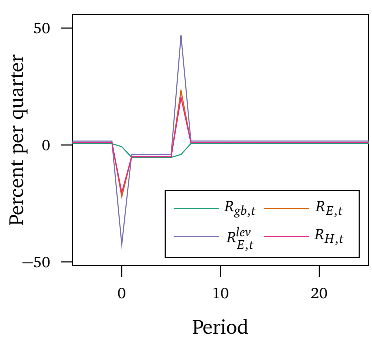
(f) Disaster risk shock to p_t (III)



(g) Disaster $b_t = 1$ (I)



(h) Disaster $b_t = 1$ (II)



(i) Disaster $b_t = 1$ (III)

subsequent period has the previously described effects—amplified by a multitude. The two effects combined—drop in productivity and increased risk for the disaster to persist—cause huge drops of business investments and residential investments in the initial period of the disaster. In the following disaster periods, expectations do not change anymore until the disaster ends so that investments are only effected by decreasing technology, capital and residential structures. The initial drop of business investments exceeds the destruction of productive capital so that Tobin's q also collapses. In the following periods the effect turns and business investments decline by less than the rate at which capital is destructed so that Tobin's q begins to slowly recover. On the other hand, since land is not destructed, house prices continue to decline as long as D_t declines. Finally, once the disaster ends, the probability for the economy to be hit by a disaster again jumps back to $p \approx 0.0072$. The massive change in expectations leads to a boom immediately after the disaster. Both investments increase and so do Tobin's q and house prices. The huge drops in Tobin's q and in house prices at the start of the disaster yield huge drops in the return rates while the boom after the disaster ends implies huge yields of both risky assets.

Summing up, the model can generate different premia for unlevered equity and housing mainly due to different elasticities for Tobin's q and for house prices. Additionally, the gap between the two risky assets can be enlarged by leverage. However, we could not achieve further improvements of the model fit, in particular for the volatility of the return on equity, by fine-tuning the parameters controlling the elasticities of Tobin's q and of house prices. Increasing the elasticity κ of Tobin's q has the desired effect and helps to generate more volatile return rates on equity. However, it also implies a too large premium on equity compared to housing and, counterfactual to the data, reduces the volatility of business investments. Decreasing the elasticity of house prices ϕ also impairs the model's fit. At odds to the data, the return on housing and its volatility decrease while the Sharpe ratio of housing further increases.[16]

The main shortcoming of the model's asset price statistics, independent of the calibration, remains the fact that the Sharpe ratios of the two risky assets turn out far too similar.[17] In order to identify the reasons for this failure, we can return to equation (4). Note that by definition all bonds, $x \in \{\text{gb}, \text{cb}, \text{hb}\}$, also satisfy

$$\mathbb{E}_t\left[M_{t+1}R_{x,t+1}^{(T_x)}\right] = 1.$$

Moreover, the condition also holds for the levered return on equity

$$\mathbb{E}_t\left[M_{t+1}R_{E,t+1}^{lev}\right] = \frac{1}{1-m_{cb}}\mathbb{E}_t\left[M_{t+1}R_{E,t+1}\right] - \frac{m_{cb}}{1-m_{cb}}\mathbb{E}_t\left[M_{t+1}R_{cb,t+1}^{(T_{cb})}\right] = \frac{1-m_{cb}}{1-m_{cb}} = 1.$$

Hence, proceeding in an analogous way as before, the Sharpe ratio of housing and of

[16] See Appendix B.

[17] Different from the non-disaster models in section 3, we did not optimize the model's fit by matching moments with regard to the 'free' parameters. First, even with more efficient methods such as Polynomial Chaos Expansions proposed by Fehrle et al. (2019) such parameter inference would still be too time consuming. Second, we argue that it is the model's structure which is too simple to disentangle the Sharpe ratios of equity and housing.

Table 11: Risk premia components

	$\frac{2\sqrt{\mathrm{Var}[M_{t+1}]}}{(\mathbb{E}[M_{t+1}])^4}$	SR_E	SR_E^{lev}	SR_H	$r_{M,\mathrm{EP}}$	$r_{M,\mathrm{EP}^{lev}}$	$r_{M,\mathrm{HP}}$	$r_{\mathrm{EP},\mathrm{HP}}$	$r_{\mathrm{EP}^{lev},\mathrm{HP}}$
				Data					
USA	–	–	0.36	1.01	–	–	–	–	0.19
				Model					
G	0.64	0.88	0.90	0.91	−0.70	−0.76	−0.66	0.99	0.98
G*	0.96	0.62	0.66	0.61	−0.62	−0.66	−0.61	0.99	0.98

Notes: SR_E : annualized Sharpe ratio of equity, SR_H : annualized Sharpe ratio of housing, $r_{X,Y}$: correlation between variables X and Y where M_{t+1} : stochastic discount factor, $EP_{t+1} := R_{E,t+1} - R_{f,t}$: ex-post equity premium, $HP_{t+1} := R_{H,t+1} - R_{f,t}$: ex-post housing premium.

leveraged equity can again be decomposed as

$$\begin{aligned}
\mathrm{SR}_E^{lev} &:= \frac{\mathbb{E}\left[R_{E,t+1}^{lev} - R_{gb,t+1}^{(T_{gb})}\right]}{\sqrt{\mathrm{Var}\left[R_{E,t+1}^{lev} - R_{gb,t+1}^{(T_{gb})}\right]}} = -\frac{\sqrt{\mathrm{Var}\left[M_{t+1}\right]}}{\mathbb{E}\left[M_{t+1}\right]} \mathrm{Corr}\left[M_{t+1}, R_{E,t+1}^{lev} - R_{gb,t+1}^{(T_{gb})}\right], \\
\mathrm{SR}_H &:= \frac{\mathbb{E}\left[R_{H,t+1} - R_{gb,t+1}^{(T_{gb})}\right]}{\sqrt{\mathrm{Var}\left[R_{H,t+1} - R_{gb,t+1}^{(T_{gb})}\right]}} = -\frac{\sqrt{\mathrm{Var}\left[M_{t+1}\right]}}{\mathbb{E}\left[M_{t+1}\right]} \mathrm{Corr}\left[M_{t+1}, R_{H,t+1} - R_{gb,t+1}^{(T_{gb})}\right].
\end{aligned} \tag{8}$$

We summarize the decomposition of the Sharpe ratios in Table 11. Note however, that we compute moments from the simulation of the model's equilibrium outcomes so that the decompositions (8) only hold for samples that are consistent with the agent's expectations in the model solution, i.e. for samples which include disasters (G*). For non-disaster samples (G), it can be interpreted at best as a rough approximation which neglects the effects from the occurrence of disasters.

Nonetheless, the major deficit of the model is obvious. The return rates and also the premia between the two risky assets are again almost perfectly correlated so that their correlations with the stochastic discount factor are practically identical. The model's relatively simple structure implies that the effects of shocks on risky return rates are aligned and may only differ in size. This fact also becomes clearly evident from Figure 1 and the above interpretation of the impulse response functions. Hence, by (8) the Sharpe ratios of the two assets must be the same. Compared to the models without disaster risk in Table 6, the introduction of disasters risk raises the standard deviation of the stochastic discount factor by a factor of 4. The model can therefore explain substantially larger Sharpe ratios and matches the value of housing from the data. Yet, it now fails to simultaneously generate the lower Sharpe ratio of equity and, in consequence, produces far to involatile return rates on equity. Finally, the perfect correlation between the risky assets still implies that the mean *and* the standard deviation of returns on total risk are the weighted averages of the two risky assets. In order to explain the different Sharpe ratios of the two assets and in order to prevent the counterfactual characteristics of the total portfolio, it would be necessary to introduce effects into the model which help to dissolve the perfect correlation between the risky return rates.

Any mechanism that increases the volatility of the retrun on equity and decreases, in absolute terms, the correlation of the return on equity with the stochastic discount factor would improve the models fit. Assuming additionally that cooperate bonds could default in normal times meets these requirements.[18] The additional source of uncertainty increases the volatility of the return on equity while the assumption of independence decrease in absolute terms the correlation between the stochastic discount factor and the return on equity.

Other mechanisms concerning housing specific characteristics could improve the model's fit in general. E.g. due to the poor divisibility of housing, there may be credit constrained households which can only invest in equity. For them it would be impossible to smooth the consumption bundle by adjusting consumption and residential investment and subsequently the equity premia would increase. Albeit, housing investment participation distributes far broader and is less concentrated towards the top quantiles than the participation at the stock market as Kuhn et al. (Forthcomming) show. This indicates that the effect is minimal at best.

Among others, Mian et al. (2013) find a large effect of housing wealth on consumption. Modeling such a channel would increase in absolute terms the correlation between house prices and the stochastic discount factor and thus between the return on housing and the stochastic discount factor, which would separate the Sharpe ratios. Theoretical foundations for a large causal effect are given e.g. by Berger et al. (2017) and Guerrieri and Lorenzoni (2017). Gertler and Gilchrist (2018) summarize this channel in a review as follows: Mortgages are the household's most common structure of debt. Hence, declining house prices increase the households leverage ratio and the resulting tightened budget constraint forces the household to reduce his consumption spending.

Last, the fact that the risk of housing wealth is more idiosyncratic increases the volatility of the return on housing on an individual level. This is neither observable in a representative agent framework nor in the aggregated data of JKKST and thus explains potentially the difference in the Sharpe ratios. The PSID-based data from Flavin and Yamashita (2002) imply a Sharpe ratio of equity of 0.35, similar to the aggregated one, but the Sharpe ratio of housing is reduced by half to 0.47. Even if this explains a large part, a differential of one third remains.

5 Conclusion

In the present paper, we study the effects of housing on asset pricing statistics, especially on risk premia, in production economies. The stylized facts for asset prices which we consider are: i) a risk-free rate in the range of 1-2.2 percent together with a low volatility, ii) return rates on equity moderately larger than returns on housing, iii) premia on risky returns over 3 percent, iii) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and iv) a Sharpe ratio of housing

[18] Gourio (2012) argues e.g. the financial crises 2008 was not a great diasaster and US-treasury bonds and bills did not default. Nevertheless, a lot of cooporate bonds defaulted.

significantly larger than the Sharpe ratio of equity and similar to the Sharpe ratio of total risk. Since we study production economies, we also check the model's compatibility to the following well-established stylized facts of housing and business cycles: i) volatility of residential investments exceeds the volatility of business investments, ii) house prices are at least twice as volatile as GDP and are positively correlated with GDP, and iv) residential investments co-moves with house prices, GDP, and business investments.

We first introduce housing into non-disaster economies with habits and capital adjustment costs a la Jermann (1998). Housing provides the household with an insurance against fluctuations in the composite good and the model's ability to generate sizeable risk premia vanishes in consequence. However, the household's desire to smooth his consumption of the composite good induces demand effects which coincide with business cycle characteristics. Limitation of the household's option to smooth consumption of the composite good helps to generate modest risk premia but also eliminates the demand effects and hence reduces the model's fit to business cycle statistics. Moreover, the risk-free rate is far too volatile in the model and the model fails to explain the empirically observed differences between the returns on equity and on housing.

Second, we extend a standard RBC model with disaster risk similar to Gourio (2012) and Fernández-Villaverde and Levintal (2018) by housing. We find that the model can reproduce return rates on leveraged equity, on housing and on government bonds which are all close to the data. Moreover, the model can also match the volatility of government bonds and housing returns but equity returns are too involatile compared to the data. Different premia and different volatilities for equity and housing in the model are the result of i) different adjustment costs for productive capital and for residential structures which result in different elasticities of stock prices and of ii) leverage on equity. However, despite different premia and volatilities between the two risky assets, the model does not allow to disentangle the Sharpe ratios. Our calibration allows close replication of the empirical Sharpe ratios of housing and of total risk while the Sharpe ratio of equity exceeds its empirical counterpart substantially. Finally, regardless of its rather simple structure, the model is also able to generate relative volatilities of business investments, residential investments and house prices which all fit the data. Moreover, the model also reproduces the empirically observed correlation between GDP and residential investments and between GDP and house prices.

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APPENDIX

Total risk premium in disaster and non-disaster risk production economies

A Stylized facts and data Resources

A.1 Stylized facts

In Table 12 we present return rates for all countries from the JKKST database. We observe the following stylized facts. First, risk premia in all countries are sizeable with equity premia between 1.17 percent in Portugal and 12.91 percent in Finland, and housing premia between 3.47 and 8.39 percent in Germany and Norway, respectively. Second, in all countries except for Italy and Portugal the return on housing is lower than the return on equity. Third, in all countries listed the volatility of returns and premia on equity exceeds the volatility of returns and premia on housing and the volatility of the risk-free rate is the smallest. Fourth, the Sharp ratio of housing is larger than of equity in all listed countries. Last, there is no systematic nexus between the return on housing and on equity.

Table 13 displays the business cycle statistics for the same countries. Note that for several continental European countries the standard deviation of residential investment exceeds the standard deviation of business investment only slightly or is even smaller. More precisely, this is the case for France, Finland, Germany, Italy, the Netherlands, Norway, Portugal, and Spain. House prices are pro-cyclical with GDP and more volatile than GDP in all countries except for Germany. Business investment and residential investment are positively correlated in all countries but Sweden and Australia, and house prices and residential investment are positively correlated throughout.

A.2 Sources

The data pertaining the rates of return, mortgage etc. are from JKKST. The source of the data pertaining the business cycle statistics is listed below:

- GDP, residential investment, non-residential investment: OECD Economic Outlook Nov 2018; Denmark: Statistics Denmark.
- House prices: OECD Real house price indices, s.a. 16.05.2019 devided by the OECD Economic Outlook Nov 2018 CPI Deflator.
- Population: FRA, USA: OECD Total population PERSA: Persons, seasonally adjusted; UK: Office for National Statistics UK, resident population: mid-year estimates (Qtly data interpolated (by the Office); Otherwise: Yearly, Worldbank, midyear (interpolated (by own calculation)).
- Home ownership rates: Japan (2007): http://www.stat.go.jp/english/index.html; USA (2007), Australia (2003): https://www.oecd.org/eco/growth/evolution

  %20of%20homeownership%20rates.pdf p.212; Otherwise (2010): EUROSTAT "Eurostat - Data Explorer - Distribution of population by tenure status, type of household and income group"

Table 12: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP
AUS	6.84	7.01	7.22	2.00	4.84	5.02	5.22
BEL	10.29	8.02	8.65	2.49	7.80	5.53	6.16
DNK	13.35	6.6	8.69	0.72	12.63	5.88	7.96
FIN	14.17	8.84	11.67	1.25	12.91	7.59	10.41
FRA	9.61	5.78	6.61	2.24	7.37	3.54	4.37
GER	9.04	4.75	5.75	1.28	7.76	3.47	4.47
ITA	4.76	5.62	5.89	1.55	3.21	4.07	4.34
JPA	5.86	5.54	6.19	0.98	4.88	4.56	5.21
NLD	8.93	7.83	8.12	1.44	7.49	6.38	6.68
NOR	13.03	9.82	10.66	1.43	11.60	8.39	9.23
PRT	2.92	6.65	6.62	1.75	1.17	4.89	4.87
ESP	7.10	4.43	4.89	0.69	6.40	3.74	4.20
SWE	12.29	8.82	10.56	0.91	11.38	7.92	9.65
CH	7.23	6.01	6.98	−0.17	7.40	6.19	7.16
UK	8.00	7.00	7.47	1.56	6.44	5.44	5.91
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$
AUS	21.53	5.71	6.02	3.32	21.32	6.22	6.27
BEL	22.99	6.04	6.37	2.84	23.08	6.61	7.02
DNK	23.84	7.72	8.91	1.56	24.33	7.24	8.93
FIN	37.72	9.13	21.58	4.52	36.90	9.64	21.39
FRA	24.11	5.52	6.95	2.55	23.98	6.18	7.39
GER	22.82	3.12	5.09	1.73	23.12	4.30	5.99
ITA	27.98	10.77	10.07	3.18	27.57	11.51	10.74
JPA	20.15	6.53	8.10	2.53	19.94	6.47	8.03
NLD	22.06	9.14	9.40	2.91	22.13	9.68	9.91
NOR	29.60	8.51	9.02	2.39	29.66	9.64	10.12
PRT	26.86	7.26	7.64	2.57	26.81	7.39	7.64
ESP	27.13	8.36	8.62	4.43	25.93	8.19	8.10
SWE	26.03	7.35	11.67	2.06	25.75	7.22	11.42
CH	21.61	4.59	8.01	2.29	21.41	4.94	7.97
UK	8.00	7	7.47	1.56	6.44	5.44	5.91
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00
	SR_E	SR_H	SR_T	$r_{\text{EP,HP}}$	$r_{\text{EP,TP}}$	$r_{\text{HP,TP}}$	
				Data			
AUS	0.23	0.81	0.83	−0.09	0.58	0.72	
BEL	0.34	0.84	0.88	−0.05	0.43	0.87	
DNK	0.52	0.81	0.89	0.37	0.81	0.83	
FIN	0.35	0.79	0.49	0.25	0.83	0.5	
FRA	0.31	0.57	0.59	0.06	0.63	0.76	
GER	0.34	0.81	0.75	0.21	0.79	0.74	
ITA	0.12	0.35	0.4	−0.24	−0.01	0.97	
JPA	0.24	0.7	0.65	0.16	0.79	0.72	
NLD	0.34	0.66	0.67	0.02	0.52	0.84	
NOR	0.39	0.87	0.91	−0.01	0.46	0.86	
PRT	0.04	0.66	0.64	0.07	0.49	0.89	
ESP	0.25	0.46	0.52	0.09	0.4	0.94	
SWE	0.44	1.1	0.85	0.01	0.8	0.5	
CH	0.35	1.25	0.9	−0.19	0.88	0.18	
UK	0.27	0.61	0.69	−0.13	0.72	0.54	
USA	0.36	1.01	0.75	0.19	0.9	0.56	

Notes: Mean percentage returns of equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity (EP), housing (HP), and the total risk premium (TP) and the corresponding standard deviations $\sigma(X)$. SR_E : Sharpe ratio of equity, SR_H : Sharpe ratio of housing, SR_T : Sharpe ratio of total risk $r_{X,Y}$: correlation between variables X and Y where $EP_{t+1} \coloneqq R_{E,t+1} - R_{f,t}$: ex-post equity premium, $HP_{t+1} \coloneqq R_{H,t+1} - R_{f,t}$: ex-post housing premium $TP_{t+1} \coloneqq R_{T,t+1} - R_{f,t}$: ex-post total risk premium. Periods: Australia 1970-2015, Belgium 1976-2015, Denmark 1995-2015, Finland 1970-2015, France 1980-2015, Germany 1991-2015, Italy 1970-2015, Japan 1963-2015, the Netherlands 1970-2015, Norway 1978-2015, Portugal 1988-2015. Spain 1971-2015, Sweden 1970-2015, Switzerland 1970-2015, United Kingdom 1969-2015, USA 1970-2015, data from JKKST, own calculations.

Table 13: Empirical business cycle statistics

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r^{P_h}_{RESI}$	r^{BUSI}_{RESI}	r^{GDP}_{RESI}	$r^{P_h}_{GDP}$
AUS	1.24	3.77	6.65	3.56	0.55	−0.12	0.56	0.35
BEL	1.02	4.21	7.19	3.54	0.62	0.22	0.47	0.31
DNK	1.42	3.52	5.95	3.96	0.60	0.28	0.66	0.75
FIN	2.21	2.87	3.15	3.00	0.73	0.40	0.67	0.66
FRA	0.95	2.75	3.17	3.19	0.65	0.64	0.81	0.48
GER	1.47	2.54	2.20	0.82	0.06	0.49	0.57	−0.16
ITA	1.44	2.66	1.67	3.73	0.25	0.37	0.46	0.15
JPA	1.59	2.41	3.84	2.70	0.31	0.27	0.45	0.55
NLD	1.37	5.86	5.57	4.08	0.40	0.31	0.48	0.35
NOR	1.46	4.68	4.51	3.72	0.60	0.28	0.31	0.56
PRT	1.58	3.37	2.58	1.87	0.40	0.56	0.64	0.50
ESP	1.33	3.48	3.42	4.17	0.43	0.69	0.77	0.61
SWE	1.51	4.46	5.20	2.95	0.42	−0.22	0.04	0.57
CHE	1.64	–	–	2.69	–	–	–	0.61
UK	1.58	2.68	5.56	4.85	0.51	0.16	0.69	0.71
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64

Notes: Business cycle statistics are from quarterly logged per capita hp-filtered (1600) data. σ_x is the standard deviation of x, r^x_y the correlation between x and y. RESI=residential investment, BUSI=non-residential investment, P_h house prices. Periods: Australia 1970-2015, Belgium 1976-2015, Denmark 1995-2015, Finland 1970-2015, France 1980-2015, Germany 1991-2015, Italy 1970-2015, Japan 1963-2015, the Netherlands 1970-2015, Norway 1978-2015, Portugal 1988-2015. Spain 1971-2015, Sweden 1970-2015, Switzerland 1970-2015, United Kingdom 1969-2015, USA 1970-2015, Data: See Appendix A, own calculations.

B Further Results

Economies with Non-Disaster risk

Higher share of land's value in new houses: We show the results from increasing the share of land in the Jermann (1998) model with housing to $\phi = 0.3$ in row K of tables 15 and 16. First, the return rates on both risky assets increase moderately while the risk-free rate decreases slightly. Nevertheless, the model can still not produce sizable risk premia. The volatility of residential investment decreases only moderately and remains far too large. On the other hand, house prices in the model become noticeably more volatile than empirically observed.

We also present the results for the two-sector model à la Boldrin et al. (2001) with $\phi = 0.3$ in rows L, M, and N of tables 15 and 16. We fine-tune the free parameters by matching moments in the same way as we did for $\phi = 0.106$ and summarize the parameters' values in Table 14. Risk premia decrease moderately in model L compared to model C and again vanish if labor supply is endogenously determined in model M. In model N with limited sectoral labor mobility changes are only negligible compared to model E. The volatility of house prices increases in all three models and, as Fehrle (2019) shows, the correlations also increase. Nevertheless, with labor market frictions house prices do not co-move with residential investment.

To sum up, an increase in the share of land's value in new houses does not affect the conclusion that the present framework can not simultaneously reproduce asset pricing statistics and business cycle statistics as observed in the data.

Table 14: Calibration

	L	M / N
$\beta^{\star}$	0.999	0.999
η	5	5
$\mu_c^{*)}$	0.72	0.49
$\mu_h^{*)}$	0.18	0.19
χ_c	0.825	0.825
χ_h	0.70	0.70
χ_n	–	0.95
a_y	1.005	1.005
a_d	1.002	1.002
ϕ	0.30	0.30
α_y	0.25	0.25
α_d	0.20	0.20
κ_y	6.25	6.25
κ_d	1.25	1.25
δ_k	0.022	0.022
δ_h	0.009	0.009
σ_y	0.0094	0.0094
ρ_y	0.966	0.966
σ_d	0.0172	0.0172
ρ_d	0.923	0.923

Notes: $^{*)}$ Endogenous by the model. **L**: C but $\phi = 0.3$. **M/N**: D/E but $\phi = 0.3$.

Habitat without habits: The effects of habits in consumption, housing and leisure, respectively, can be discussed by setting the corresponding habit parameter to zero in model E. First, rows N of tables 15 and 16 show that habits in housing have negligible consequences for the presented business cycle characteristics and only small significance for risk premia. Without habits in housing, risk premia on both assets are reduced by approximately 0.8 percentage points. Rows O of the same tables show that without habits in leisure risk premia are halved. Moreover, the volatility of house prices reduces towards its empirical counterpart but business investment becomes even less volatile and the correlation between GDP and residential investment also moves farther away from the value in the data.

Habits in consumption, see row P of tables 15 and 16, have the largest effect on the results from model E. Without habits in consumption the household's marginal utility does not fluctuate enough between different realizations of the productivity shock so that risk premia reduce drastically. While business investment becomes less volatile, the standard deviation of residential investment increases. Moreover, the correlation between house prices and residential investment becomes positive and the correlation between business and residential investment also increases substantially.

Table 15: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27
				Model			
K	4.83	4.59	4.72	4.03	0.77	0.54	0.67
L	4.84	4.61	4.70	0.50	4.33	4.10	4.19
M	2.70	2.60	2.64	1.95	0.74	0.64	0.68
N	4.92	4.69	4.78	1.39	3.49	3.27	3.36
O	4.31	4.11	4.19	1.67	2.61	2.41	2.49
P	3.40	3.32	3.36	2.05	1.34	1.26	1.29
Q	2.66	2.50	2.57	2.02	0.63	0.47	0.54
R	5.16	3.87	4.59	1.36	3.76	2.49	3.20
R^{lev}	8.24	3.87	6.30	1.55	6.62	2.30	4.70
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$
USA	16.71	3.78	6.9	2.31	16.47	4.41	7.00
				Model			
K	8.25	6.01	7.29	1.45	8.09	5.80	7.11
L	22.43	21.47	21.86	8.49	20.62	19.57	19.99
M	8.82	7.70	8.14	2.13	8.54	7.37	7.84
N	22.77	21.81	22.19	12.84	18.62	17.42	17.90
O	19.93	19.00	19.39	11.59	16.04	14.87	15.36
P	14.75	14.17	14.41	8.65	11.92	11.17	11.49
Q	8.32	6.63	7.32	3.05	7.73	5.86	6.64
R	3.71	1.68	2.73	1.51	4.33	2.59	3.45
R^{lev}	6.83	1.68	4.45	1.32	7.41	2.44	5.07

Notes: Mean percentage returns of equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity (EP), housing (HP), and the total risk premium (TP) and the corresponding standard deviations $\sigma(X)$. We employ a second order perturbation and simulated time series with 100,000 periods. **K**: B but $\phi = 0.3$. **L**:**L**: C but $\phi = 0.3,, \chi_h = 0.7$. **M**: D but $\phi = 0.3\chi_h = 0.7$. **N**: E but $\phi = 0.3, \chi_h = 0.7$. **O**: E but $\chi_h = 0$. **P**: E but $\chi_n = 0$. **Q**: E but $\chi_c = 0$. **R**: G but $\phi = 0.106$. **R**^{lev}: R but with leverage and default, housing remain not leveraged.

Housing with disaster risk

Lower share of land's value in new houses: Tables 15 and 16 display in rows R and R^{lev} the effects from lowering the share of land in our disaster economies to $\phi = 0.106$ as in Davis and Heathcote (2005). We find that returns on equity do not change by much but housing premia decrease by 0.7 percentage-points and the standard deviation of returns on housing is reduced by 0.96. The Sharpe ratio of housing exceeds its empirical counterpart. Moreover, the volatility of house prices drops below the standard deviation of GDP. The lower share of land in new houses increases the volatility of residential investment which now becomes too large.

Table 16: Simulated business cycle statistics III

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
				Data				
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.63
				Model				
K	1.05	1.10	11.44	3.26	0.99	0.98	0.98	0.98
L	2.11	1.05	1.03	4.79	−0.03	0.00	0.06	0.96
M	1.42	0.77	4.39	3.04	0.80	0.83	0.84	0.94
N	1.9	0.96	2.12	4.23	0.04	0.14	0.39	0.91
O	1.95	0.84	1.83	3.57	−0.06	0.07	0.35	0.88
P	1.89	0.65	2.16	2.73	−0.1	0.01	0.28	0.89
Q	1.44	0.71	2.95	2.17	0.11	0.35	0.52	0.91
R	1.01	2.59	8.51	0.90	1.00	0.89	0.68	0.68

Notes: σ_x is the standard deviation of x, r_y^x the correlation between x and y. Business cycle statistics from HP-filtered (1600) times series. We employ a second order perturbation and report the average outcomes from repeated simulations with 180 periods. **K**: B but $\phi = 0.3$. **L**: C but $\phi = 0.3, , \chi_h = 0.7$. **M**: D but $\phi = 0.3\chi_h = 0.7$. **N**: E but $\phi = 0.3, \chi_h = 0.7$. **O**: E but $\chi_h = 0$. **P**: E but $\chi_n = 0$. **Q**: E but $\chi_c = 0$. **R**: G but $\phi = 0.106$.

C Economies with non-disaster risk

We summarize the details for the models discussed in section 3.

C.1 Housing with Jermann (1998)

The equilibrium conditions from the optimization problem (1) for the benchmark Jermann (1998) model which is extended by housing are determined by

$$\Lambda_t = \mu_c (C_t - C_{ht})^{\mu_c(1-\eta)-1} (H_t - H_{ht})^{\mu_h(1-\eta)}, \tag{9a}$$
$$Y_t = a^{(1-\alpha_y)t} Z_t K_t^{\alpha_y}, \tag{9b}$$
$$Y_t = C_t + I_t + D_t, \tag{9c}$$
$$r_t = \alpha_y \frac{Y_t}{K_t}, \tag{9d}$$
$$P_{h,t} = \frac{1}{1-\phi} D_t^{\phi}, \tag{9e}$$
$$q_t = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}, \tag{9f}$$
$$K_{t+1} = (1-\delta_k) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t, \tag{9g}$$
$$H_{t+1} = (1-\delta_h) H_t + D_t^{1-\phi}, \tag{9h}$$
$$C_{h,t+1} = \chi_c C_t, \tag{9i}$$
$$H_{h,t+1} = \chi_h H_t, \tag{9j}$$
$$q_t = \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\left(1 - \delta_k + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right], \tag{9k}$$
$$P_{h,t} = \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\mu_h}{\mu_c}\frac{C_{t+1} - C_{h,t+1}}{H_{t+1} - H_{h,t+1}} + P_{h,t+1}(1-\delta_h)\right)\right], \tag{9l}$$

given the state variables $K_t, H_t, C_{h,t}, H_{h,t}$ and Z_t. Additionally, the log of productivity follows the exogenous AR(1)-process

$$\ln Z_{t+1} = \rho_y \ln Z_t + \epsilon_{t+1},\ \epsilon_t \sim \text{iidN}(0, \sigma_y^2).$$

Finally, GDP is defined by

$$\text{GDP}_t = Y_t + MRS_t^{H,C} H_t, \ \text{ where } MRS_t^{H,C} = \frac{\mu_h}{\mu_c}\frac{C_t - C_{ht}}{H_t - H_{ht}}.$$

We re-scale the variables by $k_t := \frac{K_t}{a^t}$, $h_t := \frac{H_t}{a^{(1-\phi)t}}$, $c_{h,t} := \frac{C_{h,t}}{a^t}$, $h_{h,t} := \frac{H_{h,t}}{a^{(1-\phi)t}}$, $y_t := \frac{Y_t}{a^t}$, $c_t := \frac{C_t}{a^t}$, $i_t := \frac{I_t}{a^t}$, $d_t := \frac{D_t}{a^t}$, $p_{h,t} := \frac{P_{h,t}}{a^{\phi t}}$, and $\lambda_t := \frac{\Lambda_t}{a^{((\mu_c+(1-\phi)\mu_h)(1-\eta)-1)t}}$. Hence, system (9) in scaled variables reads

$$
\begin{aligned}
\lambda_t &= \mu_c(c_t - c_{ht})^{\mu_c(1-\eta)-1}(h_t - h_{ht})^{\mu_h(1-\eta)}, && \text{(10a)}\\
y_t &= Z_t k_t^{\alpha_y}, && \text{(10b)}\\
y_t &= c_t + i_t + d_t, && \text{(10c)}\\
r_t &= \alpha_y \frac{y_t}{k_t}, && \text{(10d)}\\
p_{h,t} &= \frac{1}{1-\phi} d_t^{\phi}, && \text{(10e)}\\
q_t &= \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}, && \text{(10f)}\\
a k_{t+1} &= (1-\delta_k)k_t + \Phi\left(\frac{i_t}{k_t}\right)k_t, && \text{(10g)}\\
a^{1-\phi}h_{t+1} &= (1-\delta_h)h_t + d_t^{1-\phi}, && \text{(10h)}\\
a c_{h,t+1} &= \chi_c c_t, && \text{(10i)}\\
a^{1-\phi}h_{h,t+1} &= \chi_h h_t, && \text{(10j)}\\
q_t &= \mathbb{E}_t\left[\beta a^{(\mu_c+(1-\phi)\mu_h)(1-\eta)-1}\frac{\lambda_{t+1}}{\lambda_t}\left(r_{t+1} - \frac{I_{i+1}}{k_{t+1}} + q_{t+1}\left(1-\delta_k+\Phi\left(\frac{i_{t+1}}{k_{t+1}}\right)\right)\right)\right], && \text{(10k)}\\
p_{h,t} &= \mathbb{E}_t\left[\beta a^{(\mu_c+(1-\phi)\mu_h)(1-\eta)-1+\phi}\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\mu_h}{\mu_c}\frac{c_{t+1}-c_{h,t+1}}{h_{t+1}-h_{h,t+1}} + p_{h,t+1}(1-\delta_h)\right)\right]. && \text{(10l)}
\end{aligned}
$$

C.2 Moving to Boldrin et al. (2001)

We continue to present the details for the two-sector models from section 3.

Sectoral frictions with exogenous labor supply:

$$
\begin{aligned}
\max_{C_t,D_t,I_{yt},I_{dt},K_{yt+1},K_{dt+1},H_{t+1}} \quad & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{((C_t-C_{ht})^{\mu_c}(H_t-H_{ht})^{\mu_h})^{1-\eta}-1}{1-\eta},\\
\text{s.t.} \quad & Y_t = a_y^{(1-\alpha_y)t}Z_{yt}K_{yt}^{\alpha_y},\\
& Y_t = C_t + I_t,\\
& D_t = a_d^{(1-\alpha_d)t}Z_{dt}K_{dt}^{\alpha_d},\\
& I_t = I_{dt} + I_{yt},\\
& K_{yt+1} = (1-\delta_k)K_{yt} + \Phi_y\left(\frac{I_{yt}}{K_{yt}}\right)K_{yt},\\
& K_{dt+1} = (1-\delta_k)K_{dt} + \Phi_d\left(\frac{I_{dt}}{K_{dt}}\right)K_{dt},\\
& H_{t+1} = (1-\delta_h)H_t + D_t^{1-\phi}.
\end{aligned}
\tag{11}
$$

where $\eta, \alpha_y, \alpha_d, a_y, a_d, \mu_c, \mu_h > 0, \mu_c+\mu_h = 1, \delta_k \in [0,1], \delta_h \in [0,1]$ and $\Phi_y(x) = \frac{\varphi_{y,1}}{1-\kappa_y} x^{1-\kappa_y} + \varphi_{y,2}$ and Φ_d analogously.

First, if labor supply is exogenous, the system of equations derived from the optimization problem (11) for an equilibrium in period t reads

$$\Lambda_t = \mu_c (C_t - C_{ht})^{\mu_c(1-\eta)-1} (H_t - H_{ht})^{\mu_h(1-\eta)}, \tag{12a}$$

$$Y_t = a_y^{(1-\alpha_y)t} Z_{y,t} K_{y,t}^{\alpha_y}, \tag{12b}$$

$$D_t = a_d^{(1-\alpha_d)t} Z_{d,t} K_{d,t}^{\alpha_d}, \tag{12c}$$

$$Y_t = C_t + I_{y,t} + I_{d,t}, \tag{12d}$$

$$r_{y,t} = \alpha_y \frac{Y_t}{K_{y,t}}, \tag{12e}$$

$$r_{d,t} = \alpha_d P_{d,t} \frac{D_t}{K_{d,t}}, \tag{12f}$$

$$P_{h,t} = \frac{P_{d,t}}{1-\phi} D_t^{\phi}, \tag{12g}$$

$$q_{y,t} = \frac{1}{\Phi_y'\left(\frac{I_{y,t}}{K_{y,t}}\right)}, \tag{12h}$$

$$q_{d,t} = \frac{1}{\Phi_d'\left(\frac{I_{d,t}}{K_{d,t}}\right)}, \tag{12i}$$

$$K_{y,t+1} = (1-\delta_k) K_{y,t} + \Phi_y\left(\frac{I_{y,t}}{K_{y,t}}\right) K_{y,t}, \tag{12j}$$

$$K_{d,t+1} = (1-\delta_k) K_{d,t} + \Phi_d\left(\frac{I_{d,t}}{K_{d,t}}\right) K_{d,t}, \tag{12k}$$

$$H_{t+1} = (1-\delta_h) H_t + D_t^{1-\phi}, \tag{12l}$$

$$C_{h,t+1} = \chi_c C_t, \tag{12m}$$

$$H_{h,t+1} = \chi_h H_t, \tag{12n}$$

$$q_{y,t} = \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(r_{y,t+1} - \frac{I_{y,t+1}}{K_{y,t+1}} + q_{y,t+1}\left(1-\delta_k + \Phi_y\left(\frac{I_{y,t+1}}{K_{y,t+1}}\right)\right)\right)\right], \tag{12o}$$

$$q_{d,t} = \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(r_{d,t+1} - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}\left(1-\delta_k + \Phi_d\left(\frac{I_{d,t+1}}{K_{d,t+1}}\right)\right)\right)\right], \tag{12p}$$

$$P_{h,t} = \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\mu_h}{\mu_c} \frac{C_{t+1} - C_{h,t+1}}{H_{t+1} - H_{h,t+1}} + P_{h,t+1}(1-\delta_h)\right)\right], \tag{12q}$$

given the state variables $K_{y,t}, K_{d,t}, H_t, C_{h,t}, H_{h,t}, Z_{y,t}$ and $Z_{d,t}$. Additionally, the log of productivity follows the exogenous AR(1)-process

$$\ln Z_{y,t+1} = \rho_y \ln Z_{y,t} + \epsilon_{y,t+1},$$

$$\ln Z_{d,t+1} = \rho_d \ln Z_{d,t} + \epsilon_{d,t+1},$$
$$\begin{pmatrix} \epsilon_{y,t+1} \\ \epsilon_{d,t+1} \end{pmatrix} \sim \text{iidN}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_d^2 \end{pmatrix} \right).$$

Finally, GDP is defined by

$$\text{GDP}_t = Y_t + P_{d,t} D_t + +MRS_t^{H,C} H_t, \;\; \text{where } MRS_t^{H,C} = \frac{\mu_h}{\mu_c} \frac{C_t - C_{ht}}{H_t - H_{ht}}.$$

We re-scale the variables by $k_{x,t} := \frac{K_{x,t}}{a_y^t}$, $h_t := \frac{H_t}{a_y^{(1-\phi)\alpha_d t} a_d^{(1-\phi)(1-\alpha_d)t}}$, $c_{h,t} := \frac{C_{h,t}}{a_y^t}$, $h_{h,t} := \frac{H_{h,t}}{a_y^{(1-\phi)\alpha_d t} a_d^{(1-\phi)(1-\alpha_d)t}}$, $y_t := \frac{Y_t}{a_y^t}$, $c_t := \frac{C_t}{a_y^t}$, $i_{x,t} := \frac{I_{x,t}}{a_y^t}$, $d_t := \frac{D_t}{a_y^{\alpha_d t} a_d^{(1-\alpha_d)t}}$, $p_{h,t} := \frac{a_d^{(1-\phi)(1-\alpha_d)t}}{a_y^{(1-(1-\phi)\alpha_d)t}} P_{h,t}$, $p_{d,t} := \frac{a_d^{(1-\alpha_d)t}}{a_y^{(1-\alpha_d)t}} P_{d,t}$, and $\lambda_t := \frac{\Lambda_t}{a_y^{((\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-1)t} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)t}}$. Hence, system (12) in scaled variables reads

$$\lambda_t = \mu_c (c_t - c_{ht})^{\mu_c(1-\eta)-1} (h_t - h_{ht})^{\mu_h(1-\eta)}, \tag{13a}$$
$$y_t = Z_{y,t} k_{y,t}^{\alpha_y}, \tag{13b}$$
$$d_t = Z_{d,t} k_{d,t}^{\alpha_d}, \tag{13c}$$
$$y_t = c_t + i_{y,t} + i_{d,t}, \tag{13d}$$
$$r_{y,t} = \alpha_y \frac{y_t}{k_{y,t}}, \tag{13e}$$
$$r_{d,t} = \alpha_d p_{d,t} \frac{d_t}{k_{d,t}}, \tag{13f}$$
$$p_{h,t} = \frac{p_{d,t}}{1-\phi} d_t^{\phi}, \tag{13g}$$
$$q_{y,t} = \frac{1}{\Phi_y'\left(\frac{i_{y,t}}{k_{y,t}}\right)}, \tag{13h}$$
$$q_{d,t} = \frac{1}{\Phi_d'\left(\frac{i_{d,t}}{k_{d,t}}\right)}, \tag{13i}$$
$$a_y k_{y,t+1} = (1-\delta_k) k_{y,t} + \Phi_y\left(\frac{i_{y,t}}{k_{y,t}}\right) k_{y,t}, \tag{13j}$$
$$a_y k_{d,t+1} = (1-\delta_k) k_{d,t} + \Phi_d\left(\frac{i_{d,t}}{k_{d,t}}\right) k_{d,t}, \tag{13k}$$
$$\left(a_y^{\alpha_d} a_d^{(1-\alpha_d)}\right)^{1-\phi} h_{t+1} = (1-\delta_h) h_t + d_t^{1-\phi}, \tag{13l}$$
$$a_y c_{h,t+1} = \chi_c c_t, \tag{13m}$$
$$\left(a_y^{\alpha_d} a_d^{(1-\alpha_d)}\right)^{1-\phi} h_{h,t+1} = \chi_h h_t, \tag{13n}$$

$$q_{y,t} = \mathbb{E}_t\left[\beta a_y^{(\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-1} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)} \frac{\lambda_{t+1}}{\lambda_t}\right. \tag{13o}$$

$$\left.\left(r_{y,t+1} - \frac{i_{y,t+1}}{k_{y,t+1}} + q_{y,t+1}\left(1-\delta_k+\Phi_y\left(\frac{i_{y,t+1}}{k_{y,t+1}}\right)\right)\right)\right], \tag{13p}$$

$$q_{d,t} = \mathbb{E}_t\left[\beta a_y^{(\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-1} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)} \frac{\lambda_{t+1}}{\lambda_t}\right. \tag{13q}$$

$$\left.\left(r_{d,t+1} - \frac{i_{d,t+1}}{k_{d,t+1}} + q_{d,t+1}\left(1-\delta_k+\Phi_d\left(\frac{i_{d,t+1}}{k_{d,t+1}}\right)\right)\right)\right], \tag{13r}$$

$$p_{h,t} = \mathbb{E}_t\left[\beta a_y^{(\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-(1-\phi)\alpha_d} a_d^{(1-\phi)(1-\alpha_d)(\mu_h(1-\eta)-1)} \frac{\lambda_{t+1}}{\lambda_t}\right. \tag{13s}$$

$$\left.\left(\frac{\mu_h}{\mu_c}\frac{c_{t+1}-c_{h,t+1}}{h_{t+1}-h_{h,t+1}} + p_{h,t+1}(1-\delta_h)\right)\right], \tag{13t}$$

Endogenous labor supply:

$$\begin{aligned}
\max_{\dots,N_{yt},N_{dt}} \; & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{((C_t-C_{ht})^{\mu_c}(H_t-H_{ht})^{\mu_h}((1-N_t)-(1-N_{ht}))^{\mu_n})^{1-\eta}-1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = Z_{yt}K_{yt}^{\alpha_y}(a_y^t N_{yt})^{1-\alpha_y}, \\
& Y_t = C_t + I_t, \\
& D_t = Z_{dt}K_{dt}^{\alpha_d}(a_d^t N_{dt})^{1-\alpha_d}, \\
& N_t = N_{dt} + N_{yt}, \\
& N_t \le 1 \\
& \dots
\end{aligned} \tag{14}$$

where now $\mu_c, \mu_h, \mu_n > 0, \mu_c + \mu_h + \mu_n = 1$.

If labor supply is endogenous in the model as in (14), the system of equations defining an equilibrium in the scaled variables remains as in (13) with the following adjustments to the production functions and to the marginal utility of consumption and with the additional equations pinning down labor supply

$$y_t = Z_{y,t}k_{y,t}^{\alpha_y}N_{y,t}^{1-\alpha_y}, \tag{15a}$$

$$d_t = Z_{d,t}k_{d,t}^{\alpha_d}N_{d,t}^{1-\alpha_d}, \tag{15b}$$

$$\lambda_t = \mu_c(c_t-c_{ht})^{\mu_c(1-\eta)-1}(h_t-h_{ht})^{\mu_h(1-\eta)}((1-N_t)-(1-N_{h,t}))^{\mu_n(1-\eta)}, \tag{15c}$$

$$(1-\alpha_y)\frac{y_t}{N_{y,t}} = \frac{\mu_n}{\mu_c}\frac{c_t-c_{h,t}}{(1-N_t)-(1-N_{h,t})}, \tag{15d}$$

$$(1-\alpha_d)p_{d,t}\frac{d_t}{N_{d,t}} = \frac{\mu_n}{\mu_c}\frac{c_t-c_{h,t}}{(1-N_t)-(1-N_{h,t})}, \tag{15e}$$

$$N_t = N_{y,t} - N_{d,t}, \tag{15f}$$

$$N_{h,t+1} = 1-\chi_n(1-N_t). \tag{15g}$$

Limited labor mobility Finally, if the household is unable to adapt his labor supply in response to technology shocks but is committed to working hours that are contracted sector-specifically one period ahead, the conditions in (15d) and (15e) must be adjusted to

$$\mathbb{E}_t\left[\lambda_{t+1}\left((1-\alpha_y)\frac{y_{t+1}}{N_{y,t+1}}-\frac{\mu_n}{\mu_c}\frac{c_{t+1}-c_{h,t+1}}{(1-N_{t+1})-(1-N_{h,t+1})}\right)\right]=0, \tag{16a}$$

$$\mathbb{E}_t\left[\lambda_{t+1}\left((1-\alpha_d)p_{d,t+1}\frac{d_{t+1}}{N_{d,t+1}}-\frac{\mu_n}{\mu_c}\frac{c_{t+1}-c_{h,t+1}}{(1-N_{t+1})-(1-N_{h,t+1})}\right)\right]=0. \tag{16b}$$

D Housing with disaster risk

We present the details for the model with disaster risk from section 4.

Disaster Risk The economy faces a risk for great disasters which are introduced through an exogenous shock in form of a binary variable b_t which indicates disasters in case of $b_t = 1$ while $b_t = 0$ in normal times. Disasters reduce productivity but also partly destroy the stock of productive capital and of residential structures (see below). Following Gourio (2012) disasters appear with time-varying probability and size. More specifically, we assume that

$$P(b_{t+1}=1|b_t=0)=\min\{p_t,1\},\ P(b_{t+1}=0|b_t=0)=1-\min\{p_t,1\}$$

where the log of p_t follows an AR(1)-process

$$\ln p_{t+1}=(1-\rho_p)\ln\bar{p}+\rho_p\ln p_t+\epsilon_{p,t+1},\ \epsilon_{p,t}\sim \text{iidN}(0,\sigma_p^2).$$

Additionally, disasters remain persistent with probability no less than $q\in(0,1)$ so that

$$P(b_{t+1}=1|b_t=1)=\max\{q,\min\{p_t,1\}\},\ P(b_{t+1}=0|b_t=1)=1-\max\{q,\min\{p_t,1\}\}.$$

Finally, the disaster size $1-e^{\omega_{t+1}}$ at which productivity, productive capital and residential structures are destroyed by a disaster also evolves stochastically according to

$$\omega_t := \bar{\omega}e^{\hat{\omega}_t},$$
$$\hat{\omega}_{t+1}=\rho_\omega\hat{\omega}_t+\epsilon_{\omega,t+1},\ \epsilon_{\omega,t}\sim \text{iidN}\left(0,\sigma_\omega^2\right),$$

where $\bar{\omega}<0$. We slightly deviate from the treatment in Gourio (2012) in the specification of the process governing the disaster size and allow autocorrelation but restrict outcomes to $\omega_t < 0$ so that disasters always have negative effects. The specification is similar to Fernández-Villaverde and Levintal (2018).

Representative Household The household derives utility from a composite good $\tilde{C}_t$ that is represented by a Cobb-Douglas aggregate consisting of consumption C_t, housing H_t and leisure $1-N_t$, i.e.

$$\tilde{C}_t := C_t^{\mu_c} H_t^{\mu_h} (1-N_t)^{1-\mu_c-\mu_h}.$$

We assume that the household's preferences over streams of the composite good are described by a recursive utility function, as introduced by Epstein and Zin (1989), of the form

$$\tilde{V}_t = \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t \tilde{V}_{t+1}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ is the household's EIS and γ the coefficient of RRA. Note however that γ and ψ describe the household's RRA and EIS with respect to the composite good $\tilde{C}$. Since the composite good aggregator is of the Cobb-Douglas type, the consumption-based RRA is given by $\mu_c\gamma$ and the consumption-based EIS reads $\frac{1}{1-\mu_c(1-1/\psi)}$.[19] For easier notation we define $V_t := \tilde{V}_t^{1-1/\psi}$ which satisfies the recursion

$$V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}},$$

where we use, similar to Caldara et al. (2012), the notation

$$\theta := 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

In the case where $\theta = 0$, the RRA equals the reciprocal of the EIS and the household's utility reduces to the 'classical' expected discounted sum of within period CRRA utilities. Hence, θ can also be interpreted as the deviation from this 'classic' case. The representative household supplies labor services N_t and capital services K_t and receives wages W_t and capital rents r_t. He buys consumption goods C_t and invests in productive capital I_t and new houses $H_{\text{new},t}$ with relative price $P_{h,t}$. Hence, his budget constraint reads

$$W_t N_t + r_t K_t = C_t + I_t + P_{h,t} H_{\text{new},t}.$$

We assume capital adjustment costs as in Jermann (1998). Moreover, disasters result in the destruction of a fraction $1-e^{\omega_{t+1}}$ of the stocks of capital and residential structures so that the stocks accumulate according to

$$K_{t+1} = e^{\omega_{t+1} b_{t+1}} \underbrace{\left((1-\delta_k)K_t + \Phi\Big(\frac{I_t}{K_t}\Big)K_t \right)}_{=:K^\star_{t+1}},$$

$$H_{t+1} = e^{\omega_{t+1} b_{t+1}(1-\phi)} \underbrace{\left((1-\delta_h)H_t + D_t^{1-\phi} \right)}_{=:H^\star_{t+1}},$$

[19] See Swanson (2012) and Heiberger and Ruf (2019).

where $\delta_k, \delta_h \in [0,1]$ and $\Phi(x) := \frac{\varphi_1}{1-\kappa} x^{1-\kappa} + \varphi_2$. The household maximizes life-time utility V_t subject to his budget constraint and subject to the laws of accumulation for capital and housing. Hence, the first order conditions for the household are given by

$$
\begin{aligned}
W_t &= \frac{\mu_n}{\mu_c} \frac{C_t}{1-N_t}, \\
q_t &= \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}, \\
q_t &= \mathbb{E}_t \left[M_{t+1} e^{\omega_{t+1} b_{t+1}} \left(r_{t+1} + q_{t+1} \left(1 - \delta_k + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} \right)\right)\right], \\
P_{h,t} &= \mathbb{E}_t \left[M_{t+1} e^{(1-\phi)\omega_{t+1} b_{t+1}} \left(\frac{\mu_h}{\mu_c} \frac{C_{t+1}}{H_{t+1}} + P_{h,t+1}(1-\delta_h) \right)\right],
\end{aligned}
$$

where M_{t+1} denotes the model's stochastic discount factor

$$
M_{t+1} := \beta \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) \left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}} \right)^{-\theta} \quad \text{with} \quad \Lambda_t := \mu_c \frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{C_t}.
$$

Representative Firm The firm produces output Y_t from labor N_t and capital services K_t according to the production function

$$
Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha}.
$$

Labor augmenting technical progress A_t grows stochastically and is damaged during disasters such way that

$$
\begin{aligned}
A_{t+1} &= A_t a e^{z_{t+1} + \omega_{t+1} b_{t+1}}, \\
z_{t+1} &= \rho_z z_t + \epsilon_{z,t+1}, \ \epsilon_{z,t} \sim \text{iidN}(0, \sigma_z^2).
\end{aligned}
$$

The firm's first order conditions from maximization of profits $Y_t - W_t N_t - r_t K_t$ subject to the production function read

$$
\begin{aligned}
W_t &= (1-\alpha) \frac{Y_t}{N_t}, \\
r_t &= \alpha \frac{Y_t}{K_t}.
\end{aligned}
$$

Construction Sector Finally, residential investments D_t are combined with a fixed factor land $L_t \equiv 1$ in order to form new houses according to

$$
H_{\text{new},t} = D_t^{1-\phi} L_t^{\phi}.
$$

Maximization of profits $P_{h,t}H_{\text{new},t} - D_t - P_{l,t}L_t$ yields the first order conditions

$$
\begin{aligned}
P_{h,t} &= \frac{1}{1-\phi} D_t^{\phi}, \\
P_{l,t} &= \phi P_{h,t} D_t^{1-\phi}.
\end{aligned}
$$

General Equilibrium Summing up, in any period t the economy's equilibrium is characterized by the following system of equations

$$A_t = A_{t-1} e^{z_t + \omega_t b_t}, \tag{17a}$$

$$K_t = e^{\omega_t b_t} K_t^{\star}, \tag{17b}$$

$$H_t = e^{(1-\phi)\omega_t b_t} H_t^{\star}, \tag{17c}$$

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha}, \tag{17d}$$

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{17e}$$

$$W_t = (1-\alpha) \frac{Y_t}{N_t}, \tag{17f}$$

$$W_t = \frac{\mu_n}{\mu_c} \frac{C_t}{1-N_t}, \tag{17g}$$

$$Y_t = C_t + I_t + D_t, \tag{17h}$$

$$\Lambda_t = \mu_c C_t^{\mu_c(1-\frac{1}{\psi})-1} (1-N_t)^{\mu_n(1-\frac{1}{\psi})} H_t^{\mu_h(1-\frac{1}{\psi})}, \tag{17i}$$

$$P_{h,t} = \frac{1}{1-\phi} D_t^{\phi}, \tag{17j}$$

$$q_t = \frac{1}{\varphi_1} \left(\frac{I_t}{K_t} \right)^{\kappa}, \tag{17k}$$

$$K_{t+1}^{\star} = (1-\delta_k) K_t + \Phi \left(\frac{I_t}{K_t} \right) K_t, \tag{17l}$$

$$H_{t+1}^{\star} = (1-\delta_h) H_t + D_t^{1-\phi}, \tag{17m}$$

$$q_t = \mathbb{E}_t \left[M_{t+1} e^{\omega_{t+1} b_{t+1}} \left(r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \left(1 - \delta_k + \Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) \right) \right) \right], \tag{17n}$$

$$P_{h,t} = \mathbb{E}_t \left[M_{t+1} e^{(1-\phi)\omega_{t+1} b_{t+1}} \left(\frac{\mu_h}{\mu_c} \frac{C_{t+1}}{H_{t+1}} + P_{h,t+1}(1-\delta_h) \right) \right], \tag{17o}$$

$$V_t = (1-\beta) (C_t^{\mu_c} (1-N_t)^{\mu_n} H_t^{\mu_h})^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{17p}$$

given the state variables $K_t^{\star}, H_t^{\star}, z_t, \omega_t, p_t$ and b_t. The stochastic discount factor satisfies

$$M_{t+1} := \beta \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) \left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}} \right)^{-\theta}$$

Moreover, the exogenous state variables are governed by the stochastic processes

$$z_{t+1} = \rho_z z_t + \epsilon_{z,t+1},\ \epsilon_{z,t} \sim \text{iidN}(0, \sigma_z^2), \tag{18a}$$
$$\ln p_{t+1} = (1-\rho_p)\ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1},\ \epsilon_{p,t} \sim \text{iidN}(0, \sigma_p^2), \tag{18b}$$
$$\omega_t := \bar{\omega} e^{\hat{\omega}_t},\ \hat{\omega}_{t+1} = \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1},\ \epsilon_{\omega,t} \sim \text{iidN}\left(0, \sigma_\omega^2\right), \tag{18c}$$
$$P(b_{t+1} = 1 | b_t = 0) = \min\{p_t, 1\},\ P(b_{t+1} = 1 | b_t = 1) = \max\{q, \min\{p_t, 1\}\}. \tag{18d}$$

Finally, we define GDP as the sum of consumption, both investment types and the implicit rent from housing

$$\text{GDP}_t = Y_t + MRS_t^{H,C} H_t, \ \text{ where } MRS_t^{H,C} := \frac{\mu_h}{\mu_c}\frac{C_t}{H_t}.$$

We scale the variables in terms of technology A_{t-1} by $a_t := \frac{A_t}{A_{t-1}}$, $k_t^\star := \frac{K_t^\star}{A_{t-1}}$, $h_t^\star := \frac{H_t^\star}{A_{t-1}^{1-\phi}}$, $k_t := \frac{K_t}{A_{t-1}}$, $h_t := \frac{H_t}{A_{t-1}^{1-\phi}}$, $y_t := \frac{Y_t}{A_{t-1}}$, $c_t := \frac{C_t}{A_{t-1}}$, $i_t := \frac{I_t}{A_{t-1}}$, $d_t := \frac{D_t}{A_{t-1}}$, $w_t := \frac{W_t}{A_{t-1}}$, $p_{h,t} := \frac{P_{h,t}}{A_{t-1}^{\phi}}$, $\lambda_t := \frac{\Lambda_t}{A_{t-1}^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)-1}}$, and $v_t := \frac{V_t}{A_{t-1}^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)}}$. Hence, the system of equations (17) can be written equivalently in terms of the scaled variables as

$$a_t = e^{z_t + \omega_t b_t}, \tag{19a}$$
$$k_t = e^{\omega_t b_t} k_t^\star, \tag{19b}$$
$$h_t = e^{(1-\phi)\omega_t b_t} h_t^\star, \tag{19c}$$
$$y_t = k_t^\alpha (a_t N_t)^{1-\alpha}, \tag{19d}$$
$$r_t = \alpha \frac{y_t}{k_t}, \tag{19e}$$
$$w_t = (1-\alpha)\frac{y_t}{N_t}, \tag{19f}$$
$$w_t = \frac{\mu_n}{\mu_c}\frac{c_t}{1-N_t}, \tag{19g}$$
$$y_t = c_t + i_t + d_t, \tag{19h}$$
$$\lambda_t = \mu_c c_t^{\mu_c(1-\frac{1}{\psi})-1}(1-N_t)^{\mu_n(1-\frac{1}{\psi})} h_t^{\mu_h(1-\frac{1}{\psi})}, \tag{19i}$$
$$p_{h,t} = \frac{1}{1-\phi} d_t^{\phi}, \tag{19j}$$
$$q_t = \frac{1}{\varphi_1}\left(\frac{i_t}{k_t}\right)^\kappa, \tag{19k}$$
$$a_t k_{t+1}^\star = (1-\delta_k)k_t + \Phi\left(\frac{i_t}{k_t}\right) k_t, \tag{19l}$$
$$a_t^{1-\phi} h_{t+1}^\star = (1-\delta_h) h_t + d_t^{1-\phi} \tag{19m}$$

$$q_t = \mathbb{E}_t\left[a_t^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)-1} m_{t+1} e^{\omega_{t+1} b_{t+1}} \left(r_{t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1}\left(1-\delta_k + \Phi\left(\frac{i_t}{k_t}\right)\right)\right)\right], \tag{19n}$$

$$p_{h,t} = \mathbb{E}_t\left[a_t^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)-1+\phi} m_{t+1} e^{(1-\phi)\omega_{t+1} b_{t+1}} \left(\frac{\mu_h}{\mu_c}\frac{c_{t+1}}{h_{t+1}} + p_{h,t+1}(1-\delta_h)\right)\right], \tag{19o}$$

$$v_t = (1-\beta)(c_t^{\mu_c}(1-N_t)^{\mu_n} h_t^{\mu_h})^{1-\frac{1}{\psi}} + a_t^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)} \beta(\mathbb{E}_t v_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{19p}$$

where

$$m_{t+1} := \beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{v_{t+1}}{(\mathbb{E}_t v_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}.$$

Solution Method First, note that given period t's scaled state variables $k_t^\star, h_t^\star, z_t, \omega_t, p_t$ and b_t and the control variables for labor supply N_t, house prices $p_{h,t}$ and the value function v_t, all other period t variables as well as next period's endogenous state variables can be easily computed from equations (19a)-(19m). We approximate the policy functions for N_t, $p_{h,t}$ and the value function v_t by linear combinations of Chebyshev polynomials. We compute the coefficients in the linear combinations such way that the Euler equations (19n) and (19o) and the recursive equation (19p) for the value function are satisfied exactly at a sparse grid of collocation points (see Judd et al. (2014) and Heer and Maussner (2009) for details). Thereby, the expectations with respect to normally distributed random variables are computed by Gauss-Hermite quadrature.